13 Graphs

13.11C Finding Graphically the Points of Intersection of a Circle and a Straight Line Graph

On Section 13.11A you have solved simultaneous equations graphically where one of the equations is linear and the other is quadratic. We now focus on the similar case of a straight line intersecting a circle.

Worked Example 1

(a) Draw centrally-placed *x*- and *y*-axes and scale them using the ranges $-7 \leq x \leq 7$, $-7 \leq y \leq 7$.

(b) Accurately construct the locus $x^2 + y^2 = 36$.

(c) On the same graph, draw the line $y = x + 2$.

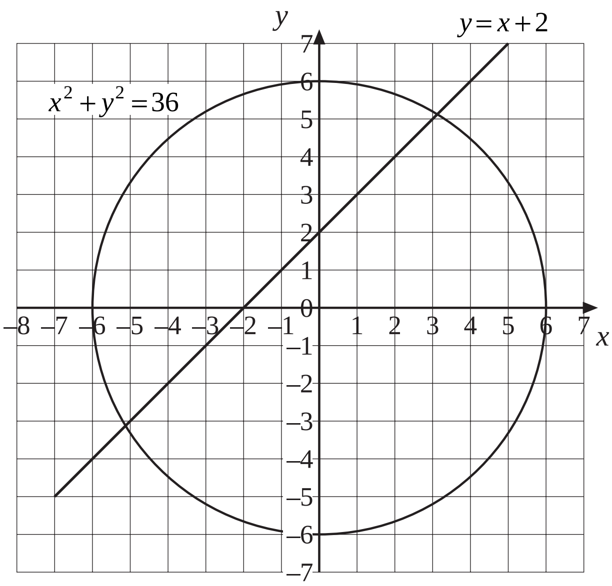
(d) Write down the *x*-and *y*-coordinates of the points where the line meets the circle.

Solution

(a, b) The diagram shows the circle $x^2 + y^2 = 36$. This has centre the origin, radius 6 units and is drawn with a pair of compasses.

(c) The line $y = x + 2$ passing through (–2, 0), (0, 2) and (3, 5) has been added to the diagram.

(d) From the graph, we read off the coordinates of the points where the line and the circle meet. These are (3.1, 5.1) and (–5.1, –3.1).



Worked Example 2

(a) Draw centrally-placed x- and y-axes and scale them using the ranges $-5 \le x \le 5$, $-5 \le y \le 5$.

(b) Accurately construct the locus $x^2 + y^2 = 16$.

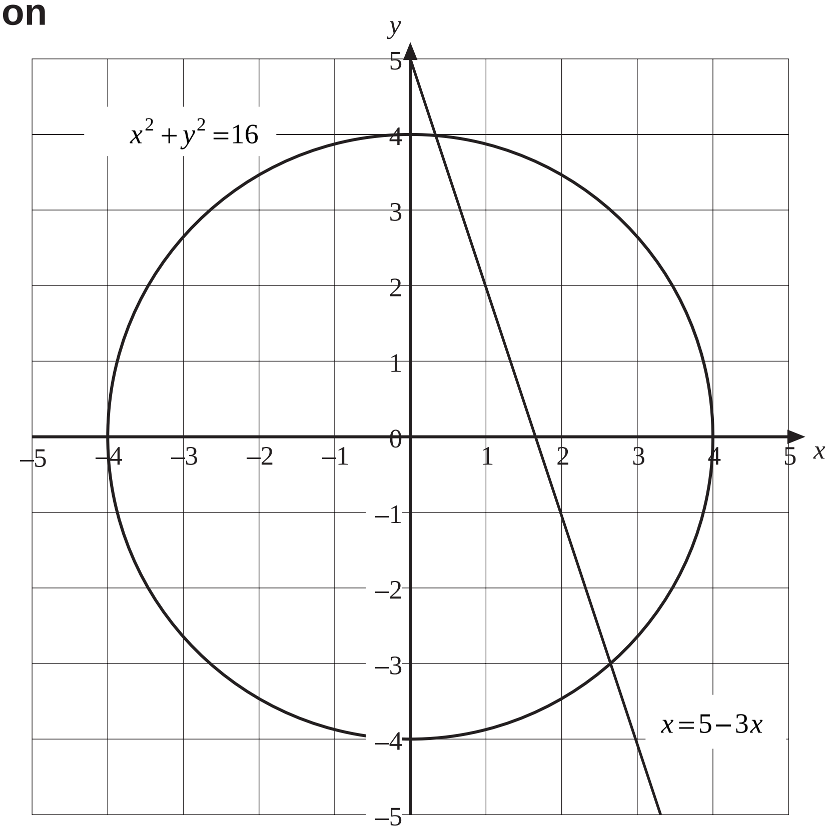
(c) On the same graph, draw the line $y = 5 - 3x$.

