An Exploratory Study of a Story Problem Assessment:

Understanding Children's Number Sense

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Abstract

The purpose of this study was to identify and describe students' use of number sense as they solved story problem tasks. Three 8- and 9-year-old students participated in clinical interviews. Through a process of holistic and qualitative coding, researchers used the number sense view as a theoretical framework for exploring how students' number sense was reflected in their numerical reasoning and verbal explanations of strategies for solving story problems. The results suggest that students' coordination of number sense constructs affords flexibility in problem-solving and deeper understanding of place value. Implications for classroom practice and mathematics education research are discussed.

Keywords: number sense, story problems, assessments

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Number sense plays a key role in early mathematics development and is, therefore, an important area of research in mathematics education. While various researchers have different definitions for the term "number sense," in this study, we use McIntosh, Bana, and Farrell's (1997) description that number sense refers to "a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations" (p. 324). This description encompasses both the intuitive nature of number sense and the educationist perspective that number sense is composed of multiple constructs that are foundational to understandings of quantities, number, relationships among numbers, and the number system (Berch, 2005; Jordan, Kaplan, Olah, & Locuniak, 2006; Resnick, Lesgold, & Bill, 1990).

As a complex construct, number sense is difficult to assess by traditional standard measures (i.e., a test or observation) because it is composed of multiple components such as students' abilities to decompose numbers, think flexibly, understand number relationships, and use place value understandings. We hypothesized that researchers and teachers could better understand students' number sense by engaging students in a story problem assessment and analyzing their strategies and verbal explanations for number sense constructs. The purpose of this study was to analyze individual students' responses for solving story problems to identify and describe their use of number sense.

Assessing Number Sense

The complexity of number sense makes it difficult for researchers and teachers to assess students' number sense. Hence, much of the recent research in number sense aims to tease out the components of number sense by operationalizing and testing various constructs of number sense (e.g., Chard et al., 2005; Clarke & Shinn, 2004; Geary, 2011). The most commonly assessed constructs in current number sense instruments include counting, quantity discrimination, number combinations, number identification, and estimation (Chard et al., 2005; Clarke & Shinn, 2004; Geary 2011; Jordan et al., 2006; Jordan et al., 2007; Jordan et al., 2009; Jordan et al., 2010; Lago & DiPerna, 2010; Locuniak & Jordan, 2008). Some measures, such as the Early Mathematics Curriculum Based Measure (Chard et al., 2005; Clarke & Shinn, 2004) and Geary's (2011) number sense tasks, contain either a missing number task or violation of counting rules task as a way to further assess students' counting skills. Estimation tasks are also often used as a number sense measure (Geary, 2011; Jordan et al., 2006; Jordan et al., 2007; Jordan et al., 2009; Jordan et al., 2010; Lago & DiPerna, 2010).

Overall, there is a wide range of tasks purported to assess students' number sense. The deconstruction of "number sense" into constructs of counting, quantity discrimination, number combinations, number identification, among other concepts and skills provide educators and researchers with information about specific aspects of students' number sense. Nevertheless, it also seems important to use this knowledge of specific constructs and tasks that make up number sense to understand *how* students are holistically engaging their number sense. This idea is not new. While the current literature on assessing number sense emphasizes the deconstruction of number sense into constructs, in the 1990s, McIntosh, Reys, and Reys (1992) developed a framework for assessing number sense holistically. We propose revisiting this approach.

McIntosh et al. (1992) stated that, “Even if it were possible to identify and assess all components, a person’s number sense may be inadequately reflected by the individual components” (p. 5). We hypothesized that a story problem assessment could provide insight into students’ thinking process and how they are using their number sense to solve these story problems.

Story Problems as Assessments of Number Sense

While story problems were used in some of the reviewed assessment studies (e.g., Jordan et al., 2007; Locuniak & Jordan, 2008) as a measure for number sense, none of these studies examined *how* students used number sense when solving story problems. For example, Jordan et al. (2006) presented four addition and four subtraction story problems to students as one of their measures for assessing students’ number sense. Students’ solutions were determined to be correct or incorrect. While evaluating correct or incorrect responses provides valuable information about a students’ current ability to produce a solution accurately, analyzing students’ solutions, strategies, and reasoning provide valuable information on students’ use of number sense when solving problems.

In this study, we propose that an analysis of students’ strategies and reasoning on story problem tasks provide educators and researchers with an understanding of how students use their number sense to solve problems. Deconstructing number sense into a variety of constructs pinpoints aspects of students’ number sense, however, story problems have the potential to then re-construct the number sense components with the purpose of assessing how students apply and coordinate the multiple constructs of number sense. In the context of their proposed framework for assessing number sense, McIntosh et al. (1992) left readers with the question, “Are number sense and problem solving different?” (p. 8). This study uses this question as a starting point to explore a story problem assessment and find out how students use number sense to problem solve.

Theoretical Orientation: The Number Sense View and Adaptive Expertise

The theoretical orientation of this study is framed in the number sense view (Baroody, Feil, & Johnson, 2007; Baroody & Rosu, 2006) and construct of adaptive expertise (Verschaffell, Luwell, Torbeyns, & Van Dooren, 2009; Hatano & Oura, 2003). These theories value the complexity of number sense and orient us to the holistic nature of number sense. Based in these theories is the idea that students' strategies and reasoning for solving story problems reveals students' interconnected number knowledge, termed "number sense."

Rather than focus on the components that make up number sense, Greeno (1991) suggested that number sense requires theoretical analysis. The *number sense view* is a theoretical lens for understanding number sense more holistically as opposed to assessing individual constructs that make up the term "number sense" (Baroody & Rosu, 2006). The number sense view emphasizes conceptual development of number concepts and is based in the idea that interconnected concepts develop mathematical meaning and understanding. The number sense view is used to explain why computational fluency, estimation, and reasoning with numbers improve as a child constructs deeper understanding of number sense concepts (Baroody, Feil, & Johnson, 2007; Baroody & Rosu, 2006). Number sense is at the heart of this interconnected knowledge that builds children's mathematical reasoning and understanding.

