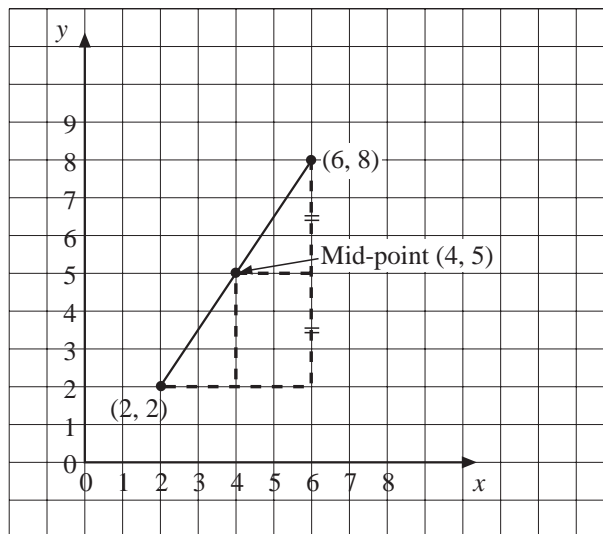


13 Graphs

13.2C Mid-Points of Line Segments

The coordinates of the mid-point between two other points may be found by drawing or by calculation.

Consider the line segment that joins the point A which has coordinates (2, 2) and the point B (6, 8). The mid-point of the line segment AB is shown in the diagram below.



The value of the x -coordinate of the mid-point of the line segment AB is the mean value of the two x -coordinates of the end points A and B.

Similarly for the y -coordinate of the mid-point, it is the mean of the y -coordinates of the end points A and B.

The coordinates of the mid-point could have been calculated directly as shown below.

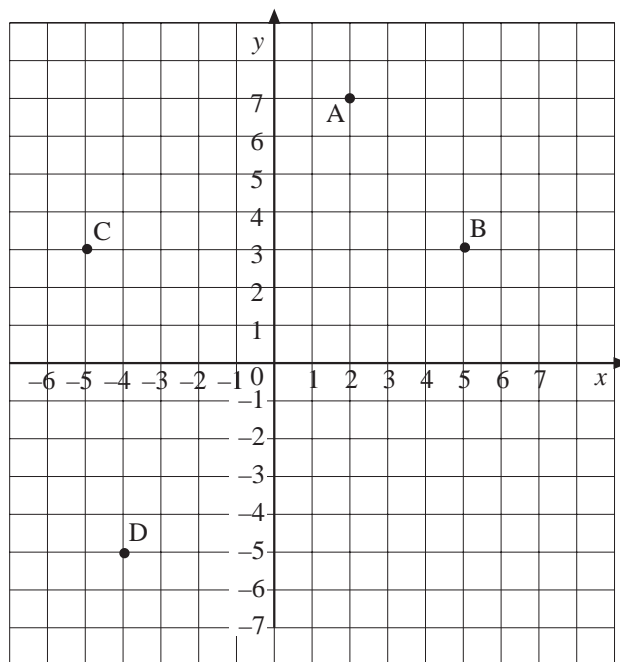
$$\begin{aligned} \left(\frac{2+6}{2}, \frac{2+8}{2} \right) &= \left(\frac{8}{2}, \frac{10}{2} \right) \\ &= (4, 5) \end{aligned}$$

Generally, for any two points, the coordinates of the mid-point of the line segment joining the points (a, b) and (c, d) is given by $\left(\frac{a+c}{2}, \frac{b+d}{2} \right)$.



Worked Example 1

The diagram shows the points A, B, C and D.



Find the coordinates of the mid-point of the line segment:

- (a) AB (b) AC (c) BD



Solution

- (a) The coordinates of A are (2, 7).

The coordinates of B are (5, 3).

$$\begin{aligned} \text{The coordinates of the mid-point of AB} &= \left(\frac{2+5}{2}, \frac{7+3}{2} \right) \\ &= \left(\frac{7}{2}, \frac{10}{2} \right) \\ &= (3.5, 5) \end{aligned}$$

- (b) The coordinates of C are (-5, 3).

$$\begin{aligned} \text{The coordinates of the mid-point of AC} &= \left(\frac{2+(-5)}{2}, \frac{7+3}{2} \right) \\ &= \left(\frac{-3}{2}, \frac{10}{2} \right) \\ &= (-1.5, 5) \end{aligned}$$