(d) Solve graphically the simultaneous equations

$$x^2 + y^2 = 16$$

$$y = 5 - 3x$$

Solution



(a, b) The diagram shows the circle $x^2 + y^2 = 16$. This has centre the origin, radius 4 units and is drawn with a pair of compasses.

(c) The line $y = 5 - 3x$ passing through (0, 5), (1, 2) and (3, –4) has been added to the diagram.

(d) From the graph, we read off the coordinates of the points where the line and the circle meet. These are (2.7, –3.0) and (0.3, 4.0).

So the approximate solutions of the simultaneous equations are

$$x = 2.7, \ y = -3.0$$

and

$$x = 0.3, \ y = 4.0$$

Worked Example 3

(a) Draw centrally-placed x- and y-axes and scale them using the ranges $-6 \le x \le 6$ $-6 \le y \le 6$.

(b) Accurately construct the locus $x^2 + y^2 = 25$.

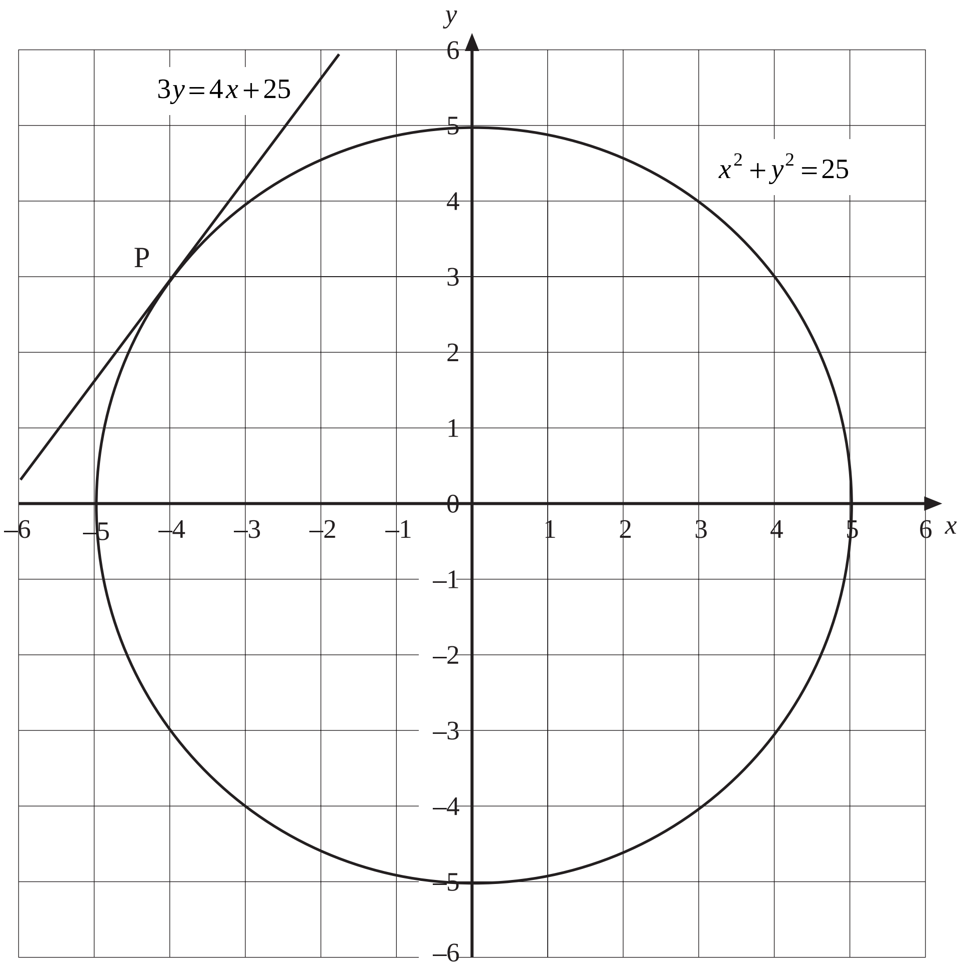
(c) On the same graph, draw the line $3y = 4x + 25$. What do you notice?

(d) Read off the x- and y-coordinates of the point P where the line meets the circle in the second quadrant.

(e) Find the equation of the perpendicular through P to the line $3y = 4x + 25$.

(f) Show that the line in (e) goes through the origin.

Solution



(a, b) The diagram shows the circle $x^2 + y^2 = 25$. This has centre the origin, radius 5 units and is drawn with a pair of compasses.