When children are stimulated to develop a strong framework of number relations and use these relations for flexible mental computation, they develop adaptive expertise (Verschaffell, Luwell, Torbeyns, & Van Dooren, 2009). Hatano and Oura (2003) define "adaptive experts" as experts who apply their schemas in adaptive and tuned ways. Adaptive expertise goes beyond the routine expertise of speed, accuracy, and automaticity of solving familiar problems. Adaptive experts tend to use creative and innovative approaches to solving problems. They understand why procedures work, invent new procedures, and flexibly apply their knowledge to new situations. Baroody et al. (2007) explain that adaptive expertise involves deep conceptual knowledge, deep procedural knowledge, and flexibility.

Study Purpose

The purpose of this study was to identify and describe students' use of number sense as they solve story problem tasks. The following research questions guided this study:

RQ 1: How is students' number sense reflected in their explanation of strategies and reasoning on story problems?

RQ 2: How do students explain their understanding of number patterns and number relationships within the base-ten place value number system as they solve story problems?

While previous number sense assessments operationalize and pinpoint students' number sense strengths and weaknesses, the story problem assessment we used in this study can help teachers and researchers analyze how students holistically engage their number sense foundations. Analyzing students' strategies through this lens can provide information about how students decompose numbers, think flexibly, understand number relationships, and use place value understanding. This valuable insight into students' applications of number sense allows teachers to reflect on students' progress and plan instruction. This lens also provides researchers with a more accurate understanding of students' conceptual understanding of numbers and their relationships.

Overview of the Methodology

This study used clinical interviews (Ginsburg, 2009; Piaget, 1976) for observing and interpreting 8- and 9-year-old students' use of number sense in solving story problems. Learning mathematics is complex and is more than obtaining the correct solution. Strategies and reasoning involved in solving story problems are key in developing strong number sense and learning mathematics. The clinical interview method considers the complexity of learning mathematics and makes it possible to focus on the student's underlying cognitive processes for solving mathematics problems (Gingsburg & Pappas, 2004; Zazkis & Hazzan, 1999).

Within the clinical interview method, this study used open-ended story problem tasks to elicit students' thinking processes. Students were asked to solve a series of carefully selected story problems and think aloud and/or explain their thinking about how they solved the problem. The use of story problems in assessing students' mathematical reasoning and strategies as well as for inferring students' cognitive processes has been used by researchers to reveal aspects of students' thinking beyond the correct/incorrect information provided by multiple-choice questions (Cai, 1995; Cai, 2000; Gingsburg & Pappas, 2004; Zazkis & Hazzan, 1999). Using clinical interviews, this study was an exploratory investigation into using story problems to identify and describe students' uses of number sense in the process of solving story problems.

Participants, Setting, and Materials

The participants were three 8- and 9-year-old students at the time of the study and were enrolled in mathematics tutoring at a regional state university in a semi-rural area of the western United States. One male and two females participated in clinical interviews in February and March of their nine-month (September-May) school year. Two students had been attending one-hour tutoring sessions once a week over a 3-month period; the third student had started tutoring services two weeks prior to our interview. One student was homeschooled for half the day and attended public school during the other half of the day. The other two students attended public school in the full-day setting. One student received special education services in mathematics at the time of the interview. Another student qualified for special education services in mathematics then was disqualified during the three months of tutoring. One student did not receive special education services at school, but was deemed “at-risk for failure” by her teacher and other school personnel due to low mathematics achievement. The sample was opportunistic and selected by two of the authors: the lead researcher and the tutoring clinic teacher. There were two separate but related aims to the clinical interviews of these students. The researcher’s aim was to pilot test story problems as a way to reveal students’ use of number sense, while the tutoring clinic teacher’s aim was to better understand her students’ use of number sense for further instruction (i.e., formative assessment). Pseudonyms are used in this paper in order to protect the identities of the children.

The clinical interview sessions took place in one of the university’s clinical instructional observation rooms, which contains a kidney table and small chairs for the researcher, tutoring clinic teacher, and student as well as an observation window for parents and research observers. Each clinical interview session lasted approximately 20 minutes with 5 minutes of introductions and paperwork with the parent and 14-18 minutes of engagement with the child in the interview. Students were participating in ongoing one-on-one teaching and learning interviews throughout their tutoring sessions with the tutoring clinic teacher. The selected episodes for this article were conducted by the researcher and highlight examples of students’ strategy descriptions that reveal important aspects of their number sense.

The materials for this study included the protocol for the clinical interviews and task sheets for students (described below in Procedures and attached in the Appendix), mathematics tools and manipulatives, video recording/audio recording devices, and parent paperwork for permission to participate in the study. The task sheets were carefully selected story problems based on Cognitively Guided Instruction problem types (Carpenter et al., 2015; Hiebert et al., 1997) and the Common Core State Standards for Mathematics problem situations (Common Core State Standards Initiative, 2010). Mathematics tools and manipulatives, specifically, base ten blocks, unifix cubes, number lines, and hundreds charts, were available for students to use as they solved the problem and/or as tools for illustrating their thinking about a problem. The audio recording device captured the students' verbal responses and utterances during the interview. Video recording devices built-in to the clinical observation rooms were utilized for this study as a secondary means for capturing further information. The video camera focused on the broad student-interviewer-workspace view to capture the movements, writing, and actions of the student and interviewer.

Data Sources and Procedures

The clinical interviews in this study involved two phases: 1) designing the semi-structured interview (e.g., task sheets, interview protocol); and 2) conducting clinical interviews with students as they solved story problem tasks on a task sheet.

