

# 13 Graphs

## 13.11A Graphical Solutions of Simultaneous Equations – One Linear and One Quadratic Function

In Section 13.10 you have solved simultaneous equations where both of the equations were linear. In this section we extend this to solve simultaneous equations where one equation is *linear* and the other is *quadratic*.



### Worked Example 1

Use a graph to find the solutions of the simultaneous equations

$$x + y = 5$$

$$y = x^2 - 1$$



### Solution

We begin by drawing the straight line  $x + y = 5$  which passes through  $(0, 5)$ ,  $(2, 3)$  and  $(5, 0)$ .

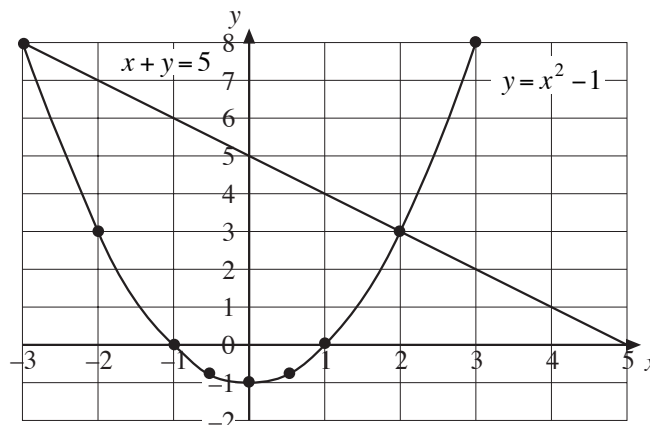
We then draw the graph of the quadratic equation  $y = x^2 - 1$ . To do this we first work out a table of values:

|               |    |    |    |    |   |   |   |
|---------------|----|----|----|----|---|---|---|
| $x$           | -3 | -2 | -1 | 0  | 1 | 2 | 3 |
| $y = x^2 - 1$ | 8  | 3  | 0  | -1 | 0 | 3 | 8 |



### Note

To improve the shape of a quadratic graph it is often useful to include additional points close to the turning point. In this case, we include the additional points  $(-0.5, -0.75)$  and  $(0.5, -0.75)$ .



The curve and the line intersect in two places  $(-3, 8)$  and  $(2, 3)$ , so there are two solutions to the simultaneous equations. The solutions are  $x = -3$ ,  $y = 8$  and  $x = 2$ ,  $y = 3$ .



## Worked Example 2

Use a graph to estimate the solutions of the simultaneous equations

$$y = 2 - 3x$$

$$y = 9 - x^2$$

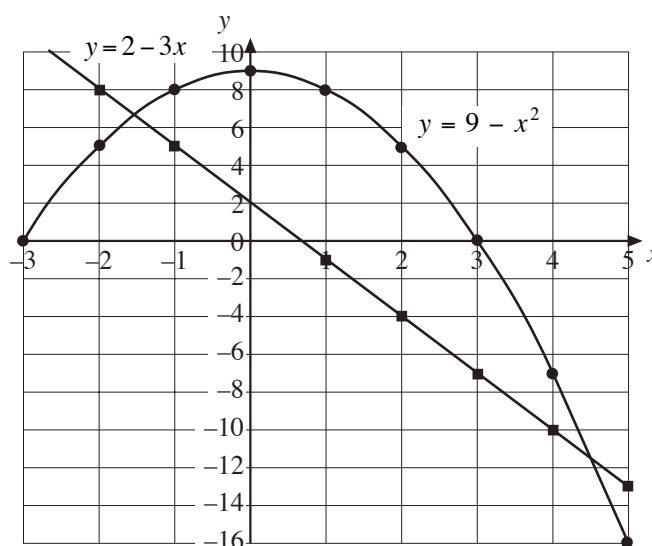


## Solution

We begin by drawing the line  $y = 2 - 3x$  which passes through  $(-2, 8)$ ,  $(0, 2)$  and  $(3, -7)$ .

We then draw the graph of the quadratic equation  $y = 9 - x^2$ . To do this we work out a table of values:

|               |    |    |    |   |   |   |   |    |     |
|---------------|----|----|----|---|---|---|---|----|-----|
| $x$           | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4  | 5   |
| $y = 9 - x^2$ | 0  | 5  | 8  | 9 | 8 | 5 | 0 | -7 | -16 |



Identifying the points on the graph where the curve and the line intersect enables us to estimate the solutions to the equations as  $x = -1.5$ ,  $y = 6.6$  and  $x = 4.5$ ,  $y = -11.6$ .



## Exercises

1. Use graphs to solve the following pairs of simultaneous equations:

(a)  $x + y = 10$

$$y = x^2 - 2$$

(b)  $y = 6 - 4x$

$$y = 12 - 2x^2$$

(c)  $x + 2y = 14$

$$y = x^2 + 2$$

(d)  $y = 2x + 1$

$$y = 25 - 3x - x^2$$

2. Use graphs to estimate the solutions to the following pairs of simultaneous equations:

(a)  $y = x + 5$

$$y = x^2$$

(b)  $y = x + 4$

$$y = x^2 + 2x$$

(c)  $x + y = 9$

$$y = 18 - x^2$$

(d)  $y = 3x - 5$

$$y = 4x - x^2$$

# Answers

## 13.11A Graphical Solutions of Simultaneous Equations – One Linear and One Quadratic Function

1. (a)  $x = 3, y = 7$       and       $x = -4, y = 14$   
(b)  $x = 3, y = -6$       and       $x = -1, y = 10$   
(c)  $x = 2, y = 6$       and       $x = -2.5, y = 8.25$   
(d)  $x = -8, y = -15$       and       $x = 3, y = 7$
  
2. (a)  $x = 2.8, y = 7.8$       and       $x = -1.8, y = 3.2$   
(b)  $x = 1.6, y = 5.6$       and       $x = -2.6, y = 1.4$   
(c)  $x = 3.5, y = 5.5$       and       $x = -2.5, y = 11.5$   
(d)  $x = 2.8, y = 3.4$       and       $x = -1.8, y = -10.4$