Lexical Ambiguities in the Vocabulary of Statistics

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Abstract

Lexical ambiguities exist when two different meanings are ascribed to the same word. Such lexical ambiguities can be particularly problematic for learning material with technical words that have everyday meanings that are not the same as the technical meaning. This study reports on lexical ambiguities in six statistical words germane to statistics classes—*arbitrary*, *model*, *random*, *range*, *uniform*, and *variable*—based on data collected from an Advanced Placement (AP) Statistics course taught in the southeastern United States. Data was collected at the beginning and end of an academic year. Of the six words studied, four—*model, random, uniform,* and *variable* –were found to have substantial lexical ambiguities.

Keywords: lexical ambiguities, vocabulary, language, secondary school students

Lexical Ambiguities in the Vocabulary of Statistics

Introduction

Language matters. While many issues regarding language may be regarded as “mere semantics” in daily life, attention to the subtleties in language can be critical for ensuring understanding of complex or unintuitive ideas (Wild, 2006). Groups differing from each other in ways such as age, culture, or gender may use similar references and vocabularies but ascribe to them substantially different meaning (Wild, 2006). Arguments for the explicit researching and teaching of vocabulary related to quantitative reasoning are not new (e.g. Henkin, 1972; Austin & Howson, 1979; Rothman & Cohen, 1989), nor are the recognition of language-based misconceptions in statistics education (e.g. Utts, 2002; Rumsey, 2009). Research on language use by students learning statistics, however, is still limited with much anecdotal evidence about language-related problems known to instructors of college-level statistics (Kaplan, Fisher, & Rogness, 2010).

Literature Review

The fact that words can have many meanings should not come as a surprise to anyone. While the different meanings associated with a particular word or term may be deftly handled in everyday communication, misconceptions can arise in students encountering technical words with meanings that are related to—but distinct from—the colloquial use of the word. This phenomenon may be described as a *lexical ambiguity* or an instance of *polysemy*; both terms refer to several meanings being associated with a single word or phrase. When the vocabulary of mathematics has lexical ambiguities, the teacher may assume that they and the students share an understanding of a concept when, in fact, there is a lack of shared understanding. In mathematics education, the recognition that reading ability and language are related to mathematics learning dates back nearly a century (Monroe & Englehart, 1931). Specific attention to the impact of lexical ambiguities on classroom learning is more recent, with increased attention dating back about four decades (e.g. Nicholson, 1977; Otterburn & Nicholson, 1976).

Using a sample of 300 secondary school students from the United Kingdom, Otterburn and Nicholson (Nicholson, 1977; 1976) reported that many common mathematical words could not be satisfactorily defined by even a simple majority of participants. While Otterburn and Nicholson stop short of explaining the cause of this deficiency, Austin and Howson (1979) speculate that, for words like *multiple*—which has a common colloquial meaning in addition to the mathematical meaning—the lexical ambiguity may be a contributing factor.

Despite the potential for misunderstandings introduced by lexical ambiguities, relatively few empirical studies have been undertaken in this area in mathematics education. Exceptions to this include work with young children (ages 3 to 8) on spatial meanings of words (Durkin, Crowther, Shire, Riem, & Nash, 1985; Durkin & Shire, 1991). At this age, a *big number* is apt to be interpreted as some number—any number—written with large numerals rather than a number with a large value (Durkin & Shire, 1991). Similarly, a *high number* may be any number written near the top of the page (Durkin & Shire, 1991).

The concept of a 'variable' is fundamental to mathematics, but it is also rife with lexical ambiguities. Dogbey (2010; Dogbey & Kersaint, 2012) provides a thorough overview of the literature relating to the meanings ascribed to 'variable' in mathematics. While there are several characterizations of the different meanings and uses of 'variable,' they all recognize both uses for which a variable represents a single number and uses for which a variable can take many values, sometimes called a *varying quantity* (2010, 2014; Küchemann, 1978, 1981; Philipp, 1992; Usiskin, 1988). When examining middle school mathematics textbooks from 1957-2009, Dogbey (2010; Dogbey & Kersaint, 2012) found that the use of 'variable' as a specific unknown was more frequent than the use of 'variable' as a varying quantity. While the multiple meanings of *variable* have been studied in the context of mathematics generally, they have not been studied in the specific context of statistics which uses the word in several ways.

A variety of mathematical terms have been the subject of other notable studies. Mamolo (2010) conducted a conceptual analysis of the symbols '+' and '1' and showed that they can have several meanings. Zazkis (1998) demonstrated lexical ambiguities associated with *quotient* and *divisor* through interview excerpts. Sacco (2013) studied three high school pre-calculus students' understandings of *domain*, *range,* and *function*. While the concept of *range* that Sacco (2013, p. 12) set out to study was its meaning of "the set of all outputs of the function *f*," one of the three participants did demonstrate range's potential for lexical ambiguities by conflating its mathematical definition with a less-mathematical one.

