Realistic real world contexts: Model eliciting activities

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"…in this activity, for the first time I feel that I'm solving it myself. No one is telling me anything to do, and I find my own way. I'm formulating it myself." (Gül)

Abstract. Researchers have proposed a variety of methods to make a connection between real life and mathematics so that it can be learned in a practical way and enable people to utilise mathematics in their daily lives. Model-eliciting activities (MEAs) were developed to fulfil this need and are very capable of serving this purpose. The reason MEAs are so effective is the series of principles that should guide the development of these activities. This study aims to determine how one of these principles, the "reality" principle, is reflected in a MEA activity and to investigate participants' opinions of the realistic context of MEA. As a result of analysing work from pre-service teachers (PTs) who engaged in process during a MEA, and data acquired in interviews conducted after the activity, it was found that these activities of realistic real life contexts played a significant role in highlighting the various advantages and challenges of MEAs that have been reported by authors of past studies. These results are used to argue for mathematics instruction to include MEAs that promote connecting real life and mathematics in the classroom, and understanding different aspects of mathematics in real life.

Keywords: Mathematical modelling, model-eliciting activities, problem solving, realistic context, reality principle

Introduction

Although it has been claimed that mathematics is a product of the human brain and thus an abstraction, it is an undeniable fact that the real world influenced the birth of mathematics and that mathematics has a connection with reality (Umay, 2007). However, most people regard mathematics as an abstract branch of science that has no connection to real life, despite its significant and close relationship with society and social life. Students around the world have difficulties using and applying mathematics, a critical life skill, in their lives (Maaß, 2005). One reason for this situation might be the traditional problem solving activities, which are frequently used at schools, and have an artificial connection with real life. This is because students who usually solve these kinds of problems tend to think that utilising real life experiences and perceptions while doing so might prevent them finding the correct solution. This creates negative outcomes for the connection of mathematics to real life and its usage in it (Greer, 1997; Verschaffel, De Corte, & Borghart, 1997). Real life problems can be used in lessons to get rid of these negative influences and to relate mathematics to real life (Freudenthal, 1968).

Mathematical modelling activities start with a real life problem and are finished by mathematising the problem and finding its solution (mathematical model) and interpreting this solution regarding real life (Yoon, Dreyfus, & Thomas, 2010). Some researchers claim that modelling activities are more suitable than traditional problem solving activities for making a connection between mathematics and real life. This is because traditional problems aim to show how to apply a specific procedure so the context is usually sparse and contrived and has to be interpreted in an unrealistic way (Lesh, Hoover, Hole, Kelly, & Post, 2000). However, real life problems are the starting point for modelling activities, and these activities are seen as an ideal way to learn mathematics and to recognise and understand the aspects of mathematics that surround us in real life (Lingefjärd, 2002; Lingefjärd & Holmquist, 2005).

Some researchers (Lesh et al., 2000) developed MEAs as a certain kind of modelling activity based on specific principles. They claim that these activities are more effective teaching instruments than traditional problem solving activities. MEAs make remarkable contributions to important skills like being able to use mathematics in real life and to connect school mathematics and daily life (Doruk & Umay, 2011). In fact, certain skills which are used while studying MEAs and are critical for achievement are regarded as similar to the characteristics that students need in their professional life after graduation (Lesh & Yoon, 2006; Lesh & Sriraman, 2005; Lingefjärd, 2006). Another important feature of MEAs is that their real life context is not artificial and these activities start with a complex real life situation students can make sense of using their own experiences (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). Since student are familiar with the realistic context of model-eliciting activities, they are encouraged to find creative solutions. These activities also develop ownership because students do not only seek the correct solution known by the teacher, but they generate their own models to solve the problem (Chamberlin & Moon, 2005). These advantages make it seem that the integration of modelling activities into mathematics lessons is the best possible way to connect mathematics with real life and to minimise difficulties in using it in life (Maaß, 2005; Mousoulides, Christou, & Sriraman, 2007). This study aimed to demonstrate the effectiveness of model-eliciting activities (MEAs) in building a correlation between mathematics and daily life and the contributions of the real life context included in these activities. In addition, the study will specify the content of MEAs and possible difficulties and obstacles caused by real life context. Thus, this study analyzed the activity process of the PTs who work with this type of MEA for the first time. The study also asked the PTs to compare traditional problem solving activities and MEAs with reference to realistic context and investigated their opinions about the effect of MEAs' realistic structure on mathematics instruction after the activities. Thus, the study aimed to make use of both PTs experiences as students who solve problems and their opinions on this issue as future mathematics instructors.

Background

Mathematical Modelling Approaches and Model and Modelling Perspective in Mathematics Education

Mathematical modelling (MM) can be described as a circular and multidimensional problem-solving process that involves translating real life problems into mathematical language, solving them using mathematical techniques and subsequently testing the solutions (Blum & Niss 1989; Haines & Crouch, 2007). In this respect, all practices that attempt to connect mathematics and real life can be described as MM. In spite of this commonality, there are different approaches to the use of MM in mathematics instruction, and a common approach has yet to be established (Kaiser & Sriraman, 2006). These approaches can be classified in two categories by the purpose of their use in mathematics: 1) MM as the objective of mathematics instruction, and 2) MM as a tool used in mathematics instruction (Galbraith, 2012). According to the first approach, MM is the ability to use the abstract concepts taught in mathematics in real life and a basic skill that students should acquire (Blum, 2002; Haines & Crouch, 2001; Lingefjard & Holmquist, 2005). The model and modelling perspective (MMP), one example of the second approach, uses the term, "model-eliciting activities," for modeling problems, and assigns importance to the use of these activities in mathematics instruction (Lesh & Doerr, 2003). Unlike the first approach, MMP suggests that six principles should be taken into consideration in the development of activities. This study adopted the MMP, and focused on the reality principle among these six principles.

Use of contexts in problem solving: Model-eliciting activities and traditional problem solving activities

This section will compare traditional problem solving activities and MEAs in their contexts. First, these points should be considered: The researchers who promote MMPs do not claim that they should replace traditional teaching methods and problem solving activities, although they see the use of MEAs in mathematics instruction as important (Erbaş, Çetinkaya, Alacacı, Kertil, Çakıroğlu, & Baş, 2014). When performed before or after the direct instruction of a subject, these activities serve for students' developing their own comprehension or make their conceptual understanding of the subject deeper (Lesh, Yoon, & Zawojewski, 2007; Yoon, Dreyfus, & Thomas, 2010). Traditionally, it is assumed that real life problems are more difficult to solve than the traditional problems in course books, and the former can be solved after achieving some progress in problem-solving instruction. The real life problems are referred to as applied problem solving in the traditional problem solving approach, and they are seen as a specific form of traditional problem solving. However, MMP regards the traditional problem solving activities as a specific form of MEAs, and presumes both that it is easier to solve meaningful real life problems and that they provide deeper and more meaningful conceptual learning (Lesh & Doerr, 2003).

The pedagogical use of problems that have a limited and artificial context is still quite common despite increasing emphasis for the use of problems in mathematics education with realistic contexts (Cooper & Harries, 2003). However, there are striking reasons to consider the use of problems with real contexts in mathematics education. First, the genuineness in these problems help raise conscious individuals who can contribute to society. Second, these problems help structure mathematical concepts in an appropriate way, strengthen mathematical understanding, and students learn by comprehending (Blum & Niss, 1991; de Lange, 1996, Pace, 2000; Cooper & Harries, 2003). The use of realistic contexts in mathematical problems makes cooperation easier for students with different points of view, creates opportunities to restructure the schemas already existing in their minds (Dapueto & Parenti, 1999) and enables students to develop sophisticated solution strategies specific to that real life context (Gravemeijer & Doorman, 1999).

