10 Equations

10.12A Algebraic Solution of Simultaneous Equations – One Linear and One Quadratic Function

In Section 10.8 you have solved simultaneous equations where both of the equations are *linear*. In this section we extend this to solving simultaneous equations where one equation is linear and the other is *quadratic*. This will normally give you a quadratic equation to solve.

Worked Example 1

Solve the simultaneous equations

$$y = x^2 - 1 \qquad (1)$$

$$y = 5 - x \qquad (2)$$

Solution

Subtract equation (2) from equation (1).

$$y = x^2 - 1 \qquad (1)$$

$$y = 5 - x \qquad (2)$$

$$0 = \left(x^2 - 1\right) - \left(5 - x\right) \qquad (2) - (1)$$

This equation simplifies to

$$0 = x^2 - 1 - 5 + x$$

so $\quad 0 = x^2 + x - 6$

We now solve this quadratic equation by factorisation.

$$0 = \left(x + 3\right)\left(x - 2\right)$$

so $\quad x + 3 = 0 \text{ or } x - 2 = 0$

therefore $\quad x = -3 \text{ or } x = 2$

These values of x are now substituted into one of the original equations to find the corresponding values of y. It is usually easiest to use the linear equation for this.

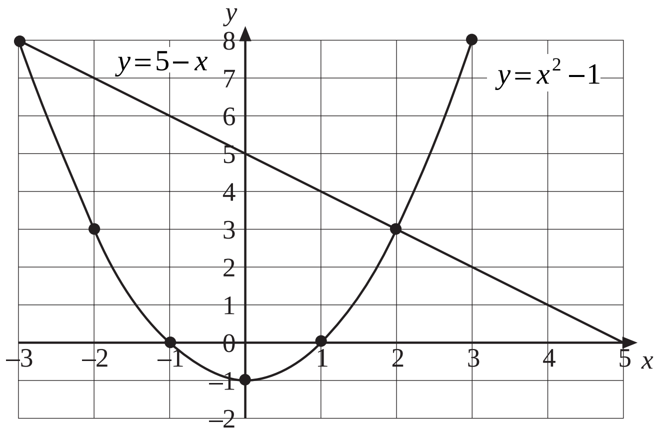
Substituting the first solution $x = -3$ into equation (2) gives $y = 5 - \left(-3\right) = 8$.

Substituting the second solution $x = 2$ into equation (2) gives $y = 5 - 2 = 3$.

The solutions are $x = -3$, $y = 8$ and $x = 2$, $y = 3$.

Note

In Worked Example 1, the first equation, $y = 5 - x$, can be represented by a straight line graph. The second equation, $y = x^2 - 1$, can be represented by a quadratic curve. When we solve this pair of simultaneous equations, we are finding the coordinates of the two points where the line and the curve intersect. This is shown on the graph, the intersection points being (–3, 8) and (2, 3).



Worked Example 2

Solve the simultaneous equations

$$y = 3x^2 - 4 \qquad (1)$$

$$y = 2x + 3 \qquad (2)$$

Solution

Subtract equation (2) from equation (1).

$$y = 3x^2 - 4 \qquad (1)$$

$$y = 2x + 3 \qquad (2)$$

$$0 = \left(3x^2 - 4\right) - \left(2x + 3\right) \qquad (2) - (1)$$

This equation simplifies to

$$0 = 3x^2 - 2x - 7$$

which does not factorise, so we need to use the quadratic equation formula to find the two possible values of x.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 3 \times (-7)}}{2 \times 3} = \frac{2 \pm \sqrt{4 + 84}}{6} = \frac{2 \pm \sqrt{88}}{6}$$

so $x = 1.896805253$

$= 1.90$ (to 2 d.p.)

or $x = -1.230138587$

$= -1.23$ (to 2 d.p.)

Substituting the un-rounded values for x into the linear equation (2) gives

$$y = 2x + 3 = 2 \times 1.896805253 + 3 = 6.793610507 = 6.79 \text{ (to 2 d.p.)}$$

and $y = 2x + 3 = 2 \times -1.230138587 + 3 = 0.539722826 = 0.54$ (to 2 d.p.).