Data Sources: Semi-Structured Clinical Interview

The first phase was designing the clinical, semi-structured interviews with a cognitive orientation towards revealing students' use of number sense. The primary sources of data in this study were students' responses to the story problem tasks and the semi-structured interview questions. The tasks consisted of four story problem situations: 1) equal groups unknown product, 2) put together total unknown, 3) take from result unknown, and 4) add to start unknown (Common Core State Standards Initiative, 2010). The four story problem situations align with the Cognitively Guided Instruction problem types, which have been widely used in research on children's thinking and processes for solving problems (e.g., Carpenter et al., 2015; Hiebert et al., 1997; Mottier Lopez & Allal, 2007). For the purposes of this paper, we have used the Common Core State Standards for Mathematics terms for these problem situations (Common Core State Standards Initiative, 2010). The four problem types were chosen in order to have a variety of problem types with differing operations and equation structures as well as varying levels of difficulty. Numbers were selected to encourage a range of strategies from the students.

Semi-structured interview questions were designed to elicit verbal responses from students as they solved the story problems and/or after they solved the problem. These questions were planned in advance (see Appendix A), but contingent upon a student's response (Zazkis & Hazzan, 1999). The interview was planned as semi-structured in order to provide room for the student's account of his/her solution method and allow for follow-up questions and variations on the planned questions (Gingsburg & Pappas, 2004; Zazkis & Hazzan, 1999). The purpose of the semi-structured interview protocol was to provide a window into students' use of number sense in solving the problem. In particular, the researchers focused on ways students' verbal responses reflected their number sense.

Clinical Interview Procedures

The second phase was conducting the clinical interviews. The interview began with the interviewer reading the first story problem to the student. The interviewer and tutoring clinic teacher observed the student's responses and mathematical behavior (e.g., physically solving the problem with manipulatives, talking aloud, whispering comments to herself) as he or she solved the problem. The interviewer then asked initial questions to elicit the student's explanation of his or her strategy for solving the problem, followed by probing questions that were asked based on the student's initial responses, and finally ended with questions that could extend the student's thinking and provide even more insight into his or her use of number sense. All three interviews were conducted by the same interviewer in order to have consistency with the way the interview was conducted (Goldin, 2000). Sessions were audio recorded for transcription and analysis and both the interviewer and tutoring clinic teacher took field notes.

Data Analysis

Following the completion of the data collection process, a formal analysis occurred in two stages: holistic coding and qualitative coding (Cai, Lane, & Jakabcsin, 1996; Cai, Magone, Wang, & Lane, 1996). During the first stage, the interviewer and the tutoring clinic teacher used the Cognitively Guided Instruction framework for students' development of problem solving strategies to holistically code students' strategies on the four tasks (Carpenter et al., 2015). This framework delineates students' typical strategies for solving problems as direct-modeling strategies, invented strategies (e.g., using counting or known facts), and standard algorithms. A direct-modeling strategy involves using concrete manipulatives or drawings to express each part of the problem. Invented strategies vary, though often involve using counting strategies or known facts to solve a problem. Standard algorithms strategy refers to using an algorithmic procedure to solve the problem. These broad categories provided a starting point for understanding how the student approached the problems.

The second stage of data analysis involved qualitative coding (Cai, Lane, & Jakabcsin, 1996; Cai, Magone, Wang, & Lane, 1996) for number sense themes within students' strategies as evidenced in their written work and verbal responses (Glesne, 2011). Counting, quantity discrimination, number combinations, number identification, number patterns, and estimation are common constructs across multiple measures in a variety of number sense assessment studies, and hence, are constructs we looked for in students' strategies and verbal explanations (Chard et al., 2005; Clarke & Shinn, 2004; Geary 2011; Jordan et al., 2006; Jordan et al., 2007; Jordan et al., 2009; Jordan et al., 2010; Lago & DiPerna, 2010; Locuniak & Jordan, 2008). These common number sense constructs were the starting point for coding and understanding how students used number sense in solving the story problem tasks. In addition, we looked for ways number sense was reflected in students' understanding and use of the base-ten, place value number system. Two coders independently coded the data both holistically and qualitatively, then discussed the codes to verify the codes and check for inter-rater reliability.

Results

The research questions for this study guided an exploratory investigation into how students' number sense was reflected in their explanation of strategies and thinking on story problem tasks. To answer the research questions, the results are organized around each of the four story problem situations. We describe each story problem and present each student's strategies for solving the problem situation. The major themes in the students' explanations of their strategies are presented along with the number sense concepts embedded in their verbal explanations.

Strategies and Number Sense Constructs by Problem Situation

Four problem situations were presented to the students: 1) equal groups, 2) put together, 3) take from, and 4) add to (Common Core State Standards Initiative, 2010). Equal groups situations involve a multiplication operation, although addition strategies are often used to solve these problem situations. Put together and add to situations are commonly known as addition problems, although they can often be solved with subtraction. The "take from" problem situation calls for the subtraction operation, although like the other problem situations, it can be solved by the inverse or a related operation. A variety of problem situations and numbers were selected in order to gain a fuller understanding of each student's strategies and use of number sense for solving story problems. The use of "I" instead of "we" is used in the subsequent descriptions of the clinical interviews to reflect the interviewer's (first author's) decisions in the process of conducting one-on-one interviews with each student.

Multiplication: Equal groups unknown product situation. The equal groups situation used in this interview involved an unknown product. The story problem was read to the student as follows:

Jessica has ____ bags of cookies.

There are ____ cookies in each bag.

How many cookies does Jessica have altogether?

This problem type encourages students to think in groups, in other words, to unitize. This is a major conceptual leap for many 8- and 9-year-old students and begins to become easier to grapple with as students gain experience with equal groups situations (Fosnot & Dolk, 2001). Unitizing is difficult because children have to consider and coordinate two units. For example, in the cookies story problem for this study, children had to consider the number of bags as well as the number of cookies in each bag.