However, one cannot expect to gain the benefits of context problems by continuing to use traditional problem solving activities. These problems only dress up a mathematical problem with context from an artificially real life—the student's job is to undress the problem and solve the pure mathematical problem, the solution of which was learned beforehand (Blum & Niss, 1991; Blum, 2002). They neither enable them to use mathematics effectively in their daily lives, nor help them relate mathematics to real life and to other fields (Greer, 1997;

Schoenfeld, 1992). Since these problems prevent students from making realistic analyses and require suspending explanations related to problem's context (Verschaffel, Greer, & De Corte, 2000). These problems have an artificial context, and they do not fulfil the task of guiding students to use their experiences and perceptive knowledge about the real world. Dealing with these kind of problems frequently throughout their schooling does not help students to rely on their perceptual knowledge when they face a problem in real life and create a realistic argument about a problem's context. On the contrary, they are an obstacle to the development of these skills (Bonotto, 2001). The contrived context in traditional problem solving activities needs to be interpreted in an unrealistic way. The overuse of these activities can create the impression that mathematics is only useful in unrealistic contexts (Lesh et al., 2000). Moreover, traditional problem solving requires the student to work in a linear fashion, making progress according to rules that will take him from the given information to the desired information (Lesh and Yoon, 2006). However, reality is not linear in the world we live (Lesh and Sriraman, 2005), and real life problems do not have linear structures. Thus, linear structured solutions are not suitable for solving real world problems (Roth & McGinn, 1997). Certain trial procedures between the givens and goals are required for a successful model to solve a real world problem. Neither the given nor the goals is defined clearly. Therefore, the students must decide on their own what an appropriate solution might be, and which data and procedures to use (Zawojewski & Lesh, 2003). In any real life context, data might be insufficient, hidden or unnecessary. Students will frequently face these kinds of problems after graduating from school (Kardos, 2003).

The main reason for mathematics educators focusing on mathematical modelling is the question, "What kind of a mathematics education should be given to equip students with mathematical knowledge and thinking skills that they can utilise in daily life?" and the concern that traditional problem solving activities might be insufficient for developing students' problem solving skills (Mousoulides, Christou, & Sriraman, 2007). A team of mathematicians (Lesh, et al., 2000) who noted the limitations of traditional problem solving activities developed the MEAs, which are activities inspired by the real life problems in certain fields where mathematics is used heavily, (e.g., business, engineering, science). In these learning activities, students are not directed to follow the artificial and narrow roads that lead to ways of thinking expressed by teachers' or books. On the contrary, they are encouraged to mathematise reality by expressing their own way of thinking, testing and revising (Lesh & Sriraman, 2005).

Six Principles for Designing Productive Model-Eliciting Activities

There are certain principles that should be considered to design effective and thought-revealing MEAs (Lesh & Doerr, 2003; Lesh et al., 2000). These are the reality (or the personal meaningfulness), the model construction, the self-assessment, the model documentation, the model generalization (or the construct shareability and reusability) and the effective prototype principle (Table 1).

Table1. Six principles for creating productive model-eliciting activities, adapted from Lesh et al. (2000).

Principle	Description
Model Construction	The problem should create the necessity to form a mathematically meaningful structure and make students clearly understand that they need to develop a model.
Reality	The problem should include a situation that students might encounter in real life, and they should be able to understand it by expanding their experiences and knowledge.
Self-Assessment	Students should be able to judge the correctness of their results on their own, and whether it is necessary to improve or revise they model they created.
Model Documentation	The problem should require students to prepare a written document during the activity and clearly demonstrate their own ideas and solutions.
Model Generalization	The model created for solution should not be applicable to a specific case, but to other similar cases, too.
Effective Prototype	As well as providing a useful prototype or a metaphor to be used to interpret other structurally similar problems, it should be simple enough to enable students to produce a reasonable solution.

The Reality Principle

The reality principle requires that the problem can happen in a realistic real life situation, that students can make sense of the problem by extending their own experiences and knowledge, that students' ideas are taken seriously during the working out of the solution, and that students are not pushed to adopt the method their teacher or their textbook thinks to be right (Lesh et al., 2003). MEAs based on this principle are realistic problems that are related to students' lives (Lesh et al., 2000). In contrast to most problem solving activities that claim to be realistic, but are not, MEAs are designed as truly realistic activities. Designers of these activities are supposed to ensure that the model developed for the problem is tested in real life and that the students who solve the problem are led to collect data from the realistic context of the problem (Chamberlin & Moon, 2005). The reality stressed in this principle is not that of adults, but the students' reality, and it refers to a situation that is important for them. It is affected by their age, environment and socio-economic status (Lesh & Caylor 2007). Thus, the reality principle suggests selecting contexts and subjects that are suitable for the interests and experiences of the students who will perform the activity in order to develop problems that will encourage them to base solutions on the extension of their knowledge (Lesh et al., 2000).

Guided by the reality principle, MEAs, which are designed as the authentic simulations of real life problems, have genuine real life context, unlike traditional problems (Lesh & Sriraman, 2005). The criteria for this principle specify that it is not sufficient for real life context to merely be included in the problem. In this respect, it is claimed that the context used in the MEAs plays a special and privileged role. In the relevant literature, there are many studies of context use (real or artificial) in math problems (Cooper & Harries, 2003; Dapueto & Parenti, 1999; Gravemeijer & Doorman, 1999). However, there is a gap in the literature regarding the detailed demonstration of the effects of the MEAs carefully chosen real life context on the problem solving process. This study focuses on the realistic context of the MEAs and will contribute to the literature in this respect.

Pre-service teachers and model-eliciting activities

The research on MEAs conducted with PTs can be classified in two groups based on their viewpoint. The first group of studies analyzes the opinions of the PTs who performed these activities (e.g., Eraslan, 2011; Karalı & Durmuş, 2015; Kayhan-Altay, Yetkin-Özdemir, & Şengil-Akar, 2014; Stohlman, 2014; Thomas & Hart, 2013). The second group of studies evaluates the PTs' engagement in the MEAs, their achievement in the modelling process, their skills and their difficulties and barriers (e.g., Carlson, Larsen, & Lesh, 2003; Eraslan, 2012). The studies in the first group show that the PTs have positive opinions about MEAs. They appreciate the problems' having multiple correct answers and being more interesting and fun. They enjoy integrating their own knowledge about real life with the problem rather than simply doing what they are told, and they recognize that MEAs require a high level of thinking and help to apply mathematics to real life. However, the MEAs' ambiguity caused the PTs to form negative opinions about these activities. On the other hand, the second group of studies indicate that the PTs had difficulty understanding the problems and planning a solution strategy, especially at the beginning of the activities, although they were able to complete them successfully.

As can be seen in the aforementioned paragraph, previous researchers mainly focused on MEAs in a general standpoint. Differently from these studies in the relevant literature, this research specificially focused on the reality principle, one of the MEAs' design principles. Thus, the study compared the general results of past studies of MEAs and this study's specific results regarding the reality principle. This study also tried to determine the role played by the reality principle in the PTs' positive and negative experiences and opinions about the MEAs stated in past studies. This study combined the two viewpoints of the studies mentioned above, and aimed to determine the outputs of the reality principle created by the PTs' engagement with the MEAs as well as their approaches to the MEAs with realistic context.

The Purpose and Research Questions

The objective of the study was to determine PTs experiences with the realistic context of the activity during the process of solving an MEA developed by the researcher and their opinions about it. To achieve this objective, the study sought answers to these research questions:

1. What are the experiences of pre-service mathematics teachers' first engagement with an MEA according to the reality principle?
2. What are pre-service teachers' opinions about the real life context of the MEA after participating in it?

Methodology

Data Collection

The sample of the present study included 24 PTs (17 female, 7 male). They were students of mathematics education at a state university in Turkey. In a Turkish context, the mathematics teaching program takes four years of education and the participants of this study are in the third year of program. Their study has included both pure mathematics courses such as Abstract Mathematics, Analysis, Algebra, Geometry and pedagogy courses such as Educational Pscyhology, Sociology of Education. There are also specific courses combining mathematics and pedagogy such as Teaching Mathematics. The present study has been conducted within the course of Teaching Mathematics offered by the researcher in spring semester of 2013-2014 academic years. The content of the course is composed of specific teaching methods and practical implications. In addition to these components, the researcher has incorporated model-eliciting activities into the problem solving unit that is already present within the course content. This unit, which includes instruction about problem-solving, includes ten lessons. It focuses on the description and importance of problems and problem solving, the classification of problems, instructions for solving word problems and strategies for solving non-routine problems. Although the PTs in the study sample had frequently encountered traditional problem solving activities during their school lives, some samples and characteristics of these problems were still discussed in the lesson. When explaining the MEAs during the introduction of problem types, it was discovered that none of the PTs had participated in this type of an activity before. Within this context, the participants examined certain examples of model-eliciting activity such as the Big Foot problem and the Volleyball problem (Lesh & Doerr, 2003). After that, they engaged in the unsubscribing problem developed by the researcher.