(c) The line $3y = 4x + 25$ has been added to the diagram.
We notice that the line $3y = 4x + 25$ is a tangent to the circle.

(d) The line $3y = 4x + 25$ meets the circle $x^2 + y^2 = 25$ at the point P (–4, –3).

(e) The line $3y = 4x + 25$ can be rewritten in the form $y = \frac{4}{3}x + \frac{25}{3}$ so has gradient $\frac{4}{3}$. The perpendicular to this line must therefore have gradient $-\frac{3}{4}$.

Its equation must therefore be $y = -\frac{3}{4}x + c$ for some constant c.

Substituting the point P (–4, –3) into this equation gives

$$-3 = \left(-\frac{3}{4}\right) \times (-4) + c = -3 + c,$$

so $$c = 0$$

Therefore the perpendicular to the line $3y = 4x + 25$ at P has equation $y = -\frac{3}{4}x$.

(f) Substituting (0, 0) into both sides of the equation $y = -\frac{3}{4}x$ verifies that this perpendicular line passes though the origin. Alternatively, the fact that $c = 0$ in part (f) implies directly that the line goes through the origin.

N.B. Part (f) illustrates the fact that, at the point of contact on a circle, the angle between tangent and radius is a right angle.

Exercises

1. (a) Draw centrally-placed *x*- and *y*-axes and scale them using the ranges $-20 \le x \le 20$, $-20 \le y \le 20$.

   (b) Accurately construct the locus $x^2 + y^2 = 289$.

   (c) On the same graph, draw the line $y = x + 7$.

   (d) Write down the *x*- and *y*-coordinates of the points where the line meets the circle.

2. (a) Draw centrally-placed *x*- and *y*-axes and scale them using the ranges $-12 \le x \le 12$, $-12 \le y \le 12$.

   (b) Accurately construct the locus $x^2 + y^2 = 100$.

   (c) On the same graph, draw the line $y = 4 - 2x$.

   (d) Write down the *x*- and *y*-coordinates of the points where the line meets the circle.

   (e) Write down the solutions to the simultaneous equations

$$x^2 + y^2 = 100$$
$$y = 4 - 2x$$

(f) *Without further calculation*, write down the coordinates of the point in the first quadrant where the line $y = 2x - 4$ meets the circle $x^2 + y^2 = 100$.

3. (a) Draw centrally-placed x- and y-axes and scale them using the ranges $-15 \le x \le 15$, $-15 \le y \le 15$.

(b) Accurately construct the locus $x^2 + y^2 = 169$.

(c) Draw the line $x + y = 17$ on the same diagram.

(d) Solve graphically the simultaneous equations

$$x + y = 17$$
$$x^2 + y^2 = 169$$

(e) Write down the coordinates of the points A and B where the line $x + y = 17$ intersects the circle $x^2 + y^2 = 169$.

(f) Find the gradient of the chord AB and hence determine the gradient and equation of the perpendicular bisector of the chord AB.

(g) Verify that the perpendicular bisector of the chord AB goes through the centre of the circle.