I selected 5 and 12, 6 and 10, and 3 and 24 as possible numbers to insert into the blanks for the story problem. The numbers were chosen in order to find out if students would use place value knowledge, decompose numbers to make the problem easier to solve, and see relationships between the first two sets of numbers. Ten and 20 are numbers that the students could easily skip count with, and I wondered if students would decompose the 12 in the first set of numbers because both 10 and 2 are friendly to use in skip counting and calculations.

Margaret. Margaret was initially presented with one equal groups task using the numbers 5 and 12. She incorrectly responded "17" because she added 5 to 12 instead of multiplying the two quantities. She had difficulty understanding the story problem situation. The tutoring clinic teacher suggested a cookie context because Margaret had experience with selling cookies to raise money. The tutoring clinic teacher thought the context would give her a relatable context for solving the problem. So I told Margaret the following story:

> Margaret has 3 boxes of cookies. Can you draw your three boxes of cookies? Let's say these are 3 boxes of cookies, and Arla ate most of them. So now there are only 3 cookies in each box. Can you put three cookies in each box? That's all that's left in each box. So now there are 3 boxes of cookies and 3 cookies in each box. How many cookies do you have now?

Margaret correctly responded "9," and said, "I counted 3 and then 3 more and then 3 more." I asked her, "when you saw the 3 more, how did you know what number came next?" Margaret showed me her fingers and explained that she counted up with them. A combination of the context provided by the interviewer and the pictorial representation of the context used by Margaret helped Margaret access the problem. I was able to see her use a counting up strategy to correctly solve 3x3. She counted from 3 up to 6, then counted on from 6 up to 9. However, this was with much scaffolding by the interviewer, and the interviewer was using this opportunity to get to know the student's thinking and what she could do.

Patrick. Patrick was presented with equal groups task three times with the three different sets of numbers (5x12, 6x10, and 3x24). He answered the first two correctly and made an error on 3x24. He used partial products with the first and third problems, meaning he decomposed the two-digit number to make the problem easier to solve by using the distributive property. For example, on the first problem, Patrick decomposed the 12 into 10 and 2 so he could first solve 5x10. Once he knew 5x10 was 50, Patrick stated that “I just added 2,” meaning 2 more bags of 5. This is where Patrick’s ability to unitize, or conceive these amounts as groups, helped him solve the problem quickly. He knew 2 bags of 5 makes 10 cookies, so he added 10 to 50 to get 60.

On the second set of numbers, Patrick had quick recall (known fact) of 6 groups of 10 and answered “60” immediately and confidently. On the third problem, which had larger numbers, Patrick correctly decomposed 24 into 20 and 4 and multiplied 20x3 and 4x3 correctly to get 60 and 12. He again used the partial products strategy. When he added the products 60 and 12 together, he made an error when he added 4 (from the 24 in the story problem) instead of 2 (from the 12 in 4x3).

Patrick seemed to rely on decomposing numbers to quickly solve the problems in his head. Since Patrick’s explanations for all three problems were succinct and easy to follow, we did not have much discussion on these problems. Though he talked through his strategies quickly, I learned that he was unitizing and thinking in groups of ten and other groupings (like groups of 5). He was able to recombine amounts accurately on the first problem. Patrick used tens and ones knowledge effectively for the partial products strategy.

Joellen. Joellen was presented with two sets of numbers for the equal groups task (5x12 and 6x10). She incorrectly answered 5x12 because she added the two amounts to get 17. The second time I read the problem to her with the new amounts of 6 bags of cookies with 10 cookies in each bag, I asked her to draw a picture. I read the first line of the story problem and she drew 6 circles. Then I read, “there are 10 cookies in each bag.” Joellen started to draw 10 circles in the first bag, but then crossed them out and instead wrote numerals “10” in each bag. She correctly responded 60 and said, “I know my times tables and I knew how to...I know what 6x10 is,” then she further explained, “you have 6 bags and each of them has 10 and then you can just count it by tens.” Her statement reflected a conceptual understanding of what 6x10 means.

At the end of Joellen’s interview, I decided to give her an additional multiplication task to find out more about her grouping knowledge. I explained that she added 5 and 12 together on her first problem and asked her “but what if you multiplied them, what would you get this time…if there are 5 bags with 12 cookies in each one?” Joellen said, “So I multiply the 12 and the 5…60…because I counted by fives.” When asked if she counted by fives 12 times, Joellen said, “Actually no, I did it 3 times because I counted the fifth one time and it was 25 so I added the 25s together and that was 50, then I know that you have to go two more fives and then that would equal 10. So it was 60.” I had trouble following her thinking so she clarified her reasoning,

> So I went 5, 10, 15, 20, 25. Then I added the two 20s and that was 40. Then I added the two 5s left and I knew that was gonna be another 10, so it’s 50 and that would only equal ten 5s and then I have two extra so that would be a 10 and that makes 60.

The doubt I had about Joellen’s ability to unitize vanished. She showed that she understood grouping ideas. Joellen counted by 5s to figure out that 5x5 is 25. Instead of continuing the counting strategy, she doubled 25 to make 50. Then, she added on two more groups of 5 (plus 10) to make 60. That type of thinking requires flexibility, understanding of relationships among numbers, unitizing, and the ability to decompose and compose numbers.

Addition: Put together, total unknown. The put together problem used in the clinical interviews had an unknown total and read as follows:

Arla has ___ red marbles and ___ blue marbles.

How many marbles does she have?

This problem type asks students to take two sets of objects and put them together, in this case red marbles with blue marbles to figure out the total number of marbles. This problem situation is more difficult than the “add to” situation, particularly for younger children, because there is no action in the story (Carpenter et al., 2015). We conjectured that my interviewees would not struggle with the non-action aspect of the problem. We selected 27 and 61, 52 and 36, and 50 and 63 as possible numbers for the problem. None of the sets of numbers require regrouping, so we wondered if students would use place value knowledge and derived facts to quickly solve these problems mentally.