The unsubscribing problem

The model-eliciting activity that was designed to be used in this research aims for students to estimate the amount of electric consumption for any time of the year and on any day by using past spending values.

Bonotto (2001) suggested that using objects, things like market receipts, bottles, bus schedules—objects that students use in daily life—in mathematics lessons might be a suitable starting point for the process of mathematising, which is an effective tool for connecting real life and mathematics in the classroom. The model-eliciting activity that is utilised in the context of this study is based on a real electric bill, so the study aims to strengthen the bond between the modelling activity and reality. To suit the requirements of the MEAs' reality principle the problem is based on the context of unsubscribing from an electric company as one does when moving from one home to another. At the beginning of the activity, the class shared their own experiences of moving in a brief discussion.

The students were given the problem presented in Appendix A, and they were asked to develop a method to estimate the values that would be seen on the electricity meter 4 days later, based on the last observed values. They were also asked to generalise this method so that any person in the same situation could use it. This satisfied the model construction and model generalization principles. The problem requires the development of a simple structure that will estimate daily consumption based on the electric bill and monthly consumption using the table given in the problem, thus satisfying the effective prototype principle. The problem statement requires that students explain the methods they developed to Mr. Erdem who wants to determine the values on the electricity meter for four days later. This satisfies the model documentation principle. The researcher assigned the students to determine the values for four days of electricity consumption using the method they developed, so they had the opportunity to test and revise their methods, and the self-assessment principle was fulfilled.

The model-eliciting activity lasted for two lessons. The PTs did not leave the classroom during the interval between these two 45-minute long lessons, deciding on their own to continue their work on the activity. Thus, the last 30 minutes of this 100-minute activity, including the break, was used for the presentation of the models from volunteer groups. The students were divided into 8 groups each of 3 people. The researcher utilised a format for the activity that was devised by English (2004). Thus the activity began with a whole-class discussion of the context of the problem (the students shared their experiences of unsubscribing from electricity when moving). Then, students were read the unsubscribing problem and the problem was briefly discussed. In the next step, students worked in groups to develop a model to solve the problem. Meanwhile, the researcher walked among the students and observed them without providing guidance. When groups asked for help, the researcher responded with thought-provoking questions intended to help them find their own ways of thinking. During the final stage of the activity, the groups presented their solutions to their classmates, described the model they developed and revised their models in some situations. To clarify and extend the ideas of the groups, the author and the remaining student groups asked questions about aspects of the presentations. The group discussions during the model developing process and the presentations of the models to the class were recorded.

After the activity, semi-structured interviews were conducted individually with nine female and two male PTs who had volunteered. Interviews were conducted in the researcher's office when the PTs were available and were audio-recorded with the interviewees' permission. During the interviews, the researcher asked pre-formulated questions, which aimed to find out PTs' ideas about the realism of modelling activity's context and its influence on the stages and aftermath of the activity (Appendix B). The researcher also asked additional questions to acquire deeper information.

To provide data triangulation, the researcher's data sources were tape recordings of the model-eliciting activity, the PTs' written documents and tape recordings of interviews held after the activity. The tape recordings, including the PTs' conversations during their MEA activity, “The Unsubscribing Problem,” along with documentation of their operations and solutions during the activities were analyzed to understand the PTs’ experiences under influence of the reality principle during their engagement with the MEA. The voice recordings of the interviews were used to determine their opinions about the realistic context of the MEAs in which they participated.

Data Analysis

The content analysis method of inductive analysis was used to analyse the data. This method is employed to reveal the concepts behind data and relations among these concepts by means of encoding (Miles & Huberman, 1994). The author of the paper and another specialist who has experience in qualitative research did a content analysis of the tape recordings, the documents and written data from the recordings of the interviews. This study’s content analysis used concepts derived from the data to encode its data (Starrus & Corbin, 1990). In this type of encoding, each line of the data is read, and the dimensions that are important for the study objectives are determined. The researcher begins the study based on the raw data and forms the codes according their meanings. These codes are classified into specific categories, which generate themes. Briefly, this inductive analysis derives codes from the raw data and subsumes them under themes. Regarding the research problem, the codification was made for the sections that were featured in written data and were determined to be a reflection of the modelling activity’s realistic context. The researchers compared the codifications they made for reliability and exchanged ideas about certain sections that they disagreed on. Then they generated the themes based on the shared codifications (Table 1).

Table 1. An example to demonstrate the analysis methods.

PT’s comment	Encoder 1	Encoder 2	Theme
….I think the real life context is important. I used to think of one single equation for the problems I solved and I thought that I had to find the result using it. But in this activity, for the first time I feel that I’m solving it myself. No one is telling me anything to do, and I find my own way. I’m formulating it myself….	The correlation with real life enables individuals to find their own solution rather than using a predetermined formula.	The correlation with real life leads to the production of unique and individualized solutions instead of the use of a predetermined method.	Directing one to create his or her own solution.

Findings

First, the researcher will present findings from the model-eliciting activity, their tape recordings and written documents. Then the findings from the interviews will be presented.

The PTs tried to make sense of the unsubscribing problem at the first stage of the model-eliciting activity, and it was determined that the problem's real life context had certain spontaneous effects on this process. The findings from tape recordings of the model-eliciting activity and written document suggest that the real life contexts encouraged PTs to consider different aspects of the problem (Table 2), consider real life on the model construction strategies (Table 3) and formulate various models (See Table 4).

Table 2. The effects of realistic context on making sense of the problem.

Spending much time and effort to understand the complex real life problem
Consulting their real life experiences to try to understand the problem
Verifying the information given in the problem by analysing data that contradicts their own knowledge
Explaining any perplexing situation that occurs while analysing the data by considering real life factors
Explaining the data given in the modelling problem through real life situations
Judging that it is impossible to find a definite and clear relation or result since the data belongs to real life
Being puzzled and struggling with the real life problem situation's ambiguity

For example, in the following dialogue PTs concentrate on a specific month of extraordinary electricity consumption, we see how two teachers make sense of the problem considering a real life experience that is not in the problem statement while analysing the table of monthly consumption.

H: Dear Zeynep, do you have a second? There is a huge gap between the two months. Here, 85 units were consumed and this month they consumed 30. Maybe they are on vacation, what do you think?

Z: Let's check. Yes, that is possible for August. But look, here in November the consumption was 100 units.

H: Okey, this means that the bill is higher in winter since they used the combi boiler. We need to calculate a general average since both winter and summer months count.

As the PTs' sample conversation shows, they explained the possible causes of the consumption difference in certain months of the year based on real life experiences when evaluating the electricity consumption values for the eight different periods given in the problem. Similar conversations are summarized in the "consulting their real life experiences to try to understand the problem" theme in Table 2.

Here is another example, showing that they decide their previous knowledge about electricity consumption is incorrect after seeing the real electricity bill in the problem:

> I: No, it is not cheaper at night again.
>
> A: Yes, look, it is even more expensive.
>
> I: It is at the cheapest level in daytime, but it must be because we use it more during the day.
>
> C: It is a smart meter.
>
> I: We thought that it would be cheaper at night. But it seems more sensible to, for example, do the laundry during in day, isn't it?
>
> C: I think it is cheaper to do it before 10.30 a.m. or something like that.
>
> I: I agree, it must be between 5 and 10. I have heard that it is cheaper between 10.00 p.m. and 10.00 a.m.
>
> C: Some people usually do the ironing after 10.30 p.m.
>
> I: Yes, it must be just a rumour.