Instances of lexical ambiguity or polysemy can present unique challenges for teachers of bilingual students because lexical ambiguities may (and do) arise with technical mathematical words being associated with words or phrases in the students' other language(s) (Austin & Howson, 1979). More than just a practical challenge for educators, instances of polysemy are an issue of equity, particularly in the context of indigenous populations (Russell & Chernoff, 2012). As students who know a language other than Standard American English learn western mathematics, there are many opportunities and challenges for both shared understanding and culturally responsive education (Gay, 2002; Russell & Chernoff, 2012).

Other work suggests that lexical ambiguities are not necessarily problematic for bilingual students and their teachers (Barwell, Barton, & Setati, 2006). Rather, the interplay between different meanings of words can be used to support students' mathematical ideas (Barwell, 2005; Moschkovich, 2003). In a review of studies of bilingual mathematics learners, Moschkovich (2006) argues that students who use two languages when learning mathematics should not be studied from a deficit perspective or only juxtaposed with monolingual students. Instead, bilingual students who use more than one language in the classroom may have cognitive advantages relative to monolingual students (Moschkovich, 2006). More recent work by Planas (2014) continues this trend of research with bilingual students that does not emphasize difficulties. By viewing language as a resource, Planas illustrates how bilingualism can lead to mathematical learning opportunities for students. This growing body of research indicates that bilingual students and their teachers have both challenges and opportunities for mathematical learning, but these students must be viewed as mathematical learners in their own right rather than viewed as lacking traits associated with monolingual students.

In statistics education, much of the work on lexical ambiguities has been undertaken by Kaplan et al. (2009, 2010). As part of a multi-stage research project exploring aspects of language in introductory statistics courses, Kaplan et al. published results related to use and definitions of *association, average*, *confidence*, *random*, and *spread*. In their study, a pilot sample consisting of 41 college students with a median age of 19 from one institution and a large-scale sample of 777 students from two institutions were used; results were based on a random sample of responses from 100 of the college students in the large-scale sample. Students completed a questionnaire near the end of the semester (Kaplan et al., 2009) that asked them to define and use in a sentence the everyday and statistical meanings of three of the aforementioned five words. The choice of three words was to reduce the time required to complete the questionnaire, and 35 versions of the questionnaire were used to ensure adequate coverage and different orders of the words (Kaplan et al., 2010). Germane to this study are the findings related to *random*.

In their study with college students, Kaplan et al. (2010) found that only 5% of the 60 students were able to give a correct definition of the word *random*. The remaining students gave responses that were coded as “By chance (vague),” “Without order or reason,” “Unexpected, not predictable, not planned,” or “Without bias, representative, fair.” Based on this work, Kaplan et al. (2014) developed an intervention to convey the statistical meaning of *random.*

There are many statistical words with a potential for lexical ambiguity. In their paper, Kaplan et al. (2009) provide a table with 36 words that they suspect have lexical ambiguity. Pierce and Fontaine (2009) provide an additional six words which they suspect have lexical ambiguity, namely *line plot*, *bar graph*, *data, estimate, pattern, pictograph,* and *model* in their table of words with potential lexical ambiguities. Watson and Kelly (2008) report that *variation* may have lexical ambiguities. Working with students in a Hebrew language statistics course, Lavy and Mashiach-Eizenberg (2009) report that the Hebrew equivalents of *life expectancy, standard deviation, expected value,* and *significance level* have suspected lexical ambiguity. After working with students and reading the statistics education literature, this researcher believes that *arbitrary, conditional, consistent, given, individual, information, outlier, probability, significant, uniform, variability,* and *variable* are still more words which may have lexical ambiguity in the context of statistics. These words are provided in the same vein as Kaplan et al. (2009) and Pierce and Fontaine (2009), i.e. in an attempt to make visible to researchers and teachers as many possible lexical ambiguities as can be identified even though they are not necessarily empirically studied here.

Methodology

The research question motivating the study was, "What meanings do students ascribe to statistical words (*arbitrary*, *model*, *random*, *range*, *uniform*, and *variable*) that have other, non-statistical meanings?" This study was conducted using a constructivist epistemology (Crotty, 1998; Ernest, 1994; Thompson, 2014) wherein students' responses drove the analysis rather than beginning with conceptions grounded in the literature or the researcher's ideas. Data was collected from students in an Advanced Placement (AP) Statistics course. Inductive analysis (Hatch, 2002) served as the basis for the data analysis.

Participants

Twenty-four students enrolled in an AP Statistics course served as participants in this study. The academic year in which this study was conducted was the first year that the AP Statistics course was offered. All but a few students had completed Algebra II prior to entering the course, and the remaining students were co-enrolled in both AP Statistics and Algebra II. Students' ages were between 14 and 18. Of the 21 students who sat for the AP Statistics exam from this class, 18 received passing scores.

The curriculum of the AP Statistics course is designed to be comparable with the consensus-curriculum (Tintle, VanderStoep, Holmes, Quisenberry, & Swanson, 2011) introductory statistics course taught at colleges and universities in the United States (Roberts, Scheaffer, & Watkins, 1999; Scheaffer, 1997). Major topics covered in AP Statistics include *Exploring Data*, *Sampling and Experimentation*, *Anticipating Patterns*, and *Statistical Inference* (College Board, 2010).