The solutions are $x = 1.90$, $y = 6.79$ and $x = -1.23$, $y = 0.54$ (to 2 d.p.).

Worked Example 3

Solve the simultaneous equations

$$y = x^2 + 2x - 3 \qquad (1)$$

$$y = 2x \qquad (2)$$

Solution

Subtract equation (2) from equation (1).

$$y = x^2 + 2x - 3 \qquad (1)$$

$$y = 2x \qquad (2)$$

$$0 = \left(x^2 + 2x - 3\right) - \left(2x\right) \qquad (2) - (1)$$

This equation simplifies to

$$0 = x^2 - 3$$

so $x^2 = 3$

hence $x = \pm\sqrt{3}$

These values for x are now substituted into the linear equation.

Substituting the first solution, $x = \sqrt{3}$, into equation (2) gives $y = 2\sqrt{3}$.

Substituting the second solution, $x = -\sqrt{3}$, into equation (2) gives $y = 2\left(-\sqrt{3}\right) = -2\sqrt{3}$.

The solutions are $x = \sqrt{3}$, $y = 2\sqrt{3}$ and $x = -\sqrt{3}$, $y = -2\sqrt{3}$.

Exercises

1. Solve the simultaneous equations

   (a) $y = x^2 - 4$, $y = 3x$
   (b) $y = x^2 + 5x$, $y = 2x + 10$
   (c) $y = 2x^2 + x - 3$, $y = 3x + 1$

2. Solve the following simultaneous equations, giving the values of x and y correct to 2 decimal places.

   (a) $y = x^2$, $y = 3x - 1$
   (b) $y = x^2 + 8$, $y = 5 - 6x$
   (c) $y = 2x^2 + 12x - 4$, $y = 2x - 1$

3. Solve the simultaneous equations

   (a) $x + 2y = 2$, $x^2 + 8y = 8$
   (b) $4x + y = 19$, $5x^2 + 8y = 101$
   (c) $4x + y = 18$, $5x^2 + 2y = 57$

4. (a) Copy and complete the table below.

x	–3	–2	–1	0	1	2	3	4
$y = 9 - x^2$								

   (b) Draw centrally-placed x- and y-axes and scale them using the ranges $-3 \le x \le 4$, $-8 \le y \le 10$.

   (c) Draw the graph $y = 9 - x^2$.

   (d) On the same graph, draw the straight line $y = 6 - 2x$.

   (e) Write down the coordinates of the points where the line $y = 6 - 2x$ intersects the curve $y = 9 - x^2$.