When the PTs in this group recognized that the fee for night consumption was higher when evaluating the data on the real bill given in the problem, they questioned their former knowledge which contradicted with this situation, and they finally decided that the data in the problem was correct. This experience is categorized under the theme "verifying the information given in the problem by analysing data that contradicts their own knowledge"

An analysis of the data derived from the conversations of the PT groups during the process of understanding the problem indicated that the realistic context also had some effects that initially seemed negative along with the positive effects mentioned above. To understand the complicated real life problem, the PTs spent more time and struggled more than they would with traditional problem solving activities. Some groups temporarily decided that it was impossible to solve the problem or did not know what to do since they believed that would not be able to find a solution due to the problem's ambiguity, which resulted from the fact that it was taken from real life.

As seen from the PTs' discussion about a solution, the context seems to motivate a variety of real-life considerations as given in Table 3.

Table 3. The effects of realistic context on model construction process.

Consulting real life experiences in unclear situations
Deciding on the mathematical process to be followed regarding the real life situation
Considering real life situations when deciding on the solution strategies
Considering factors related to real life while planning the solution
Correcting false steps by noticing real life factors
Considering real life situations when modifying the solution
Trying to interpret results from the data by adjusting to real life

For example, a group's conversation shows that they are considering real life situations that are not in the problem statement while trying to decide on the solution strategies:

> B: How much did he spend, excluding the vacation? They are almost the same.
>
> M: The values are almost equal, as you say, since in summertime the consumption level is routine and there are no additional expenses.
>
> B: Let's say consumption is stable during the summer. But they may use the air conditioner in summertime.
>
> A: But he did not.
>
> B: Let's look at winter consumption. How did the consumption change during the winter? It rose. Before winter, it is stable. In winter, it increases.
>
> M: It must be the electric heater.
>
> A: Consumption increases as the weather gets colder.
>
> B: Yes, the consumption increases but on what scale? By which formula?

As seen in this example, the PTs explained the increase in electricity consumption in winter considering the use of electric heating, which was not given in the problem, when they were planning their solution. These types of experiences with model planning are categorized under the “considering factors (not given in the problem) related to real life while planning the solution” theme.

An analysis of the reports presenting the models created at the end of the activity and oriented to the solution of the problem indicates that the real life context in the model-eliciting activities results in creation of different models considering different aspects of reality. As seen from the different models created by the PTs, the context seems to motivate a variety of formulations as given in Table 4.

Table 4. Different models created by the PTs for Unsubscribing Problem.

The model based on the stable change between the seasons
The model based on the average daily consumption in the moving season
The model based on the average of the months near Mr. Erdems’s moving
The model that does not consider the very low monthly consumption, using the average and ratio
The model of calculating the daily consumption amount through total past consumption

The model based on the stable change between the seasons: One group of PTs evaluated the monthly consumption values on the table and determined that consumption varied with the seasons. They thought that there was a stable increase or decrease across the months of the seasons. Assuming that there was a relation between the temperature increase and electricity consumption, they claimed that there would be a stable decrease in spring, when the temperature started to increase, beginning from the first month. They calculated the daily consumption in a spring day by relation $\frac{B-10}{30}$ - where B is the unit of consumption in the previous month. Similarly, they assumed that there would be a 10-unit stable increase in monthly consumption, anticipating that electricity consumption would increase in autumn and winter due to the decrease in temperature (Figure 3).

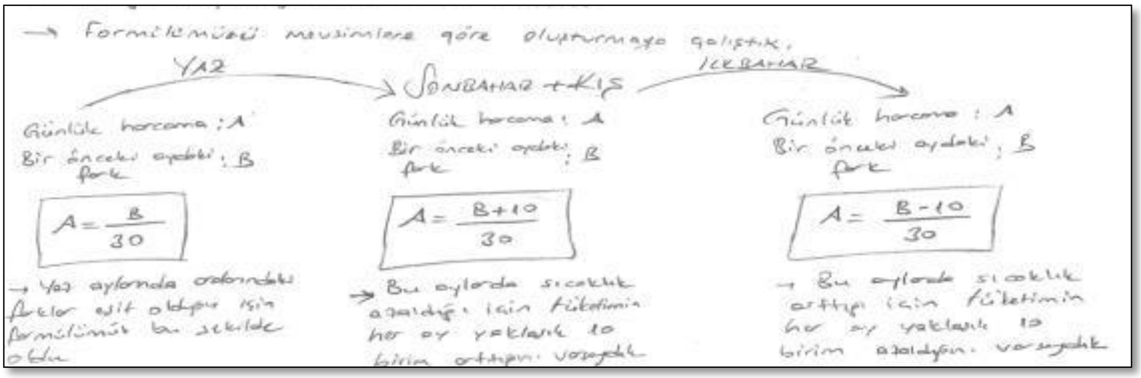

Figure 3. The model based on the difference between the seasons (Utilising stable change)

The model based on the average daily consumption in the moving season: Another group first created a graphic for the daytime consumption values shown on the table and recognised that there was a change in consumption between seasons. Then the PTs accepted that more sensitive results could be acquired by deciding on the average consumption values from the season of Mr. Erdem's moving in previous years, than calculating the average consumption for all months. The group developed a model based on calculating the average daily consumption amount in this season (Figure 4).

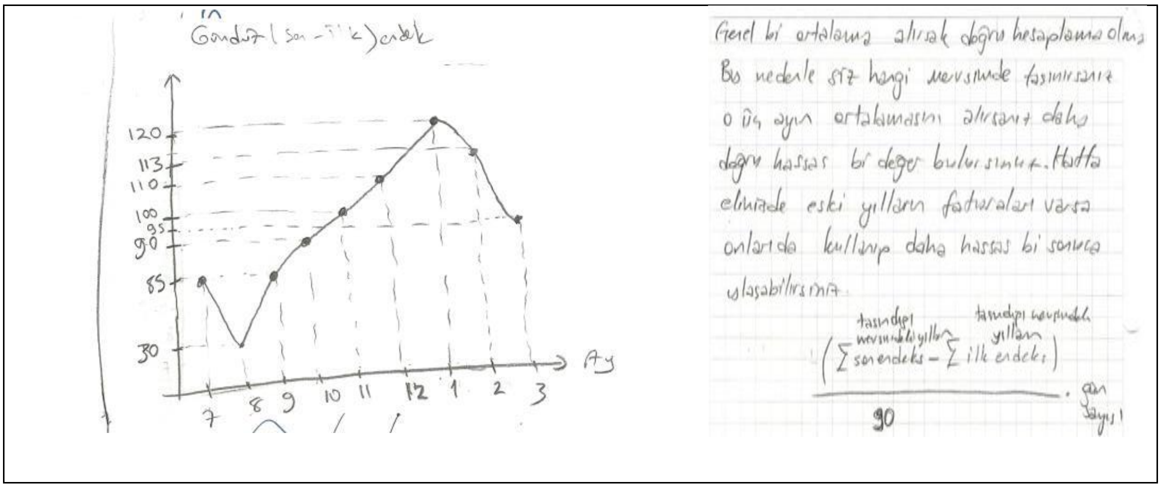

Figure 4. The model based on the average daily consumption in the moving season.

The model based on the average of the months near Mr. Erdem's moving: Another proposed solution was to make use of the average consumptions of previous three months before Mr. Erdem's moving, disregarding the seasonal differences (Figure 5).