Margaret. For the put together problem, Margaret was presented with 27 and 61. She wrote 27 on her paper with 61 directly below it. She added the ones place then the tens place to correctly find the solution 88. When asked how she figured out each addition column she explained that she just knew 7 plus 1. To solve for 6 plus 2 she started at 6 then counted up 2. Margaret started to warm-up at this point in the interview due to her success with this problem, so I moved on to the next problem without much more probing. I witnessed her use of a counting up strategy again. Her use of the algorithm on this problem led me to wonder if she would use the algorithm on the subtraction and addition problems coming up.

Patrick. Patrick was presented with two sets of numbers for the “put together” tasks (27 and 61; 227 and 61). He figured them out quickly and answered both correctly. He combined tens and ones mentally for both sets of numbers. It is not clear if he lined them up in his head like an algorithm or if he was decomposing the numbers. He did not talk about amounts, rather just digits and combining the digits to make a number. For example he said, “I added the 2 to the 6 and the 1 to the 7." The 2 and the 6 represent values of 20 and 60. When asked to solve 227 plus 61, he said he knew that the 61 and 27 are 88 and that there is nothing in the hundreds place except the two. Patrick's language in that instance indicated understanding of place value and that the numeral "2" represents hundreds (i.e., 200). Also, this verbal explanation showed that he thought of 227 as 27 with a 2 in the hundreds place.

Joellen. Joellen was presented with three sets of numbers for the "put together" task (27 and 61; 52 and 36; 50 and 63). Joellen solved all three problems correctly. On all three problems she started in the tens place then computed the ones place. She used a tens and ones strategy. Joellen's language showed that she was considering place value. For example, when solving 27 plus 61 Joellen said, "Well I know that 8…I mean 6 plus 2 is so that's what I did for the tens place that would be an 80. I know what 7 plus 1 is and that's 8." When she solved 52 plus 36, she said, "Well I know that 5 plus 3 is 8 and so it would just be an 80 in the tens place then…6 plus 2 is 8 so it would be 88." For the problem 50 plus 63, Joellen explained, "Well, because I know what 5 plus 6 is and that's 11 and then you have the extra 3 so that would be 113." When I asked her how she computed each place in the number, she explained that she used known facts: "Um…I just knew it because whenever I was in class and I was learning my adding I always tried to remember 5 plus 3."

Subtraction: Take from, result unknown situation. The "take from, result unknown situation" is the common subtraction situation found in many elementary mathematics lessons. The problem for the interview was read as follows:

Arla had ___ marbles.

She gave ___ to Jessica.

How many marbles does Arla have left?

Subtraction is often difficult for elementary students and provides important insight into students' understandings of number (Van de Walle, Karp, & Bay-Williams, 2015). This type of story problem situation can show students' skills with flexibly moving back and forth between subtraction and addition as well as indicate their ease with counting backwards and/or number combinations. We chose 18 and 6, 14 and 8, 48 and 22, and 345 and 140 as possible number sets for the "take from" problem situation. The first two sets of numbers would let us know how well the students are working within smaller amounts and give us a good sense of their fluency with addition and subtraction combinations. None of the number sets require regrouping. Again, we wanted to see if students would flexibly use their knowledge of numbers. As numbers get cumbersome, it is appropriate to use algorithms in these examples. Our purpose, however, was to see how students use their number sense and figure out problems quickly.

Margaret. Margaret was presented with two sets of numbers for the "take from" problem situation (18 and 6; and I chose an easier set of numbers, 10 and 5) both of which she answered incorrectly. When asked to figure out how many marbles Arla had left after she gave 6 of her 18 to me, she replied "14." She did not write the algorithm on her paper this time, but her explanation indicated algorithmic thinking: "Well I did the same thing. I put 8 and 6 on top of each other and did 8 plus 6 and 1 plus nothing is 1." Margaret added the two quantities instead of subtracted. She described the "1 plus nothing is 1," but her answer only reflected one ten, which came from the 14 derived from 8 plus 6.

I decided to try easier numbers: 10 minus 5. I hoped that she would recognize a known fact or have an understanding of the relationship between the two numbers. I gave her the story problem again, this time "Arla had 10 marbles and gave 5 away." She responded "15" and explained, "5 plus nothing is 5 and 1 plus nothing is 1." Again, Margaret added the two quantities instead of subtracted and her response showed algorithmic thinking as she started in the ones place with “5 plus nothing” then moved to the tens place with “1 plus nothing.” I moved on to the last task and made a note to come back to another subtraction situation.

At the end of Margaret’s interview, I tried the “take from” problem situation again, but this time with a cookie context, as suggested by the tutoring clinic teacher. I asked Margaret to draw a box of cookies, and I said, “there are 10 cookies in this box. This time Margaret ate most of this box…she ate 5 of them. How many does Arla get?” She drew 10 circles in the box to represent her cookies. She crossed out 5 of the cookies, counted the cookies that were left, and responded that 5 were left. She correctly got the answer to 10 minus 5 when she directly modeled the problem situation with a picture representation.

I tried another subtraction problem and again prompted her to draw one box with 10 cookies. I said, “Margaret gave me 2 cookies, how many are left?” Margaret crossed out 2 cookies on her picture, counted what was left and responded “8.” So I said, “Okay, so now there are 8 cookies left, right? You gave 3 of them to Arla. How many are left now?” She crossed out 3 cookies and counted 5 left. I asked “Then you gave two of them to your mom. How many are left now?” She crossed out 2 and responded, “3.” That was the first instance that Margaret quickly recognized the amount that was left, rather than counting what was left. I said, “then you gave one to your brother. How many are left now?” Margaret crossed out the cookie and quickly said “2.”