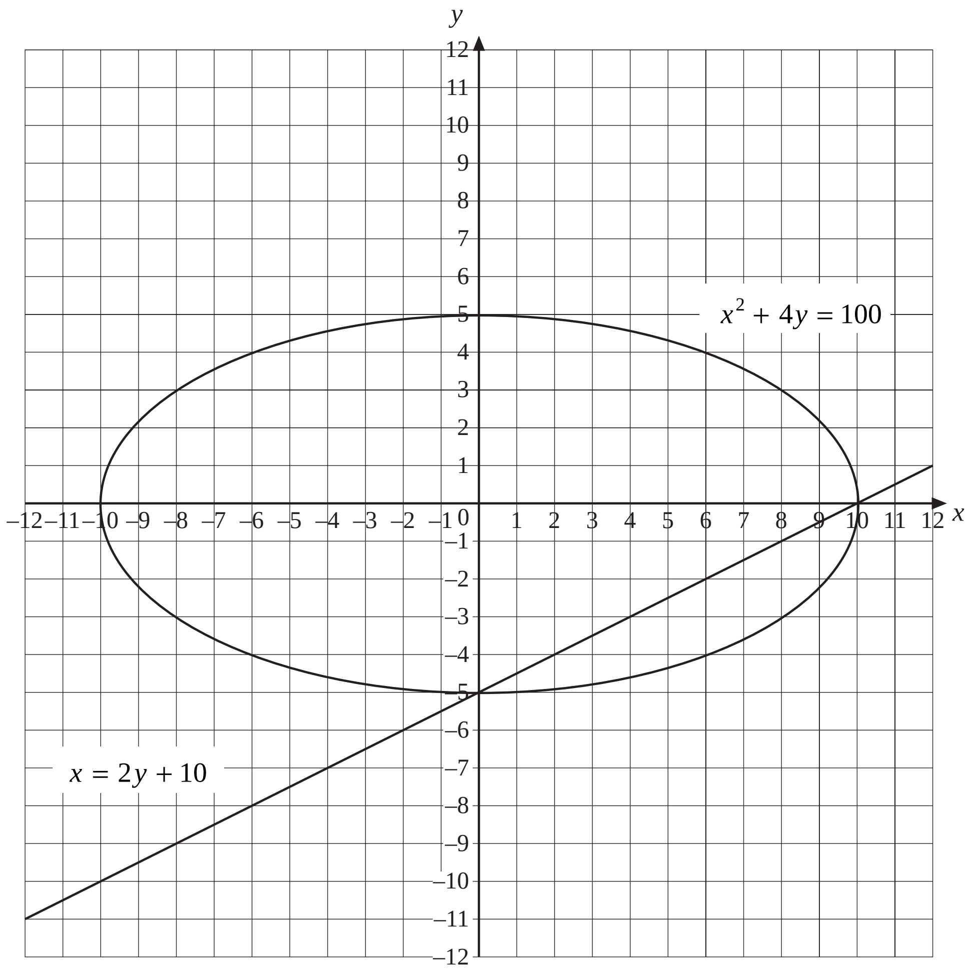
   (f) Use an algebraic method to solve the simultaneous equations

$$y = 9 - x^2$$
$$y = 6 - 2x$$

   (g) Use the solutions found in part (f) to confirm your answer to part (e).

Extension Question

(a) The diagram shows an *ellipse*. The equation of this ellipse is $x^2 + 4y^2 = 100$. The line $x = 2y + 10$ has also been drawn on the diagram.



Write down the coordinates of the points where the line $x = 2y + 10$ intersects the ellipse $x^2 + 4y^2 = 100$.

(b) Use an algebraic method to solve the simultaneous equations

$$x^2 + 4y^2 = 100$$
$$x = 2y + 10$$

and hence confirm your points of intersection in part (a).

(c) Use an algebraic method to solve the simultaneous equations

$$x^2 + 4y^2 = 100$$
$$6y + x = 30$$

and hence find the points of intersection of the ellipse with the straight line $6y + x = 30$.

Answers

10.12A Algebraic Solution of Simultaneous Equations – One Linear and One Quadratic Function

1. (a) $x = 4,\ y = 12$ and $x = -1,\ y = -3$

   (b) $x = -5,\ y = 0$ and $x = 2,\ y = 14$

   (c) $x = 2,\ y = 7$ and $x = -1,\ y = -2$

2. (a) $x = 2.62,\ y = 6.85$ and $x = 0.38,\ y = 0.15$

   (b) $x = -0.55,\ y = 8.30$ and $x = -5.45,\ y = 37.70$

   (c) $x = 0.28,\ y = -0.43$ and $x = -5.28,\ y = -11.57$

3. (a) $x = 0,\ y = 1$ and $x = 4,\ y = -1$

   (b) $x = 3,\ y = 7$ and $x = 3.4,\ y = 5.4$

   (c) $x = 3,\ y = 6$ and $x = -1.4,\ y = 23.6$

4.

x	–3	–2	–1	0	1	2	3	4
$y = 9 - x^2$	0	5	8	9	8	5	0	–7

   (e) $(-1, 8)$ and $(3, 0)$

   (f) $x = -1,\ y = 8$ and $x = 3,\ y = 0$

Extension Question

(a) $(0, -5)$ and $(10, 0)$

(b) $x = 0,\ y = -5$ and $x = 10,\ y = 0$

(c) $x = 0,\ y = 5$ and $x = 6,\ y = 4$