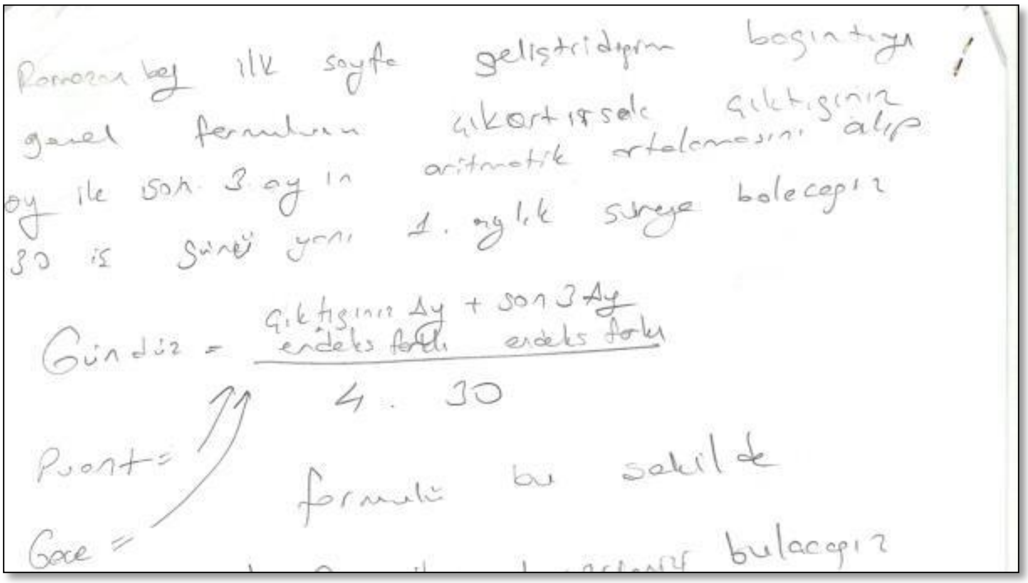

Figure 5. The model based on average consumption for the months near Mr. Erdem's moving

The model that does not consider the very low monthly consumption, using the average and ratio: Some groups pointed out that the August value on the table was very low and suggested that this consumption belonged to the months when there was nobody at home for various reasons such as vacations. Thus they claimed that the lowest values should have been excluded from the calculation, while determining daily consumption amounts by calculating the averages of consumption amounts in previous months (Figure 6).

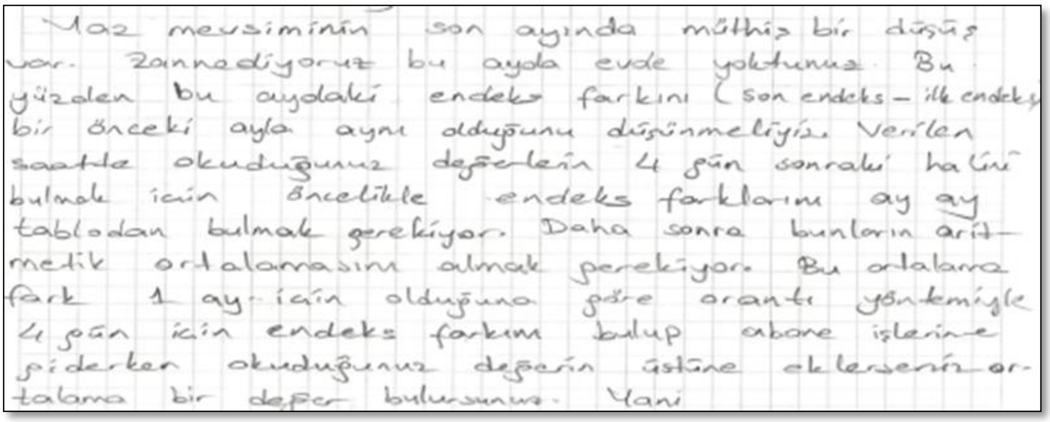
Yaz mevsiminin son ayında müthiş bir düşüş var. Zannediyoruz bu ayda evde yoktunuz. Bu yüzden bu aydaki endeks farkını (son endeks – ilk endeks) bir önceki ayla aynı olduğunu düşünmeliyiz. Verilen saatte okuduğunuz değerlerin 4 gün sonraki halini bulmak için öncelikle endeks farklarını ay ay tablodan bulmak gerekiyor. Daha sonra bunların aritmetik ortalamasını almak gerekiyor. Bu ortalama fark 1 ay için olduğuna göre orantı yöntemiyle 4 gün için endeks farkını bulup abone işlerine giderken okuduğunuz değerin üstüne eklerseniz ortalama bir değer bulursunuz. Yani

Figure 6. The average and ratio model excluding months of extraordinary consumption

The model of calculating the daily consumption amount through total past consumption: This model is the last of the solutions developed for the unsubscribing problem. It has a basic structure compared to the others, and it is based on the principle of determining the consumption amount in one day by averaging on all the past consumption values (Figure 7).

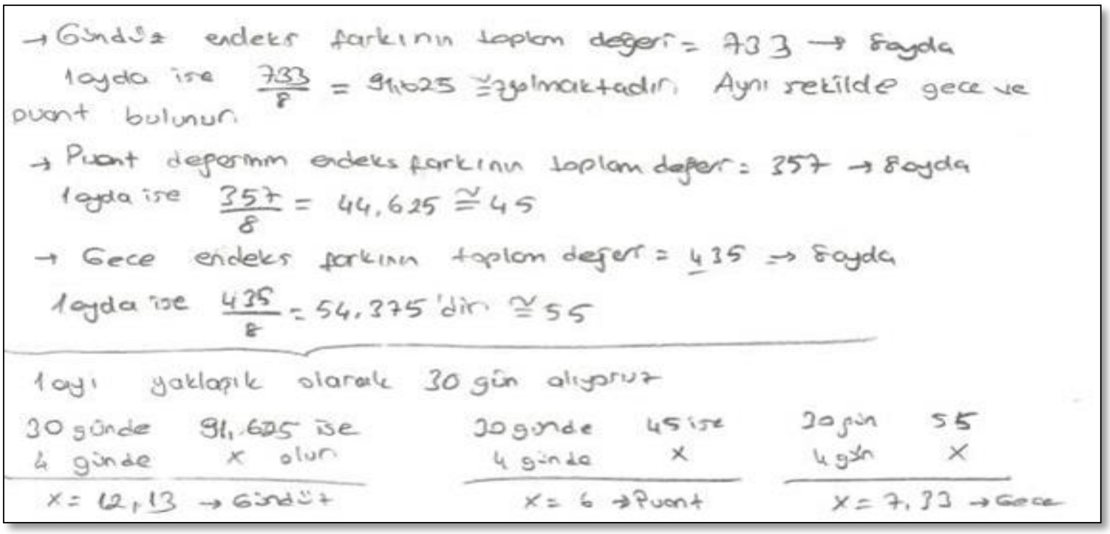
→ Gündüz endeks farkının toplam değeri = 733 → 8 ayda
1 ayda ise $\frac{733}{8} = 91{,}625$ yapılmaktadır. Aynı şekilde gece ve puant bulunur.
→ Puant değerinin endeks farkının toplam değeri: 357 → 8 ayda
1 ayda ise $\frac{357}{8} = 44{,}625 \cong 45$
→ Gece endeks farkının toplam değeri = 435 → 8 ayda
1 ayda ise $\frac{435}{8} = 54{,}375$'dir $\cong 55$
1 ayı yaklaşık olarak 30 gün alıyoruz
30 günde 91,625 ise / 4 günde x olur → x = 12,13 → Gündüz
30 günde 45 ise / 4 günde x → x = 6 → Puant
30 gün 55 / 4 gün x → x = 7,33 → Gece

Figure 7. The model based on determining the daily consumption through total consumption.

After the model eliciting activity, interviews were held with the PTs. They compared this activity with traditional problem solving activities in terms of realism (Table 5), solution process (Table 6) and structure (Table 7). These comparisons generally featured the differences naturally created by the real life context in the MEAs.

PTs who compared these activities in terms of realism stated that they thought the model-eliciting activities were more realistic (Table 5).

Table 5. The model eliciting activities are more realistic because….

Model Eliciting Activities	**Traditional Problem Solving Activities**
It contains familiar elements from real life and uses them for solution.	May have contexts that are not possible in real life.
The problem context is true to life.	Are not considered problems in real life.
The context of the activity reminds me of my own experiences.	Are created by the placement of a mathematical structure into real life context.
It is a problem that we can confront in real life more frequently.	Have turned out to be classical in time.
We build the solution ourselves.	Are generated with the aim of writing a question.
We make use of our own experiences for solution.	Students solve just for the sake of solving them (to prepare for exams) disregarding the real life context.
The solution that we find can be used in real life.	
The solutions to modelling problems are actually necessary.	
The problem is for everyone.	