The tutoring clinic teacher tried a problem with bigger numbers and no picture to see what Margaret could do. The teacher asked, “You have 24 cookies and my dog came along and…ate 6 of them. How many do you have left?” After a pause, Margaret said 19. She incorrectly counted backwards with her fingers. Once she stopped using the drawing to directly model the problem situation and had to deal with larger numbers, she incorrectly answered the problem.

Patrick. Patrick was presented with two sets of numbers for the “take from” task (375 and 140; 327 and 152 because I wanted to see what he would do with regrouping) and answered both correctly. The 375 minus 140 problem was the first instance that Patrick wrote something down on his paper. He wrote the numbers vertically and solved the algorithm. He solved the subtraction in the ones place first, then the tens, then the hundreds.

For 327 minus 152, Patrick wrote the numbers vertically and performed the algorithm with regrouping. Since he performed the algorithm correctly and quickly, I asked him how he found the difference between the numbers in each column in the algorithm. He used relationships among numbers, benchmark numbers, and known facts (like 5+5) to solve the differences quickly. Patrick flexibly thought about the relationship between 7, 2, and 5 to solve 7 minus 2 in the ones place. Patrick explained, “Um…7 and 2…well I knew that 5 needs 2 to get to 7.” To find the difference between 12 and 5 in the tens column Patrick said, “I knew that 5 needs 5 to get the 10, then I add 2 to 5.” He incremented 5+5 is 10 then two more to get to 12 to make a difference of 7 between 5 and 12. His strategy for the problem was using an algorithm, but his explanations for subtracting in each place show that he was using number relationships, incrementing, and counting on strategies to find the differences between numbers.

Joellen. I presented Joellen with two sets of numbers for the “take from” tasks (18 and 6; 345 and 140), which she solved correctly. To figure out 18 minus 6, Joellen used her knowledge of the relationships among 8, 6, and 2. She first thought about 18 as a whole and said, “9 plus 9 is 18 and then I thought of the 8.” When I asked her how the 9 and the 9 helped her figure out what to do next, she said “because it added up to 18.” I inferred that she was getting familiar with 18 and thinking about what it is composed of. Once she did that, she focused on the 8 in 18 and then she explained, “I thought of the 8 and if you plus 2 to the 6 it would have been 8 so I just minused it and then it was 12.”

For the second set of numbers, Joellen figured out 345 minus 140 immediately in her head. She said, “if you minus the 1 from the 3 it would have been 2 and then 40…44…you would minus a 40 from a 45 it would be 5 so you’d have 205.” She started in the hundreds place and subtracted those digits. Then, she looked at 45 and 40 and knew the difference between those is 5. She thought of the tens and ones together as a number which is much more efficient than doing 4 minus 4 and 5 minus 0 separately.

In the subtraction tasks, Joellen used her place value knowledge, but also used her knowledge of number relationships. She thought about the relationships among 8, 6, and 2 as a fact family to help her solve 18 minus 6. Embedded in this is her understanding that addition and subtraction are related. She used addition to find the difference between 6 and 8. In the second problem, Joellen used her place value understanding and subtracted the hundreds first, then realized that she knew the relationship between 45 and 40 and could quickly see a difference of 5 between those two numbers.

Addition: Add to, start unknown situation. The “add to, start unknown” situation is typically a difficult problem type for elementary students because the unknown is at the beginning of the problem rather than the end of the problem (Carpenter et al., 2015). The story problem used in the interviews read as follows:

Ben had some pennies.

Alex gave him ___ more. Now he has ___ pennies.

How many pennies did Ben begin with?

I chose 8 and 10, 12 and 36, and 75 and 100 as possible number sets for this problem. The first set's numbers are only 2 apart. In the second set, 12 is a multiple of 36. The third set is often recognizable to 8- and 9-year-old students in the United States as counting by 25s or quarters (U.S. currency). With a difficult story problem situation, I thought these number choices would allow the students easier access to the problem while exhibiting their knowledge of number relationships.

Margaret. I asked Margaret to solve the "add to, start unknown" task with the 8 and 10. I asked her what Ben started with if Alex gave him 8 more pennies and now he has 10. She incorrectly responded "18," after using her fingers to count up from 10. When asked how she figured it out she explained, "I did 10 plus 8." I asked, "What made you want to add those together?" After a long pause I rephrased my question and asked, "Is there anything in the problem that said 'put those numbers together?" Margaret shook her head and did not have an answer. She answered the question incorrectly, but her strategy for solving it again showed me that she tends to rely on the counting up strategy for most addition problems.

Patrick. Patrick was presented with two sets of numbers (18 and 30; 75 and 100) for this problem and performed both correctly. This was only the second instance that Patrick used paper and pencil to solve a problem. To solve how many pennies Ben started with if Alex gave him 18 and now he has 30, Patrick drew a line with an arrow and labeled 32 on one end and 18 on the other end. Then, horizontally, he wrote 21, then 30, then 12. He attempted to explain a school strategy saying "Um… they teach me problems like that at school. I don't know what they call it, but they…like…the big number is missing you subtract. No when the big number is missing you add and when the small number is missing you subtract." I asked if he was using a number line, but he said, “They call it a number family.” He struggled to explain what he did and why it works.

When I gave him the numbers 75 and 100 he went back to using a strategy based in number sense. He again used an incrementing strategy to solve the difference between numbers. Patrick explained, “I knew that 70 needed 30 to get to 100, but then I saw the 5…and 25 plus 5 is 10.” He used 100 as a benchmark or friendly number this time. Compensation was also embedded in his reasoning for why it was not 70 plus 30, rather 25 and 70. Patrick saw the 5 in 75 and realized that it needed to be 25 instead of 30.