Procedure

Students were asked to define and use in a sentence words that may have lexical ambiguities throughout the academic year. Words were chosen from suggestions given in Kaplan et al. (2009), Pierce and Fontaine (2009), Lavy and Mashiach-Eizenberg (2009), and the author's own experiences. Because the list of words of interest included 59 terms, only for a small subset was pre- and post-instruction data collected: *arbitrary*, *model, random, range, uniform,* and *variable*. Limiting the study to a subset of terms was done to lessen this study's disruption to the instructional time. These six were chosen for this study because the author noticed some students using them (with the exception of arbitrary) and because they received considerable attention throughout the course (more so than other terms such as *margin* and *pictograph*).

The extent to which these terms have been previously studied varies. *Model*, though it was included in Pierce and Fontaine's (2009) table of words with potential lexical ambiguities, was not part of an empirical study they conducted. *Range* (Sacco, 2013), *random* (Kaplan et al., 2009, 2010, 2014), and *variable* (Dogbey, 2010; Dogbey & Kersaint, 2012; Küchemann, 1978; Usiskin, 1988) have been previously studied. However, Sacco (2013) studied *range* along with *domain* in the context of functions, not in the statistical sense studied in this paper. *Random* was previously studied in its statistical sense (Kaplan et al., 2009, 2010, 2014) and was included to allow for comparisons to be made between the participants in this study and previous studies. The different uses of *variable* have been studied (Dogbey, 2010; Dogbey & Kersaint, 2012; Küchemann, 1978; Usiskin, 1988), though not in the context of its particular use in statistics. Both *arbitrary* and *uniform* were identified by the researcher as having potential lexical ambiguities and rather than having been identified by the literature previously discussed.

For each underlined word, write a **definition** in your own words and use the word in a **sentence**. Variable Definition: Sentence:
Define and use in a sentence the word variable as you use it in **daily life**. Definition: Sentence: Define and use in a sentence the word variable as it is used in **statistics**. Definition: Sentence:

Figure 1. An example of the questionnaires used in this study. The first (top) half represents the type used in the pre-phase of data collection, and the second (bottom) half represents the type used in the post-phase of data collection.

Prior to students encountering the word in the course, they were given a questionnaire (example given in Figure 1) asking them to define and use in a sentence some of the words. Asking students to define words and use them in a sentence in both the statistical and daily-life contexts was inspired by Kaplan et al. (2009, 2010). Students were verbally instructed that they should provide the everyday meanings of the words (as they ostensibly should not have developed a well-formed statistical meaning prior to the instruction). This same procedure was near the end of the course after the AP Statistics Exam had taken place.

Data Analysis

The data was analyzed using an iterative coding process (Miles & Huberman, 1994). Specifically, after typing the data, all of the definitions and sentences were read. Then, the researcher assigned each definition and sentence a code or codes that summarized the salient points. These provisional codes were derived from the data rather than the literature or other sources of meanings. For each word, this resulted in a list of many provisional codes. Many of these initial codes were so specific as to only apply to one sentence or definition, thus limiting their utility. The data for each term word was reread and the sentences and definitions were recoded, this time seeking somewhat more general codes which applied to more than one definition or sentence well. For example, the initial coding for *range* resulted in 17 different codes such as; a subset of these from the pre-instruction definitions are shown in Table 1, all of which were later assigned the code “Formula.” Note that the definitions in Table 1 are not equivalent, but they all specify some type of formula for identifying the maximum—regardless if this formula is correct. After broader codes had been established, the data was reread seeking more specific codes, which applied to more than one definition or sentence. When codes could not be sufficiently grouped under a broader code, an “Other” code was finally assigned. This process established the final codes presented below.

Table 1. Examples of original codes for *range* from pre-instruction definitions. Each of these codes was later collapsed under the broader code “Formula.”

ID	Definition	Original Code
03	The largest number minus the smallest number in a set.	Correct Statistics Definition
16	The highest number in a set of numerical data.	Maximum
19	The difference between the first & the last number	Difference
21	The distance between two numbers.	Distance

For this study, the researcher interpreted students’ written responses which were brief snippets that cannot fully capture the various meanings and conceptions that students ascribe to these terms. Many responses were clear and easily understood by the researcher. However, there were some responses whose meaning could not be understood by the researcher; such responses are categorized as “unknowable by the researcher” (UR). This category was reserved for responses which were not blank and consisted of more than stray marks on the paper. Examples of UR responses include just writing the term to be defined, when handwriting could not be discerned by the researcher or student, and other off-task activities.

For reporting results, all students were assigned a random ID number that is the same across words, i.e. Student 01 represents the same person in each of the six word sections. All responses are included verbatim without fixing spelling or grammar mistakes. Names which were included by students in their responses (e.g. including the name of a friend in a sentence) were changed to protect anonymity of participants. Some students gave multiple responses which were coded independently; therefore, some counts of codes do not sum to the displayed total. In the selected responses included in the tables below, student responses are reported verbatim, including any capitalization, punctuation, misspellings, etc.; errors are indicated using [sic].

Results

Results illustrating the lexical ambiguities ascribed to each of the six terms—*arbitrary, model, random, range, uniform,* and *variable*—are now presented. A table illustrating the counts of each code and example student responses accompanies each term.