Some of the PTs, who thought that the MEAs were more realistic than the traditional problem solving activities they had encountered, said that this was because mathematical structures were artificially integrated with real life context in traditional problem solving activities. For example one PT commented, "*For instance, age problems might be a kind of problem that we confront in daily life, but the other one (model-eliciting activity) is more true to life. The first one is true to life, too, but, I don't know how to say, it seems that there is a structure, and they tried to put that structure in the problem and adapt it to life*." (Gül) Another PT contrasted the MEAs with the stereotypical problems students solve at school, noting that they had structures that everyone can see in daily life. This PT said, "*The unsubscribing problem (model-eliciting activity) is more realistic. It is a problem that everyone might face. But the other problems are not like that. To give an example, it is not old men, but students who solve those problems. They are stereotyped and students solve them just for the sake of solving them*." (Demir)

In the following example, a PT says that the MEAs are more realistic since traditional problem solving activities used the context in an unrealistic way and ignore real life factors during the solution of problem.

"*Sometimes traditional problems don't make any sense. It might be a train problem or a pool problem, let's say. Somebody fills the pool first; then he empties it from the bottom. Why do you do that? In labour problems, for example, it says "how long does it take a worker to do this job?" Normally, workers have breaks. Also, all jobs are difficult in themselves. Is it possible to find a standard solution for all of them? We are supposed to solve these problems by ratios, which means that the solution is 'This number of workers do this work in this period of time, so that number of workers do it in that period of time.' It is not realistic at all. For this reason, modelling problems are much more realistic*". (Hazine)

Certain themes were discerned from the statements of PTs who attended the modelling activity about their comparisons between the traditional problem solving activities and their experiences in MEA process. Below is Table 6 showing these themes.

Table 6. A comparison in frame of the solution process

Model-Eliciting Activities	**Traditional Problem Solving Activities**
Seeing oneself in the problem	Looking at the problem from outside
Building the solution starting from the base	The way that was known beforehand; usage of a model-stereotype
Adding your own interpretation regarding the variables in real life	Not thinking of anything except for the given information; cancelling the real life factors
Individual solutions; there is no one clear solution; tries for different solutions	Clear answer; the standard and foreknown solution
The aim is to solve a real problem using mathematical operation	The aim is to perform a mathematical operation
Solving the real life problem using mathematics	Injecting mathematics into real life; re-solving it through mathematics
A natural desire to solve the problem	The obligation to solve it for exam preparation
The solution becomes easier when its connection with real life is considered	When real life experiences are regarded, it might give incorrect results
Real life should be regarded for the ideal solution	Real life should be ignored to find the single correct answer

Here, some PTs compared MEAs with traditional problem solving activities in terms of working out the solution. The first PT said that real life factors are ignored in traditional problem solving activities, while it is necessary to consider many factors from real life to solve MEAs: *"In model-eliciting activity, some other factors are counted, too. For example, we thought about excluding summer months. Environmental factors are counted, like the daily consumption. In other problems, we exclude most of the factors related to real life, but in these activities, we are supposed to think about every detail, like the difference between the seasons which we considered for a better solution."*(Demir) Another PT expressed the opinion that MEAs' real life context enabled problem solvers to adopt the problem, and successfullly solving them required one to regard the problem as personal. This PT said,*"While solving ordinary problems, students cannot see themselves in the problem. They cannot put themselves in that person's shoes –the person in the problem. However, while solving this unsubscribing problem, I thought that it was something that we all experience in our lives. I had this problem myself, since we move very frequently. Another point is that we cannot solve the problem if we don't consider ourselves in it. Otherwise, it would be an ordinary solution. It would only be a mathematical operation, not a realistic solution*." (Gönül)

The PTs' interviews compared the two problem solving activities in terms of structure, the themes of which are shown in Table 7:

Table 7. Comparison in structural terms

Model-Eliciting Activities	**Traditional Problem Solving Activities**
Context is important for the problem-solver	Context is not important for the problem-solver
Makes one think	Are oriented to develop operational skills
Data and the way to solve the problem are ambiguous	The data is clear
You put yourself in the shoes of the person in the problem	Difficult to comprehend because you cannot put yourself into the problem
Are beneficial for developing the skill to use mathematics in life	Limitedly benefit for the aim of using mathematics in life.

The PTs thought that the problem-solving activities they had previously performed were more addressed to using operational skills, while the MEAs were structured to improve their thinking skills. According to the PTs, the underlying reason for this was the fact that traditional problems require finding a predetermined solution considering the given and asked, and just the opposite is the case in MEAs. For example one PT commented, "*The other problems were mostly directed to develop operational skills. These ones are taken from real life and require more thinking. It makes us think 'How should I do it?' The normal problems I solved before had an obvious way of solution because there was a certain formula related to those problems and we applied that formula. But there was no specific formula for these problems. We didn't know which way to follow, so we applied different solutions. We made various attempts to solve the problem. There might not be one single solution for these problems because every student might have their own way of solving it. The other problems we used to solve had a definite result.*" (Demir)

During the interviews, the PTs were asked about the effects they realized during their engagement in the MEA of the realistic context, with the purpose of supporting the findings from tape recordings of the MEA. They interpreted what kind of effects they realised regarding the real life context during their engagement in the model-eliciting activity. The themes obtained from these comments are presented in Table 8.

Table 8. Opinions on the effects of realistic context.

Motivating the student to consider certain factors related to real life.
Considering the connection between the problem and real life as much as possible to find an ideal and realistic solution.
Making the problem-solver feel like he or she is in the problem.
Motivation to use the solution in his or her own life.
Directing one to create his or her own solution.

For instance, the statements by a PT who participated in an MEA for the first time described an important experience provided by the realistic context, "*I used to think on one single equation for the problems I solved and I thought that I had to find the result using it. But in this activity, for the first time I feel that I'm solving it myself. No one is telling me anything to do, and I find my own way. I'm formulating it myself.*" (Gül) These opinions of the PTs, the study interpreted that the realistic context motivated them to create their own solution through the choosing of their own methods and procedures during the MEA, which also enhanced their self-confidence.

The PTs also expressed their opinions about model-eliciting activity with realistic context's effects on mathematics learning. These opinions show that the PTs thought the real life context of the MEAs will make positive contributions to mathematics instruction (Table 9).

Table 9. Opinions on realistic context's effects on learning mathematics.

It makes students understand the necessity of mathematics and increases their interest.
It helps students to see application of mathematics in real life.
It contributes to permanent learning.
It helps students to use mathematics in their lives.
It develops sophisticated thinking and problem solving skills.

For instance, one PT expressed that the realistic context will enhance the permanence of learning by enabling students to build a correlation between the problem and their personal lives and creating the opinion that this new knowledge can be used in real life. This PT said, *"The more you put yourself in the problem, the better it sticks to your mind. This way, mathematics will be more permanent for you. The operations you make to solve a problem that is not taken from life will disappear after a while. Because we think that they will be useless. But in this way, the knowledge is permanent as we think that we will make use of them."* (Kartal)

Below is a list of themes obtained from PTs' statements in the interviews after the activity regarding their evaluations on the activity and their affective comments (Table 10):

Table 10. Affective comments about the model-eliciting activity.

Affects	Source of Affects
It made me feel good	For the first time I solved a problem through a way I developed myself.
I liked this activity	We found our own solution and not a formulaic solution.
I was surprised	I saw that I could make use of mathematics for unsubscribing from an electric company.
I thought that it was weird	We were seeing it for the first time.

It caught our attention.	İt was realistic and a facet of everyday life
At first, I was worried	I wouldn’t be able to understand the long verbal part, but then I saw that it helped me to understand the question.

As Table 10 shows, the PTs first experience of MEAs within a real life context generally generated positive feelings. On the other hand, the ambiguity of the problem situation due to the fact that it came from real life gave them temporary negative feelings (Themes 4 and 6).