Joellen. Joellen was presented with the same two sets of numbers for this task (18 and 30; 75 and 100). She incorrectly solved the first one with the answer 22 (correct answer was 12). She exhibited understanding of the story problem as she said, “It says Alex gave him some more and now he has 30. So it said how many pennies did Ben begin with, so you would just have to minus that.” In this subtraction instance, starting in the tens place made it more difficult for Joellen to solve in her head. She said, “It’s because if you minus the 1 from the 2—I mean the 1 from the 3—it would have been a 2. And then the 8, you’d have to get 2 more to get to 10.” Her response shows that she thought about the 0 in terms of a ten, but she did not consider how that might affect the rest of the number.

She solved the other problem, with 75 and 100, correctly. At first she said, “He has 100…30…no 25!” Joellen explained her self-correction, “Because when I said 30…because then he would have to have it then it would be 110. So then I’m like no 25 because then you just use the 5 to get to the 80 and then you have to go through the 80s and the 90s to get to 100.” Joellen was using an incrementing strategy (Van de Walle et al., 2015). It seemed that at first she focused on the tens and knew that if she had 70, she needed 30 more to get to 100. She quickly changed her response to 25 because she knew the 5 in 75 would change her answer from 30 to 25. She explained that adding 5 to 75 would take her to 80. And, then knew that she needed to go through two decades to get to 100, meaning her leaps of 5 and 20 yield her answer of 25.

Discussion

This study used story problems to examine how students holistically engaged their number sense to solve a problem. An analysis of students' verbal explanations of strategies for solving four story problem situations provided insight into how students used and applied their number sense. As expected, the three students in this study used various strategies depending on the problem situation, numbers in the problem, interviewer's questions, and, in some cases, the context of the problem. Overall, it was easier to assess and understand students' number sense when they used direct modeling or invented strategies. Nevertheless, even the students' verbal explanations of the standard algorithm procedures also provided insight into their number sense and knowledge of the base-ten place-value system. The subsequent sections present each student's typical strategies and the ways they engaged with and coordinated number sense constructs.

Margaret: Moving Beyond Counting

Adaptive expertise was not evident in Margaret's story problem assessment, primarily due to lack of interconnected knowledge (Baroody et al., 2007; Baroody & Rosu, 2006). Typically, Margaret incorrectly used the standard algorithm, but often correctly solved the problem situation with a direct modeling strategy when provided scaffolding by the interviewer. During both the multiplication problem situation and the subtraction situation, I reworded the problem situations within a context that Margaret could relate to, that of boxes of cookies. In addition to providing a relatable context, I encouraged her to draw a picture in order to help her understand the problem and in order for me to see what she could do with quantities. The context and the drawings led Margaret to directly model each problem and better show her number sense and number knowledge. Previous research shows that children frequently do not understand standard algorithms (e.g., Hiebert & Wearne, 1996; Kamii, 1989), as was the case for Margaret with these problem situations. The standard algorithm for addition had been taught and valued as an important method in Margaret's school mathematics classroom, which influenced her belief in the power of this strategy. She applied the addition algorithm to all four problems. At this point in her learning, direct modeling was a more natural strategy for her and led to her success in solving problems correctly. However, she only did this when prompted by the interviewer with a context or with a prompt to draw the situation. Her need for a meaningful context and her reliance on an addition algorithm would not necessarily come out in a typical number sense assessment. The clinical interview method provided the flexibility to learn what Margaret could do and what she knows as opposed to only what she does not know.

In both her unsuccessful attempts in using the standard algorithm as well as in her success in solving problems with direct-modeling, the story problem assessment also highlighted that Margaret relied on counting using her fingers and drawings, and sometimes, counting up from the larger number. For example, as Margaret represented the subtraction situations, she counted the circles as she drew them, counted the circles as she crossed them out, then counted how many were left. When solving addition problems, Margaret typically counted up by ones from the larger number. Her counting strategies could be the starting point for helping her develop strategies based in number sense. However, Margaret's reliance on counting strategies is inefficient as she works with larger numbers. In a longitudinal study of 5- to 8-year-old students, Jordan et al. (2008) examined the change in students' frequency of finger use on number combinations in relation to change in their accuracy. They found that finger counting was positively correlated with accuracy in kindergarten (at age 5), but by the end of second grade (at age 8), there was a significant negative correlation between finger use and accuracy. Based on Margaret's strategies and verbal explanations, number sense constructs that she needed to develop include decomposing numbers and using number relationships. Development in these areas could help Margaret engage her number sense more holistically and expand her repertoire of strategies beyond only counting.

Patrick and Joellen: Invented Strategies

Both Patrick and Joellen most often engaged with a variety of invented strategies based in decomposing and composing numbers and using tens and ones. Prior research indicates that invented strategies are a characteristic of conceptual understanding, and children who use invented strategies are more successful with transferring their mathematics knowledge to new situations (Boaler, 2012; Carpenter et al., 1998; Verschaffel et al., 2009). Patrick and Joellen's invented strategies highlighted their understanding of the base-ten place-value system and strong reasoning with numbers, which were key to their success with the story problem tasks.

Though the broad category of invented strategies was a common theme for these two students, Patrick more often used benchmark numbers while Joellen's strategies involved incrementing. Though both students used invented strategies, the differences in their explanations for solving problems reflected the differences in their use of number sense. The story problem assessment provided insight into these interesting differences in the two students' approaches. For instance, Patrick used a partial products strategy to solve the multiplication situations. He decomposed the two-digit numbers and worked with tens and ones separately to make the problem easier to solve. His verbal explanations revealed number sense understandings of unitizing, place value, number relationships, and flexibility with numbers. He coordinated decomposition of numbers with grouping ideas and was able to quickly re-compose the amounts to find his solution.