Arbitrary

In proof-based mathematics courses at the college level, one may encounter the word *arbitrary* when the specific value in question is not important. Due to the numerous definitions ascribed to *variable* that are discussed above, students' understanding of *arbitrary* was investigated as a potential alternative word to *variable* in certain contexts. Responses, both before and after instruction, indicated overwhelmingly that students do not know the word *arbitrary*. While a few students knew the word as a synonym of *capricious*, many responded with remarks which were not understandable by the researcher (UR) or by explicitly saying that they did not know the word. Total counts for each code are given in Table 2 and examples of responses are given in Table 3. The instructional staff for the course did not make special efforts to use or teach using the word *arbitrary* which explains the lack of change or growth in students' understanding of it. Instead, we note that these students were generally unfamiliar with the word *arbitrary* and so it has few, if any, lexical ambiguities associated with it in secondary school students. This may make *arbitrary* a good candidate for use in some statistics courses when referring to certain types of variables that are not typically thought of as either independent or dependent.

Table 2. The number of responses for each code for *arbitrary*.

	Pre		Post			
Code	Definition	Sentence	Daily Life Definition	Daily Life Sentence	Statistics Definition	Statistics Sentence
Did Not Know	6	2	1		1	
Unimportant	4	2	2		2	
Unnecessary	2	1	1	1	2	1
Random			4		3	
Other	4	5	7	6	4	11
UR	3	5		7	1	
Total	19	15	15	14	13	12

Note: Zeroes are omitted for clarity.

Table 3. Examples of how responses were coded for the word *arbitrary*.

		Pre		
ID	Context	Prompt	Code	Response
01	N/A	Definition	Unimportant	Of no importance. Should be disregarded.
04	N/A	Definition	Did Not Know	I don't even know
16	N/A	Definition	Other	not pertaining to the expectation
20	N/A	Definition	Unimportant	Irrelivent[sic] or redundant
05	N/A	Sentence	UR	You're[sic] existence is arbitrary.
16	N/A	Sentence	Other	The favorite foods listed were rather arbitrary.
20	N/A	Sentence	Unimportant	Some details are considered arbitrary for simplicity.
		Post		
ID	Context	Prompt	Code	Response
05	Daily Life	Definition	Other	on purpose
08	Daily Life	Definition	Other	based on choice or whatever
21	Daily Life	Sentence	Other	The numbers were arbitrary.
22	Daily Life	Sentence	UR	I arbitrarily hit a homerun.
07	Statistics	Definition	Unnecessary	not needed for distinction
20	Statistics	Definition	Other	unrelated or spurrius[sic] correlation
21	Statistics	Definition	Random	random, chosen by chance
12	Statistics	Sentence	Other	The outlier was arbitrary.
21	Statistics	Sentence	Other	We arbitrarily chose 8

Model

Model-building is one key aspect of statistics. In an introductory statistics course, students are exposed to several types of models including probability models and regression models. However, students may not perceive these tools as part of a modeling process that explains real life. Based on the responses, five different meanings of *model* were apparent: as Examples, used Generally, as Objects, as Visual aids, and as People. The codes Object and Visual are similar, but enough responses of each dictated that they be considered separately. Total counts for each code are given in Table 4 and examples of responses are given in Table 5.

Table 4. The number of responses for each code for *model*.

	Pre		Post			
Code	Definition	Sentence	Daily Life Definition	Daily Life Sentence	Statistics Definition	Statistics Sentence
Example	2		4		2	
General	6	7	7		10	10
Object	2	7	6	11	1	
Visual	7	4	2		4	2
Person		2	4	8		
Other	1				2	6
UR	3			2	1	
Total	21*	20	21*	21	19*	18

Note: Totals marked by a * indicate the column does not sum to the total because some responses received multiple codes. Zeroes are omitted for clarity.

Table 5. Examples of how responses were coded for the word *model*.

Pre				
ID	Context	Prompt	Code	Response
08	N/A	Definition	General	Something that represents something else
16	N/A	Definition	Example	An example.
23	N/A	Definition	Other	A simulation of something.
20	N/A	Sentence	Person	Jeff is a model for iphones.
07	N/A	Sentence	Visual	The pie chart is a model of the number of students who liked music

Post				
ID	Context	Prompt	Code	Response
03	Daily Life	Definition	General	A representation of some phenomena
14	Daily Life	Definition	Example	Demonstration/Example
22	Daily Life	Sentence	Object	I have a model of the solar system in my science room.
01	Statistics	Definition	General	A representation of something statistical
09	Statistics	Definition	Visual	A graph or visual example of data.
17	Statistics	Definition	Other	the small version of the experiment
03	Statistics	Sentence	Other	We flipped coins to model the probability of having a male or female child.
09	Statistics	Sentence	Other	The model show's[sic] the regression equation

Even in the data collection after instruction, responses were generally vague and referred to physical objects and other visual aids. However, six students explicitly referenced regression or probability models in their sentences in the Statistical context suggesting that some students were beginning to understand statistics as a model-building activity.