Discussion

Our study found that, due to the influence of the real life context in the activity, PTs who studied model-eliciting activity consulted their own real life experiences while they were trying to understand the problem situation. As we can see in Table 2 and 3, they considered real life factors that were not given in the problem while they were devising a solution in the model developing process, and they tried to interpret the results they acquired by adjusting them to real life. English and Lesh (2003) claimed that MEAs’ context would guide students to develop the solution, and it would help them to decide whether a solution strategy was good or bad. During the model-eliciting activity within the context of this study, it was clearly observed that this notion was correct. Students made use of this realistic context while making sense of the problem situation or developing a model for solution. They also considered this context while making decisions.

Another benefit of modelling problems is that it helps different students find different solutions (Lingefjärd, 2002). Use of MEAs in past studies showed that students developed different models to solve the same problem (Bukova-Güzel, 2011; Doruk, 2010; English, 2010. Similarly, our study proves that different groups studying the same problem situation put forward different solutions based on different approaches (See Table 4). Since our study’s participants had not experienced modelling activities at school until their third year at university, and had only dealt with traditional problem solving activities until that age, the efficiency of MEAs in this field becomes clearer. As the study shows, MEAs not only allow different solution approaches, they also facilitate each group to develop their own solutions, as a result of its realistic context. As English (2010) states, the genuine problems in modelling activities serve as an instrument which helps students generate mathematical ideas and go beyond easily applying procedures they already know.

In their comparisons between MEAs and traditional problem solving activities in realistic terms, PTs stated that the realistic context of model-eliciting activity created a natural desire in them to solve the problem, while they solved the other problems only as an exam preparation. It was interesting for the students to solve a real life problem in which they could see themselves. An important factor of this activity which made students feel like they were in the problem is that the context of these activities is realistic. As Lesh and Doerr (2003) stated, MEAs' realistic structure motivates students to develop mathematical models to solve complicated problems in real life, like applied mathematicians do. However, high stake assessments, which consist of short, formulaic test items and include artificial contexts with mathematical operations are still used in many countries (Cooper & Harries, 2003) although encouragement for the use of problems with realistic contexts has been rising in recent years. The use of such tests in student selection is an obstacle in moving forwards within mathematics education. The replacement of these exams with those including questions that realistically reflect the connection between real life and mathematics would make positive contributions to mathematics education.

Some studies in the past have shown that it was very common among students (and even PTs) to disregard real life knowledge and realistic arguments, not taking them into consideration and ignoring them while answering school mathematics problems. Those studies also propose that interpreting the answers of certain problematic word problems would help overcome this problem (Greer, 1997; Verschaffel, De Corte, & Borghart, 1997). The participants in our study think that the realistic context of model-eliciting activity requires the prioritisation of the connections between the problem and real life to find a realistic solution (See Table 8). Because of their realistic contexts, MEAs contribute to this stated problem positively. Students will gain the belief, vision and skills required to make realistic interpretations while dealing with mathematical problems if they are given the opportunity to take part in this kind of activity during schooling life. In this way it can prevent students from developing the idea -throughout their school life- that mathematics is disconnected from real life and it stops them from gaining the habit of ignoring the facts while solving a math problem. With their realistic context, the MEAs are tailor-made for this task, and using them from pre-school education to university will contribute to this aim.

The results of this study suggest that almost all the points (e.g., PTs appreciate the problems having multiple correct answers and being more interesting and fun, they enjoy integrating their own knowledge about real life with the problem rather than simply doing what they are told.) expressed by the PTs in past studies (Eraslan, 2011; Kayhan-Altay et al., 2014; Stohlman, 2014; Thomas & Hart, 2013) about MEAs' benefits for mathematics instruction can be related to their realistic real life context. Moreover, the PTs thought that some benefits, which were not included in past studies, could be provided by the realistic context, which will contribute to the findings of the studies in relevant literature. These benefits include: (1) enabling students to put themselves in the shoes of the person in the problem, leading them to find their own solutions by thinking about how they would do it in their own lives, (2) making them feel that mathematics is necessary in life and will be useful for them, (3) helping students to apply mathematics in real life and (4) supporting permanent learning.

The study determined that PTs think it is easier to make a connection between mathematics and life through the realistic context of modelling activity. Some of the MEAs' effects noted by the PTs during the interviews may be the factors that facilitate building a connection between mathematics and the real world. These effects include the real life context, reminding them of their personal experiences and motivating them to make use of these experiences to solve the problem, and to consider the relationships between the problem and real life. When students work on real data, and they deal with real life problems which make sense to them, it becomes easier for them to contextualize mathematics (Greer, 2000). The mathematising process is an effective instrument for building bridges between mathematics and the real world (Bonotto, 2001; Greer, 1993); this process is one of the key components of MEAs and an essential element of this process is a realistic context. As stated by Lingefjärd and Meier (2010), these transitions between reality and mathematics are an important part of the modelling cycle. They are also a key element of mathematical literacy, which is an important skill for society. Allowing students to participate in MEAs frequently in their school years would help them to learn this transition. Students will be able to see the mathematics around them, grasping the role of mathematics in the phenomena that surround them in their daily lives and strengthen their mathematical identity thanks to modelling activities (Lingefjard & Holmquist, 2005), which are an excellent way of recognising and understanding different aspects of mathematics in real life (Lingefjärd, 2002).

Researchers anticipate that students generating their own solutions in modelling activities will improve self-confidence and ownership (Blum & Niss, 1991; Chamberlin & Moon, 2005). While the participants were practising model-eliciting activity during our research, they said that the real world context of the activity was effective at directing them to produce their own solution rather than recalling one they had learned from someone else. Therefore, the realistic context of MEAs contributes to the improvement of self-confidence for students who do these activities. Furthermore, in the interviews after the model-eliciting activity, students expressed their positive feelings about the real life context of the activity. This supports Maaß's (2005) claim that the connection between the model-eliciting activities and reality generates positive attitudes. To see the patterns, make relations and make their own decisions in problem situations that are based on real life shows the "learner" that mathematics is a part of life and makes it more fun and interesting (Umay, 2007).

Since there are no single and clear solutions in MEAs similar to real life problems, and there are various assumptions that should be considered, the students were surprised when they firstly encountered the MEAs (unsubscribing problem) in this study. Some students even decided that this problem was impossible to solve. Eraslan (2011) analyzed the views of pre-service mathematics teachers about model-eliciting activities. In this study, he reached similar findings and posited that this situation resulted from students' doing only what their teachers asked them to do in certain learning environments where the traditional problem solving activities were commonly used. He also suggested that students' ability to generate ideas did not develop well because of this.

Like Eraslan (2011), Thomas and Hard (2013) argue that the PTs' perception that the MEAs were "frustrating" may be attributable to their own experiences as elementary school learners of mathematics. Thus, MEAs should be used along with the traditional problem solving activities either before the direct instruction of a topic, as suggested by Lesh et al. (2000), or after instructing it, as suggested by Yoon and Dreyfus (2010). On the other hand, another factor that some PTs found "frustrating" may be the weakness of the relation between the context of these activities and the PTs' experiences and social environment. Yet the reality principle suggests choosing contexts and subjects that are connected with the interests and experiences of the students with whom the activities will be performed (Lesh et al., 2000). In MEAs, the realistic context of the activity is supposed to be connected to the students' experiences. This is because, as students stated during interviews, this helps students to internalise the problem and see themselves in the context of the problem. Otherwise, it is an obstacle to engagement with the mathematical modelling activity if students do not have any or only limited experience with the real life context of the problem (Blomhøj & Jensen, 2003).