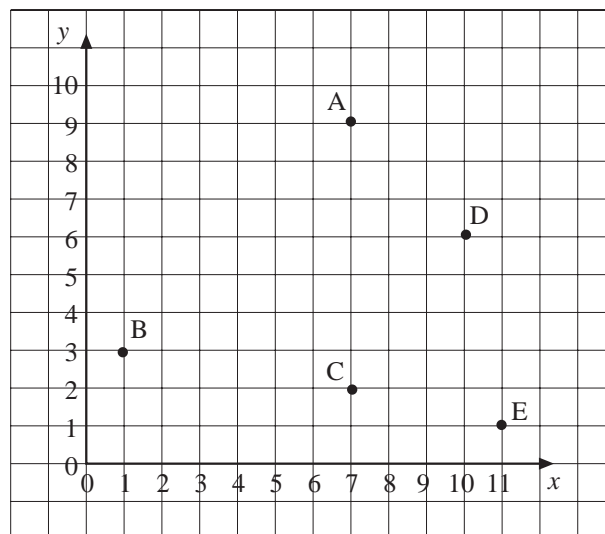
- (c) The coordinates of D are $(-4, -5)$.

$$\begin{aligned} \text{The coordinates of the mid-point of BD} &= \left(\frac{5 + (-4)}{2}, \frac{3 + (-5)}{2} \right) \\ &= \left(\frac{1}{2}, \frac{-2}{2} \right) \\ &= (0.5, -1) \end{aligned}$$



Exercises

- Draw a set of axes and mark on them the points A and B which have coordinates $(1, 4)$ and $(7, 6)$.
 - Draw the line segment AB and mark its mid-point.
 - Write down the coordinates of the mid-point of AB.
- The diagram shows the points A, B, C, D and E.

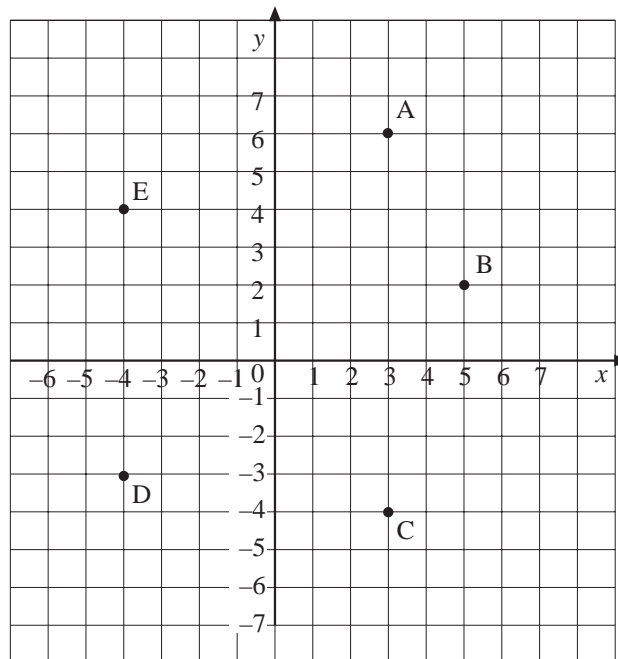


Find the coordinates of the mid-point of the line segment:

- | | | | |
|--------|--------|--------|--------|
| (a) AB | (b) AC | (c) AD | (d) AE |
| (e) BE | (f) CD | (g) DE | (h) CE |
- Determine the coordinates of the mid-point of the line segment joining the two points given in each case.

(a) $(4, 7)$ $(8, 11)$	(b) $(6, 2)$ $(18, 8)$
(c) $(3, 2)$ $(9, 4)$	(d) $(6, 3)$ $(10, 11)$
(e) $(4, 1)$ $(3, 4)$	(f) $(6, 6)$ $(1, 7)$
(g) $(2, 15)$ $(13, 2)$	(h) $(24, 2)$ $(13, 3)$

4. The diagram shows the points A, B, C, D and E.

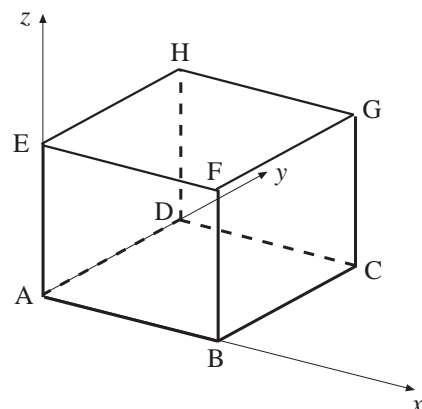


Determine the coordinates of the mid-point of the line segments below.