Random

Of the six words included in this study, *random* elicited the greatest variety of responses. Many of the senses in which *random* was defined or used in a sentence had only one or two uses among all of the responses, so the Other category is larger for this word than in the other words. Still, there are more codes for *random* than for the other words because students repeatedly used the word in related but distinct senses which would be lost by collapsing codes into a single cover word.

Prior to instruction and in the Daily Life context after instruction, students tended to define *random* as something that was unexpected, unpredictable, or had no pattern, order, or relationship. In the Statistics context after instruction, students tended to define *random* as something that is by chair, has a fair selection process, or is unbiased. When using *random* in a sentence, two uses dominated in all contexts: slang and in reference to random selection or assignment. Total counts for each code are given in Table 6 and examples of student responses are given in Table 7.

Table 6. The number of responses for each code for *random*.

	Pre				Post	
Code	Definition	Sentence	Daily Life Definition	Daily Life Sentence	Statistics Definition	Statistics Sentence
By Chance			1		6	
Fair					4	
Unexpected	1		8			
Unpredictable	3		2			
Unbiased	1				6	2
No Pattern, Order, or Relationship	6		3		2	
Random Selection, or Assignment		4			3	16
Slang		3		18		
Other	9	10	9	3	2	3
UR			2		1	1
Total	19*	17	21*	21	20*	20*

Note: Totals marked by a * indicate the column does not sum to the total because some responses received multiple codes. Zeroes are omitted for clarity.

Table 7. Examples of how responses were coded for the word *random*.

ID	Context	Prompt	Code	Response
			Pre	
ID	Context	Prompt	Code	Response
01	N/A	Definition	Unpredictable	When a possibility is impossible to predict all the time
03	N/A	Definition	Unbiased	Chosen without bias or preference, no thought to it
12	N/A	Definition	No Pattern, Order, or Relationship	not in any order
19	N/A	Definition	Other	Something chosen for no particular reason.
18	N/A	Definition	Other	Something spontanious[sic]
07	N/A	Sentence	RSA	The pool of students was randomly selected
16	N/A	Sentence	Other	The results of the survey were random.
			Post	
ID	Context	Prompt	Code	Response
06	Daily Life	Definition	Unexpected	something that happens unexpectedly
19	Daily Life	Definition	Unpredictable; Other	Unpredictable or out of the ordinary
24	Daily Life	Definition	Other	Something that happened out of the blue
01	Daily Life	Sentence	Other	When you flip a coin the outcome is random.
02	Daily Life	Sentence	Other	we will make the driver random so you don't know who it is
12	Daily Life	Sentence	Slang	He randomly started dancing
13	Statistics	Definition	Fair	Every individual having the same probability of being selected
06	Statistics	Definition	RSA	Randomization to make inference or do cause and effect or exemplify the population.
12	Statistics	Definition	By Chance	happens by chance
23	Statistics	Sentence	Other	I plotted the random variable.
19	Statistics	Sentence	RSA; Unbiased	Data should be collected randomly to eliminate bias.

Note: RSA indicates "Random Selection or Assignment."

Based on the responses given in Table 7, randomness is a difficult concept for students to define precisely and accurately. Of the 21 students who sat for the AP Statistics exam from this class, 18 received passing scores. This is evidence that the students in this class demonstrated learning of college-level introductory statistics and yet still possessed misconceptions about randomness, a fundamental statistical concept. In an introductory statistics course such as AP Statistics, randomness is closely connected with its use in random selection and random assignment preceding hypothesis tests. In these cases, randomization tends to be fair—giving an equal chance of selection to each item in the sample space or sampling frame—and so students may come to think that all randomization must be so. Similarly, it is possible that, due to the limited exposure students have to biased estimators in an introductory course, students may come to equate unbiasedness as a property of the randomization process instead of as a property of the estimator they are using.

Range

In statistics, the *range* is a single value that measures the variability in a dataset and is computed by taking the difference between the largest and smallest values. In general, students in this study were familiar with this use of *range* before entering the course. However, many students limited their responses to just discussing the formula or aspects thereof and stopped short of using the range in a context as would befit its use to measure variability. When coding responses, two primary codes emerged: Formula, for focusing on computational aspects, and Variability, when connecting the range in some way to measuring variability. Total counts for each code are given in Table 8 and examples of student responses are given in Table 9.

Table 8. The number of responses for each code for *range*.

	Pre		Post			
Code	Definition	Sentence	Daily Life Definition	Daily Life Sentence	Statistics Definition	Statistics Sentence
Formula	10	8	5	2	10	9
Variability	5	5	3	4	4	4
Other	4	3	6	8		
UR	2	3	1	1	1	1
Total	20*	19	14*	15	14*	14

Note: Totals marked by a * indicate the column does not sum to the total because some responses received multiple codes. Zeroes are omitted for clarity.

Table 9. Examples of how responses were coded for the word *range*.