Thomas and Hard (2013) claimed that PTs had certain negative experiences and opinions about the MEAs since they struggled with the ambiguity of MEAs. Likewise, Eraslan (2012) found that the PTs were confused about how to start to solve the problem due to the ambiguity of the MEAs. This study determined that this situation was a result of the reality principle. The realistic context caused the PTs to take a long time to understand the problem and confused them at the beginning of the activity. PTs attending the model-eliciting activity said that both problem and its solution were ambiguous and complicated, unlike the traditional problems. In contrast to previous research literature, this study claims that this ambiguity which causes the problem-solvers to struggle and get puzzled is in fact an opportunity for a positive experience. This is because, like real life problems, the MEAs' givens and goals or solutions are not clear and their interpretation is the major challenge (Zawojewski & Lesh, 2003). This situation is an outcome of the realistic situation, and it creates an opportunity to enable students who practise model-eliciting activities during their school education to gain valuable life skills. Reality in the world is not linear in any terms, and it cannot be restrained by a single theory (Lesh & Sriraman, 2005). Last century, Dewey asserted that education was supposed to be life itself and lessons at school needed to be as realistic as life outside the school (Dewey, 1897). It seems that linear and one-solution traditional problem solving activities would be insufficient to achieve this goal. MEAs are more suitable for this purpose since they include complex test procedures and various solutions specific to the problem-solver. A significant element of MEAs advantage in this field is their realistic real life contexts, as this research demonstrates. These activities should be implemented if we want students' mathematical knowledge to be permanent even after they graduate (Lesh & Sriraman, 2005). Students can find the opportunity to develop their skills that are critical to society through the integration of modelling activities into lessons at school (Maaß, 2005; Blum & Niss, 1991).

This study attempted to determine the effects of the reality principle by examining a single MEA. In addition, this study was conducted with PTs who had only performed traditional problem solving activities for a very long time (until their junior year at university). The apparent negative experiences, which were indicated by the findings and caused by the ambiguity of the MEAs, may have resulted from PTs' experiences in mathematics learning in their primary school years (Eraslan, 2011; Thomas & Hard, 2013). For this reason, the study suggests that future research be conducted with students in the early years of primary education and with more activities to determine more clearly whether the negative effects of ambiguity are caused by PTs' previous habits in learning mathematics and problem-solving.

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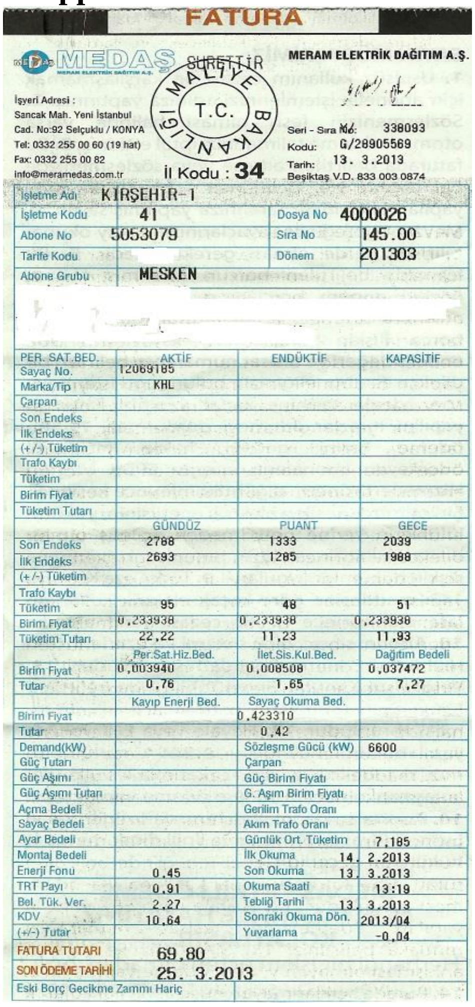
Appendix A



FATURA

MEDAŞ — MERAM ELEKTRİK DAĞITIM A.Ş.

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İl Kodu : 34

Seri - Sıra No: 338093
Kodu: G/28905569
Tarih: 13. 3.2013
Beşiktaş V.D. 833 003 0874

İşletme Adı	KIRŞEHİR-1		
İşletme Kodu	41	Dosya No	4000026
Abone No	5053079	Sıra No	145.00
Tarife Kodu		Dönem	201303
Abone Grubu	MESKEN		

PER. SAT.BED.	AKTİF	ENDÜKTİF	KAPASİTİF
Sayaç No.	12069185		
Marka/Tip	KHL		
Çarpan			
Son Endeks			
İlk Endeks			
(+/-) Tüketim			
Trafo Kaybı			
Tüketim			
Birim Fiyat			
Tüketim Tutarı			
	GÜNDÜZ	PUANT	GECE
Son Endeks	2788	1333	2039
İlk Endeks	2693	1285	1988
(+/-) Tüketim			
Trafo Kaybı			
Tüketim	95	48	51
Birim Fiyat	0,233938	0,233938	0,233938
Tüketim Tutarı	22,22	11,23	11,93
	Per.Sat.Hiz.Bed.	İlet.Sis.Kul.Bed.	Dağıtım Bedeli
Birim Fiyat	0,003940	0,008508	0,037472
Tutar	0,76	1,65	7,27
	Kayıp Enerji Bed.	Sayaç Okuma Bed.	
Birim Fiyat		0.423310	
Tutar		0,42	

Demand(kW)		Sözleşme Gücü (kW)	6600
Güç Tutarı		Çarpan	
Güç Aşımı		Güç Birim Fiyatı	
Güç Aşımı Tutarı		G. Aşım Birim Fiyatı	
Açma Bedeli		Gerilim Trafo Oranı	
Sayaç Bedeli		Akım Trafo Oranı	
Ayar Bedeli		Günlük Ort. Tüketim	7,185
Montaj Bedeli		İlk Okuma	14. 2.2013
Enerji Fonu	0.45	Son Okuma	13. 3.2013
TRT Payı	0,91	Okuma Saati	13:19
Bel. Tük. Ver.	2,27	Tebliğ Tarihi	13. 3.2013
KDV	10,64	Sonraki Okuma Dön.	2013/04
(+/-) Tutar		Yuvarlama	-0,04
FATURA TUTARI	69,80		
SON ÖDEME TARİHİ	25. 3.2013		
Eski Borç Gecikme Zammı Hariç			

Table 1. Previous billing values.

Period		DAY	PUANT	NIGHT
2012 07	First Index	2045	970	1594
	Last Index	1960	928	1554
2012 08	First Index	2075	977	1608
	Last Index	2045	970	1594
2012 09	First Index	2160	1020	1655
	Last Index	2075	977	1608
2012 10	First Index	2250	1065	1705
	Last Index	2160	1020	1655
2012 11	First Index	2350	1115	1775
	Last Index	2250	1065	1705
2012 12	First Index	2460	1170	1850
	Last Index	2350	1115	1775
2013 01	First Index	2580	1230	1929
	Last Index	2460	1170	1850
2013 02	First Index	2693	1285	1989
	Last Index	2580	1230	1929
Puant: Electricity consumption between 17.00 and 22.00				

Mr. Erdem wants to terminate his electricity subscription. He will be moving early Monday morning, and he will travel from Kirsehir to Rize in the vehicle carrying his belongings. On Thursday and Friday he will be taking care of business in the city where he is moving on Monday and the electric company will be closed at the weekend. So he will unsubscribe 4 days before moving, on Wednesday. He determined the values on the electric meter shown on Table 2. However; if he unsubscribes using these values, the cost of four days electricity will be paid by the apartment's next occupant. Mr. Erdem has an organised file where he keeps all the apartment's bills. A bill from this file is shown above. Spending for the previous eight billing cycles is shown on Table 1. Mr. Erdem wants to determine the values for the four days after the meter reading based on the values on Table 2 and unsubscribe after paying his bill in full. Your task:

- Calculate the values for 4 days later on Mr. Erdem's electricity meter.
- Develop a mathematical tool (i.e. relation, function, formula) which can be used for situations like this and is as sensitive as possible. This tool is supposed to provide information on a subscriber's spending on any time or any days in the future (estimating that all the previous bills and values of the person moving his/her house are available).

Table2. The values that Mr. Erdem sees on his electricity meter before he unsubscribes:

Day	Puant	Night
2838	1358	2065

Appendix B The following semi-structured questions were used to focus the discussion.

1. Will you please describe your general impression of the unsubscribing problem?

2. Can you compare this activity with traditional problem solving activities?

3. Can you compare the real life context in the unsubscribing problem to the contexts of the problems you have seen previously in mathematics courses?

4. Is it important for students' mathematical problems to be related to real life? Why?

5. How does it benefit students when the real life context in the problem is realistic?