Answers

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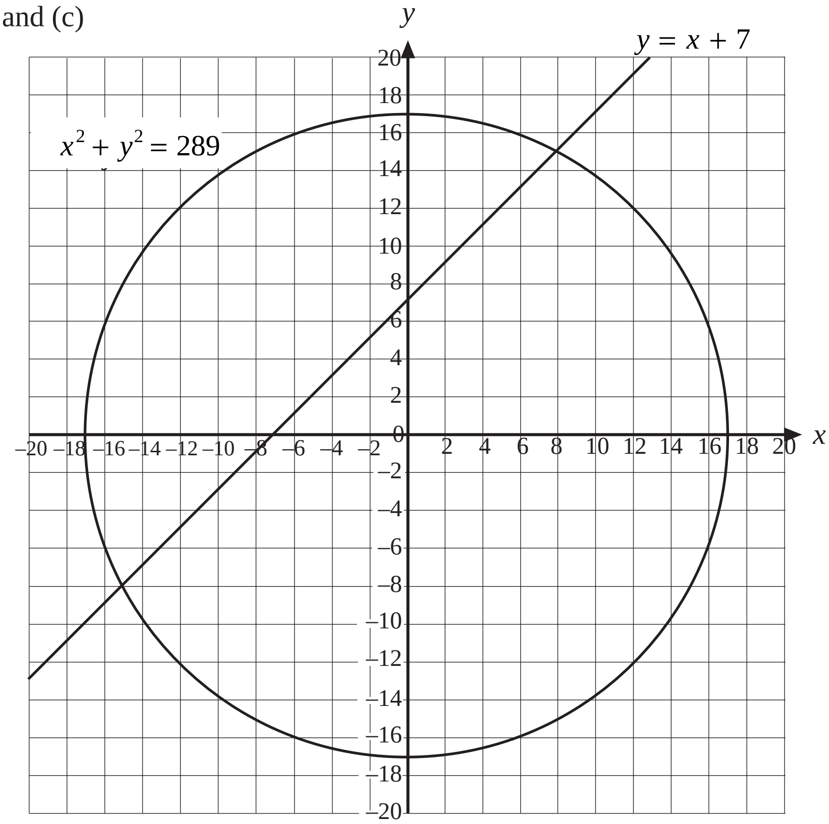
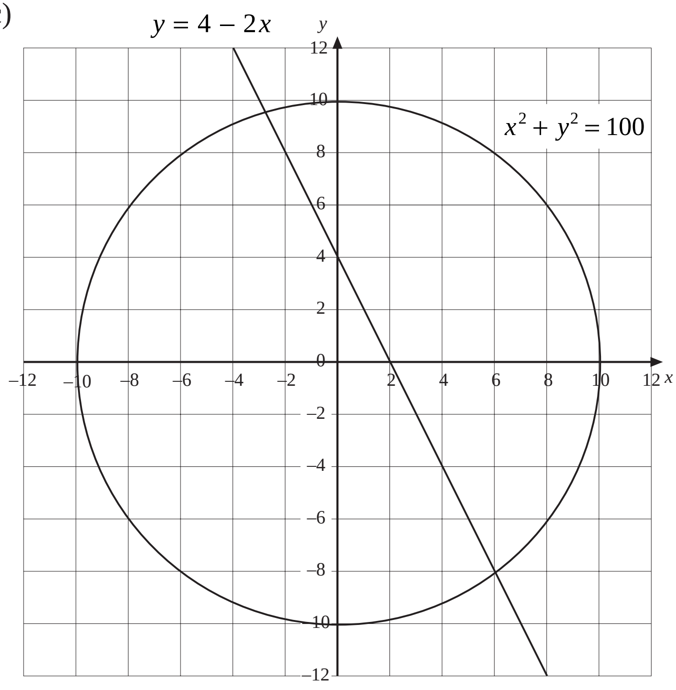
1. (a), (b) and (c)



(d) The line and the circle meet at (8, 15) and (–15, –8).

2. (a), (b) and (c)

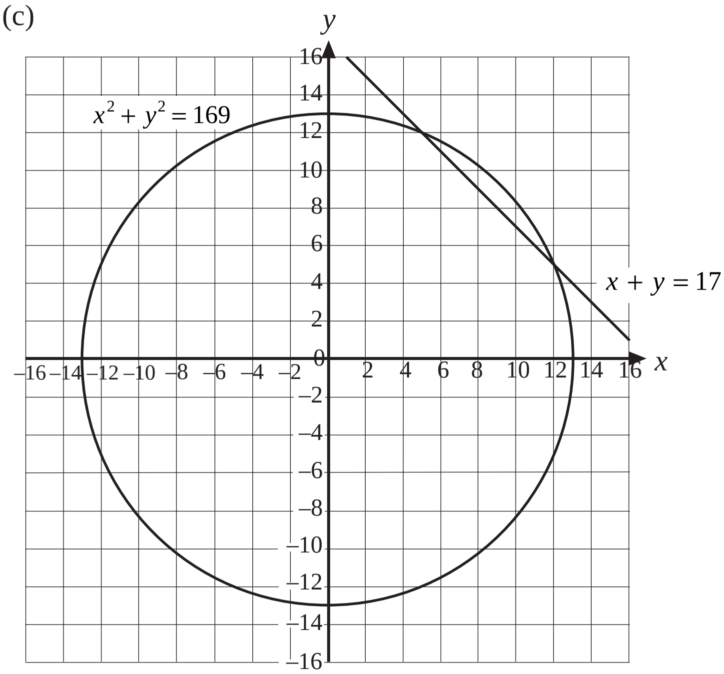


(d) (6, –8) and (–2.8, 9.6)

(e) $x = 6$, $y = -8$ and $x = -2.8$, $y = 9.6$

(f) (6, 8) and (–2.8, –9.6)

3. (a), (b) and (c)



(d) $x = 12$, $y = 5$ and $x = 5$, $y = 12$

(e) (12, 5) and (5, 12)

(f) AB has gradient –1.
The perpendicular bisector of AB has gradient 1 and equation $y = x$.