ID	Context	Prompt	Code	Response
		Pre		
01	N/A	Definition	Formula	All numbers from lowest to highest
08	N/A	Definition	Variability	how much variety something has.
13	N/A	Definition	Other	y value
08	N/A	Sentence	Other	His vocal range was spectacular.
04	N/A	Sentence	Variability	The range of school grades is 0-100.
13	N/A	Sentence	Other	The domain and range is (4,3).
		Post		
ID	Context	Prompt	Code	Response
11	Daily Life	Definition	Formula	(max-min)
08	Daily Life	Definition	Variability	spread from begining[sic] to end.
02	Daily Life	Sentence	Other	I shoot guns at the range
19	Daily Life	Sentence	Variability	Her phone conversations range from 5 min - 60 min
04	Statistics	Definition	Formula	The highest value - lowest value
16	Statistics	Definition	Variability	How many numerical values a set of data can take on.
03	Statistics	Sentence	Variability	The range of my Boxplot helped my compare it to the other Boxplot
16	Statistics	Sentence	Formula	The lowest value was 1, the highest was 5, the range is 4.

Several students gave responses that did not correctly define or use *range* as a single value, for example, “All numbers from lowest to highest” and “The range of school grades is 0-100.” Both of these responses, found in Table 9, illustrate partial understanding of the range: namely that the highest and lowest values are important. This partial understanding can be leveraged to teach students that the range, as used in statistics, is a single number that takes into account the highest and lowest values to measure variability.

Other than the incorrect specification of the formula for or use of the range, there were not many other uses of *range* given. Notably, the use of *range* along with *domain* when teaching functions was present in only a single student's response at the beginning of the academic year. This suggests that students are capable of compartmentalizing differing uses of mathematical words, particularly when the definitions are more than subtly different.

Uniform

In introductory statistics courses such as AP Statistics, treatment of bell-shaped distributions such as the Normal and t-distributions frequently appear in the curriculum. As such, students may have limited exposure to the uniform distribution. In this class, three meanings of *uniform* dominated: sameness, evenness, and clothing. Responses that were coded as *Same* focused on data that lacked variability, as in "The data was uniform, with no variation" and "same throughout." Responses that were coded as *Even* focused on data that had followed a particular pattern, as in "distributed evenly throughout." When used in sentences, students frequently described Graphs or Distributions, leading to two more major codes. Total counts for each code are given in Table 10 and examples of student responses are given in Table 11.

Table 10. The number of responses for each code for *uniform*.

	Pre		Post			
Code	Definition	Sentence	Daily Life Definition	Daily Life Sentence	Statistics Definition	Statistics Sentence
Same	9	4	8	3	9	1
Even	3				5	
Clothes	1	2	10	14		
Graph	1	6				8
Distribution		1		1	1	8
Other	4	1	1		4	1
Total	16*	14*	18*	18	18*	18

Note: Totals marked by a * indicate the column does not sum to the total because some responses received multiple codes. Zeroes are omitted for clarity.

Table 11. Examples of how responses were coded for the word *uniform*.

ID	Context	Prompt	Code	Response
		Pre		
ID	Context	Prompt	Code	Response
17	N/A	Definition	Even; Graph	The eveness[sic] and smoothness of data or graph
10	N/A	Definition	Even	Neat, even.
02	N/A	Definition	Other	a pattern
07	N/A	Definition	Other	There's no pattern
19	N/A	Definition	Other	A set of data that is closely related to each other.
01	N/A	Sentence	Same	The mixture was uniform
16	N/A	Sentence	Graph; Other	The graph was uniform, because most of the data was similar.
		Post		
ID	Context	Prompt	Code	Response
07	Daily Life	Definition	Same	same throughout
09	Daily Life	Definition	Other	Straight and well kept together.
06	Daily Life	Sentence	Clothes	Dang, those are some sweet uniforms; I like the orange pants
24	Statistics	Definition	Same	Completely the same
10	Statistics	Definition	Even	Even, flat.
19	Statistics	Definition	Other	Equal to, no variance
03	Statistics	Sentence	Same	The data was uniform, with no variation
19	Statistics	Sentence	Graph	A histogram of data showed that the data was uniform.
01	Statistics	Sentence	Graph	The bar graph was uniform in shape
24	Statistics	Sentence	Distribution	The distribution is uniform [student then included a rough sketch of a uniform histogram]

The use of *uniform* by school-aged children to refer to clothing is not surprising, particularly in the Daily Life context. The other primary uses—referring to sameness or evenness—represent competing ideas. While the use of *uniform* in the sense of sameness was found in all response contexts, its use in the sense of evenness was only found in the Statistics context. One possible explanation for this is that, in a histogram showing a uniform distribution, the bars will all be (about) the same height and will look even. However, the student who correctly sketched a histogram for a uniform distribution as part of their response for the

Sentence prompt defined *uniform* as "Completely the same." The responses therefore indicate a tension between different meanings of the word *uniform*.

Variable

After repeated reading and coding of both the pre and post data, the following response categories were established: Mathematics, Science, Other, and UR. Mathematics refers to a student defining or using in a sentence the word *variable* in a manner consistent with its typical use in mathematics classes, i.e. as one of the aforementioned uses such as a number to be evaluated or a placeholder for a number. Statistics refers to use typical in science classes, i.e. as a factor influencing another factor. Total counts for each code are given in Table 12, and examples of student responses are given in Table 13.