Joellen used a known fact (6x10) to successfully solve a multiplication situation. Her explanation for why she used a known fact highlighted her conceptual understanding of what multiplication means. When Joellen was given a problem and she did not immediately know the solution (e.g., 5x12), she holistically engaged her number sense and coordinated among multiple constructs. First she counted by fives up to 25. She then used her sense of unitizing and doubled that amount to make ten groups of 5. She added ten to that amount because she knew she still needed to consider two more groups of 5; decomposing 12 into 10 and 2 is embedded in this last step of her strategy. In sum, the story problem assessment allowed us to see how Joellen coordinated counting, unitizing, relationships among numbers, and decomposition of numbers to solve this problem quickly in her head.

Many of Patrick's other explanations highlighted his comfort with using invented strategies and standard algorithms depending on the numbers and problem situation. This type of flexibility and ability to decide on a strategy based on the numbers is an important construct of number sense and adaptive expertise, but not easily measured. Using four different problem situations with a variety of numbers allowed me to see this aspect of Patrick's thinking and reasoning with numbers.

Joellen's verbal explanations contained key language that allowed me to see a solid understanding of place value concepts. For example, when Joellen explained her solution of 88 for 27 plus 61 she said, "Well I know that 8…I mean 6 plus 2 is so that's what I did for the tens place so that would be an 80. I know what 7 plus 1 is and that's 8." In addition, her verbal explanations also highlighted her comfort with movement among numbers and their spatial relationships, which is likely why incrementing was a common strategy in her interviews. It is important to note that this relationship between number sense concepts and base-ten place-value understandings is a symbiotic relationship. Prior research purports that invented strategies, such as those Joellen engaged in, contribute to the development of base-ten place-value understandings while the base-ten place-value understandings aid her fluency with various strategies (Carpenter et al., 1998; Fuson, 1990).

Employing and Coordinating Number Sense Constructs

In the case of these three students, the findings suggest that the students' construction of number sense concepts involved multiple aspects of number sense and entailed the coordination of number sense constructs. The number sense view and adaptive expertise theories (Baroody et al., 2007; Baroody & Rosu, 2006; Verschaffel et al., 2009) explain why conceptual understanding of numbers and operations was connected to efficient mental mathematics and strong numerical reasoning. For example, Patrick employed invented strategies that allowed him to quickly and efficiently solve several problems and explain his numerical reasoning. However, when he used a school-taught method that did not make sense to him, he had difficulty explaining his reasoning and why his solution was correct. Margaret often relied on only one construct of number sense. Her counting strategies limited her success with many story problems. Hence, the coordination of number sense constructs allowed students to make meaning of the numbers, use relationships among numbers, and successfully solve problems. The story problem tasks provided a window into how students employed and coordinated number sense constructs. Analyses of students' strategies and verbal explanations on the story problems allowed us to pinpoint key aspects of number sense, while also understand how students holistically used their number sense.

Educational Implications

The results of this exploratory study suggest that story problem tasks are a useful number sense assessment tool for classroom teachers and mathematics education researchers. Story problems are commonly used in classrooms or on tests to find out if a student can solve a computation problem correctly or incorrectly. When students' strategies and verbal explanations are analyzed for number sense constructs, teachers gain insight into how students are engaging their number sense. Such an assessment and analysis provide valuable information for planning mathematics instruction.

In addition, this assessment explored the ways students used number sense in solving story problems and provided initial codes and themes for analyzing number sense within the context of problem-solving situations. This exploratory study of three students lays the groundwork for further investigation and development of codes and themes for analyzing number sense within a holistic view. Similar to prior number sense assessments, this study also dissected number sense into constructs to pinpoint the components of number sense students used to solve story problems. The analysis in this study was unique in that it went a step further than previous number sense assessments because it sought to understand how students coordinate the components of number sense to solve story problems. The story problem assessment took a holistic look at students' use of number sense and how the various components of number sense were engaged together as students solved story problems. Knowing the *what* regarding the constructs of number sense without the knowledge and insight into *how* students engage their number sense is like having a detailed list of a construction project's building materials without being able to observe the construction crew at work – in both cases one cannot accurately decipher the steps and process that lead to the final product.

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APPENDIX

Interview: Tasks and Interview Questions

TASK#1
Equal Groups, Unknown Product
Jessica has ___ bags of cookies. There are ___ cookies in each bag. How many cookies does Jessica have all together? (5, 12, 60) (6, 10, 60) (3, 24, 72)

TASK#2
Put Together, Total Unknown
Arla has _____ red marbles and ______ blue marbles. How many marbles does she have? (27, 61, 88) (52, 36, 88) (50, 63, 113)

TASK#3
Take From, Result Unknown
Arla had _____ marbles. She gave ____ to Jessica. How many marbles does Arla have left? (18, 6, 12) (14, 8, 6) (48, 22, 26) (345, 140, 205)

TASK#4
Add To, Start Unknown
Ben had some pennies. Alex gave him _____ more. Now he has _____ pennies. How many pennies did Ben begin with? (12, 18, 30) (24, 12, 36) (25, 75, 100)

Interview Questions
Initial Questions
- How did you solve the problem?
- Tell me what you were thinking?

Probing questions based on what the child says or does
- Can you tell me how you counted?
- Can you tell me why you separated those blocks?
- Why did you start with that number when you counted?

Follow-up questions/further probing
- Can you do this problem a different way?
- What if you did this without tools and did it in your head? What would you do?
- What do you notice?

FOR DATA ANALYSIS
Strategies Children Might Use
- direct modeling
- invented strategies

  - counting strategies
  - incrementing
  - combining hundreds, tens, and ones strategy
  - compensation
  - known facts
- standard algorithm

Number Sense Focus

- counting (skip counting, counting patterns, number line)
- place value (decomposing numbers; uses hundreds/tens/ones)
- number relationships (compensation, flexibility with numbers, one number related to another)
- estimation
- quantity discrimination (knew the answer because of size of quantities)
- used a number line (concrete or mental)
- flexibility with numbers