- | | |
|--------|--------|
| (a) AB | (b) AE |
| (c) BD | (d) BC |
| (e) AC | (f) DC |
| (g) DE | (h) CE |
5. Determine the coordinates of the mid-point of the line segment joining each pair of points listed below.
- | | |
|------------------------|------------------------|
| (a) (2, 3) (4, -6) | (b) (-2, 1) (3, -6) |
| (c) (-2, -3) (-8, -10) | (d) (-2, 4) (5, -6) |
| (e) (-3, -2) (4, 7) | (f) (6, -2) (8, -10) |
| (g) (2, -6) (7, -5) | (h) (8, -3) (-10, -11) |
6. The diagram shows a cuboid.

The coordinates of the vertices are listed below.

- | | |
|-------------|-------------|
| A (0, 0, 0) | B (7, 0, 0) |
| C (7, 6, 0) | D (0, 6, 0) |
| E (0, 0, 5) | F (7, 0, 5) |
| G (7, 6, 5) | H (0, 6, 5) |

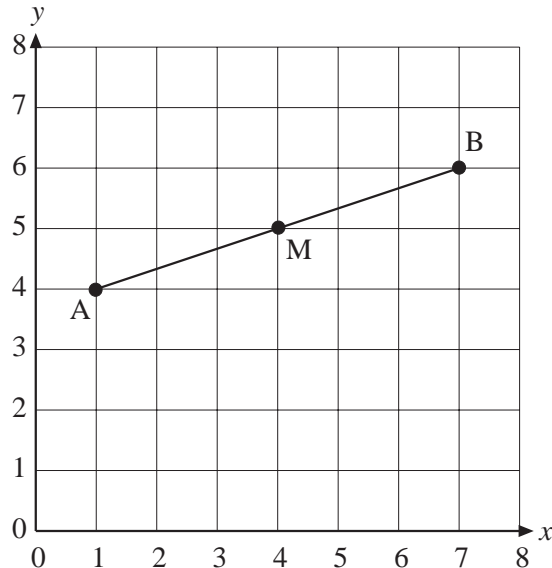


- (a) Explain why the coordinates of the mid-point of DG are (3.5, 6, 2.5).
 - (b) Determine the coordinates of the mid-point of each of the following line segments.
 - (i) AB
 - (ii) CD
 - (iii) DH
 - (iv) FG
7. The points A, B and C have coordinates (4, 2, 1), (6, 10, 7) and (4, 8, 11).
- (a) Determine the coordinates of the mid-points of AB and AC.
 - (b) The mid-point of AB is joined to the mid-point of AC by a line segment. Determine the coordinates of the mid-point of this line segment.

Answers

13.2C Mid-Points of Line Segments

1. (a) and (b)



- (c) The mid-point $M = (4, 5)$

2. (a) (4, 6) (b) (7, 5.5) (c) (8.5, 7.5) (d) (9, 5)
 (e) (6, 2) (f) (8.5, 4) (g) (10.5, 3.5) (h) (9, 1.5)
3. (a) (6, 9) (b) (12, 5) (c) (6, 3) (d) (8, 7)
 (e) (3.5, 2.5) (f) (3.5, 6.5) (g) (7.5, 8.5) (h) (18.5, 2.5)
4. (a) (4, 4) (b) (-0.5, 5) (c) (0.5, -0.5) (d) (4, -1)
 (e) (3, 1) (f) (-0.5, -3.5) (g) (-4, 0.5) (h) (-0.5, 0)
5. (a) (3, -1.5) (b) (0.5, -2.5) (c) (-5, -6.5) (d) (1.5, -1)
 (e) (0.5, 2.5) (f) (7, -6) (g) (4.5, -5.5) (h) (-1, -7)
6. (a) $D = (0, 6, 0)$ and $G = (7, 6, 5)$

In 3-dimensions, to find a mid-point we average the coordinates, just as in 2-dimensions.

The mid-point of DG is therefore $\left(\frac{0+7}{2}, \frac{6+6}{2}, \frac{0+5}{2}\right) = (3.5, 6, 2.5)$

- (b) (i) (3.5, 0, 0) (ii) (3.5, 6, 0)
 (iii) (0, 6, 2.5) (iv) (7, 3, 5)
7. (a) (5, 6, 4) (4, 5, 6) (b) (4.5, 5.5, 5)