Table 12. The number of responses for each code for *variable*.

	Pre		Post			
Code	Definition	Sentence	Daily Life Definition	Daily Life Sentence	Statistics Definition	Statistics Sentence
Math	18	17	9	9	7	
Science	2	1	3	4	6	10
Other		1	5	4	3	5
UR		1				
Total	20	20	17	17	16	15

Note: Zeroes are omitted for clarity.

Table 13. Examples of how responses were coded for the word *variable*.

		Pre		
ID	Context	Prompt	Code	Response
7	N/A	Definition	Mathematics	An undefined number in an equation represented by a letter.
15	N/A	Definition	Mathematics	An undefined value.
1	N/A	Definition	Science	A factor that affects the outcome of a situation
12	N/A	Sentence	Mathematics	The variable, x, represented 2.
13	N/A	Sentence	Mathematics	The equation contains the varible[sic] x.
1	N/A	Sentence	Science	The independent variable is the variable that is purposely changed.
		Post		
ID	Context	Prompt	Code	Response
6	Daily Life	Definition	Mathematics	something you are looking for, a letter in an equation
9	Daily Life	Definition	Science	factor, part of something
16	Daily Life	Sentence	Mathematics	The variable in the equation is unknown.
1	Daily Life	Sentence	Science	There's so many variables to take into account
13	Statistics	Definition	Other	How much it varies (spread)
6	Statistics	Definition	Science	something that has an effect on the study
17	Statistics	Sentence	Science	The variable was not the same in the experiments
14	Statistics	Definition	Mathematics	A value that is solved for
18	Statistics	Sentence	Other	The data was very variable

At the beginning of the course, students overwhelming defined and used in sentences *variable* in a way that is consistent with its use in mathematics. At the end of the course, the scientific use of *variable* was more represented than at the beginning. This is not surprising given that students were enrolled in a statistics class during the intervening time. What is particularly notable is that, when asked how they used *variable* in statistics class, many students defined it as used in mathematics but used it in a sentence as in science.

Summary

For five of the statistical words described above (excluding *arbitrary*), lexical ambiguities were present and described. While these five terms each have a precise statistical meaning, many definitions and sentences for the statistics context of the questionnaire collected at the end of the course, were vague or otherwise incorrect. However, students' responses and course performance suggest that they are able to be successful despite the persistence of—or in some cases new instances of—lexical ambiguities at the post-instruction data collection. Overall, these results illustrate a variety of lexical ambiguities present for the selection of statistical terms.

Discussion and Conclusion

Students' languages are complex and living, leading to lexical ambiguities. In many everyday circumstances, nuances of meaning may not be important, and lexical ambiguities are not problematic. When using technical language, particularly by novices, lexical ambiguities may lead to an unnoticed lack of shared understanding between students and the instructor. Building upon the work of Kaplan et al. (2009, 2010), this work represents an attempt to expose further lexical ambiguities that may appear in the statistics classroom. With the widespread adoption of the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and related standards and curricula, statistical content represents about one-fifth of the mathematics standards in grades 6-12 (Usiskin, 2014). The introduction of this new material into the framework of existing mathematics classrooms may lead to more lexical ambiguity-fostered miscommunication between students and teachers if assumptions about the meaning of technical words are not made explicit.

In this study, lexical ambiguities were shown to exist with the words *model*, *random*, *uniform*, and *variable*. Corroborating the findings of Kaplan et al. (2009, 2010, 2014) with college students, *random* has a particularly common use as a colloquial word with a meaning similar to its use in statistics, and few students seem to know a solid technical definition for it. *Range* was not found to have any strong lexical ambiguities, though many students exhibited a misconception that the range is not one value. Likewise, *arbitrary* was not found to have lexical ambiguities, though this is attributed to students' general unfamiliarity with the word; this suggests that it may be a candidate for use in mathematics and statistics classrooms to address the many competing definitions of the word *variable*.

Of the 21 participants in this study who sat the AP Statistics exam, 18 passed. This passing rate (85%) was considerably higher than the national average passing rate of 60% for the exam that year (The College Board, 2014), yet many of the students still demonstrated difficulty with terms in this study by giving vague or incorrect definitions and sentences. While the specific reason for this is not known, several factors may contribute to this tension. First, the level of mastery of statistical terms for students not included in this study is unknown—perhaps these participants, even without a complete understanding of statistical terms, still compare favorably to other students. Of course, this cannot be known without much more data. Unlike the AP Statistics Exam with its possibility of college credit, participation in this research study was not high-stakes for students. The researcher and instructional staff asked students to take their responses to the questionnaires seriously—and students appeared to do so—but there is a tremendous difference in context between low-stakes, in-class questionnaires and a high-stakes national assessment. Lastly, the AP Statistics Exam assesses proficiency with solving statistical problems, and perhaps lexical ambiguity was not problematic for students within the defined context of the exam.

There are implications for this research in the areas of pedagogy and assessment. In the area of pedagogy, these results illustrate that students made appreciable gains in their understanding of statistical terms even without direct instruction. Students—at least at the AP-level—may not need direct instruction in statistical terms to be successful on traditional course outcomes. This does not apply to all statistical terms, however. Students' use of the word *random* still exhibited misconceptions about the nature of statistical randomness even though they distinguished it from the colloquial usage. Activities should be developed which demonstrate randomness in more complicated ways than situations which are predicated upon equal (or fair) probabilities for all units under consideration. As statistical terminology is studied further, more words for which such activities would be appropriate will likely be identified.

That the students who participated in this study performed well on the AP Statistics Exam and yet still demonstrated lexical ambiguities about important statistical terms has implications for the teaching of subsequent courses, too. These results illustrate that a single introductory statistics course may not be sufficient for ensuring that students have a command of statistical language. Therefore, teachers of subsequent courses—either further statistics courses or courses for which statistics is a prerequisite—should be aware of the manner in which students are using statistical language to avoid miscommunications and misconceptions arising. This recommendation is particularly important for institutions where many students have earned college credit through a high school statistics course such as AP Statistics: not only is it possible that students have not fully mastered the meanings of statistical terms, but many courses that students collectively took may result in a wide variety of meanings within a single class. Future research could investigate how students' understanding of statistical terms and their meanings develop over time.

While this study focused on students, lexical ambiguities may be particularly problematic for teachers of statistics. In the United States, repeated calls have been made to increase the statistical preparation of mathematics teachers (e.g. Conference Board of the Mathematical Sciences, 2001, 2012). Current recommendations call for several statistics courses—different from a traditional introductory statistics course—for pre-service middle and high school teachers

(Conference Board of the Mathematical Sciences, 2012; Franklin et al., 2015). However, the preparation that many teachers have received has not been consistent with these calls and recommendations, resulting in many mathematics teachers being tasked with teaching statistics content without sufficient preparation (Conference Board of the Mathematical Sciences, 2012; Franklin et al., 2011; Franklin et al., 2015). The meanings ascribed to statistical terms by such teachers are likely to result in lexical ambiguities which may be different from the lexical ambiguities experienced by their students. Terms like *variable* have different meanings in mathematics and statistics in addition to daily life and may be an unanticipated challenge for some teachers. While data has not yet been collected on pre- and in-service statistics teachers' lexical ambiguities, training teachers to be aware of the lexical ambiguities—both for them and their students—may result in improved statistics pedagogy.

The results of this study further suggest that assessments, particularly research instruments, may benefit from attention to potential lexical ambiguities associated with terms used in items. By not attending to potential lexical ambiguities, items may unintentionally assess several concepts simultaneously, i.e. an item assessing students' understanding of a concept and the statistical terms used to describe it. While an examinee's understanding of statistical terms may be of interest to researchers and rightly assessed, students may respond to some items in ways that indicate that they do not understand a concept when they do if they are confused by lexical ambiguities associated with some terms. With the increasing trend of computer- and internet-based assessments, novel ways of presenting items to students may be devised to ameliorate these issues (e.g. dynamically providing students with one or more meanings for a term).

This study is not without limitations, particularly the limited nature of the data collection afforded by the questionnaires pictured in Figure 1. These questionnaires were based on the design of Kaplan et al. (2010) who used them with much larger sample sizes. With smaller sample sizes, better options might include asking students to write *all* definitions that they can think of instead of artificially restricting them to two. In addition to asking for definitions, Sacco (2013) asked students to rate their confidence in their definition. Other improvements might include collecting public social media posts for colloquial uses and individual and group interviews with students.

Additionally, the meanings ascribed to terms in the mathematical register is but one aspect of mathematical learning (Moschkovich, 2002). As more students and teachers encounter statistics and its register, future studies focusing on student discourse (Gee, 1999; Moschkovich, 2002, 2007) will allow for a richer, more authentic understanding of the opportunities and challenges afforded by language in the statistics classroom.

Prior to data collection, Küchemann's (1978) categorization of the different uses of *variable* was going to be applied to students' responses in addition to establishing codes derived from the data. In this categorization, the literal symbol is identified as having one of six roles: letter evaluated, letter ignored, letter as object, letter as specific unknown, letter as generalized number, and letter as variable (Dogbey, 2010; Küchemann, 1978, 1981). However, student responses were generally too imprecise to make this a meaningful activity (see above examples of student responses). Future directions for research on lexical ambiguities include collecting richer data which will enable connections to be made with existing categorization schemes to better understand how students use words in specific contexts. In the absence of such categorization schemes, richer data may allow for their creation.

Due to both changing curricular emphases and the natural evolution of language, lexical ambiguities will always be present in the mathematics classroom. While the high school students in this study had similar ambiguities to the college students in Kaplan et al.'s (2009, 2010) study, this may not always be the case. Understanding the language of each classroom and individual is the only sure way to recognize lexical ambiguities, develop interventions, and begin to help students avoid misconceptions and miscommunication. As statistics continues to have an increasing role in mathematics standards and in daily life, students are likely to enter the classroom with more lexical ambiguities for statistical terms. A deeper understanding of how students and their teachers negotiate these multiple meanings is necessary to ensure students are successful in mathematics, statistics, and in the real world.

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