

# 13 Graphs

## 13.5A Gradients of Parallel Line Segments

Parallel lines have the same gradient.



### Worked Example 1

The coordinates of the points A, B, C and D are listed below.

$$A (2, 4) \qquad B (8, 7)$$

$$C (-1, -5) \qquad D (5, -2)$$

- (a) Show that the line segments AB and CD are parallel.  
 (b) Are the line segments AC and BD parallel?



### Solution

$$(a) \quad \text{Gradient of AB} = \frac{7 - 4}{8 - 2} = \frac{3}{6} = \frac{1}{2}$$

$$\text{Gradient of CD} = \frac{(-2) - (-5)}{5 - (-1)} = \frac{3}{6} = \frac{1}{2}$$

The line segments have the same gradients and so must be parallel.

$$(b) \quad \text{Gradient of AC} = \frac{(-5) - 4}{(-1) - 2} = \frac{-9}{-3} = 3$$

$$\text{Gradient of BD} = \frac{7 - (-2)}{8 - 5} = \frac{9}{3} = 3$$

The line segments have the same gradient and so are parallel.

The results in Worked Example 1 mean that the quadrilateral ABDC is, in fact, a parallelogram.



### Worked Example 2

The points

$$P (2, 7)$$

$$Q (5, 8)$$

$$R (10, 6)$$

$$S (1, 3)$$

are joined to form a quadrilateral.

By considering the gradients of the sides, identify the type of quadrilateral.



## Solution

$$\text{Gradient of PQ} = \frac{8 - 7}{5 - 2} = \frac{1}{3}$$

$$\text{Gradient of QR} = \frac{6 - 8}{10 - 5} = \frac{-2}{5}$$

$$\text{Gradient of RS} = \frac{3 - 6}{1 - 10} = \frac{-3}{-9} = \frac{1}{3}$$

$$\text{Gradient of SP} = \frac{7 - 3}{2 - 1} = \frac{4}{1} = 4$$

The sides PQ and RS are parallel. The shape PQRS must be a trapezium as it has only two sides which are parallel.



## Exercises

- Draw the line segment joining the points (4, 7) and (5, -1).
  - Draw the line segment joining the points (-1, 3) and (0, -5).
  - Are these lines parallel?
  - Calculate the gradient of each line.
- Calculate the gradient of the line segment joining the points (1, 4) and (6, 3).
  - Calculate the gradient of the line segment joining the points (3, 6) and (8, 5).
  - Are the two line segments parallel?
- In each case below the coordinates of the points A, B, C and D are listed. Determine whether or not AB is parallel to CD.

(a)	A (1, 2)	B (6, 7)	C (3, 6)	D (8, 11)
(b)	A (2, 4)	B (3, 1)	C (8, 2)	D (6, 3)
(c)	A (7, 2)	B (8, 10)	C (9, 1)	D (11, 3)
(d)	A (-1, 8)	B (3, -2)	C (7, 11)	D (11, 1)
- The vertices A, B, C and D of a quadrilateral have coordinates

A (4, 7)	B (8, 9)	C (10, 1)	D (6, -1)
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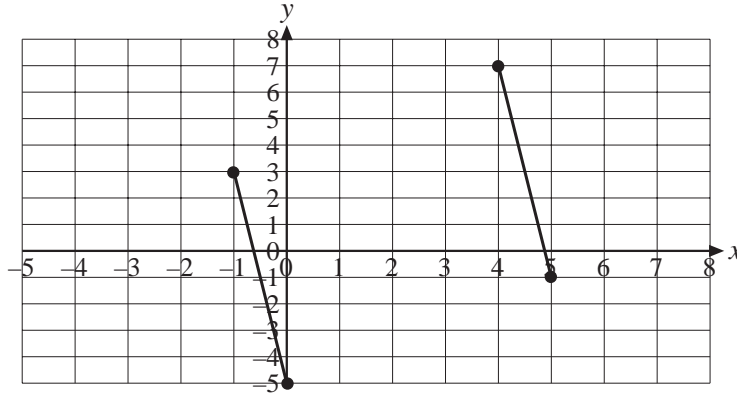
  - Calculate the gradient of each side of the quadrilateral.
  - Calculate the length of each side of the quadrilateral.
  - What is the name of this type of quadrilateral?

5. A square has vertices P, Q, R and S.  
P (0, 2)            Q (-2, 4)            R (0, 6)            S (2, 4)
- (a) Show that PQ and RS are parallel.  
(b) Show that QR and PS are parallel.  
(c) Calculate the length of each side.
6. A trapezium has vertices at the points A (4, 3), B (7, 12), C (2, 2) and D (8, 20).  
Which are the two parallel sides?
7. (a) Calculate the gradient of the line segment joining the points with coordinates (2, 2) and (7, 5).  
(b) A second line segment joins the points (3, 6) and (13, y).  
If the two line segments are parallel, determine the value of y.
8. A line segment joins the points A (2, 3) and B (5, -3).  
(a) Calculate the gradient of AB.  
(b) The line segment CD is twice as long as AB and parallel to it.  
If point C has coordinates (-2, 1), determine the coordinates of point D.

# Answers

## 13.5A Gradients of Parallel Line Segments

1. (a) and (b)



- (c) Yes      (d) They both have gradient  $-8$ .
2. (a) Gradient  $= \frac{-1}{5}$       (b) Gradient  $= \frac{-1}{5}$       (c) Yes
3. (a) Yes      Gradient of AB = gradient of CD = 1  
 (b) No      Gradient of AB =  $-3$       Gradient of CD =  $-\frac{1}{2}$   
 (c) No      Gradient of AB = 8      Gradient of CD = 1  
 (d) Yes      Gradient of AB = gradient of CD =  $-2.5$
4. (a) Gradient of AB  $= \frac{1}{2}$ ;      Gradient of BC =  $-4$   
 Gradient of CD  $= \frac{1}{2}$ ;      Gradient of DA =  $-4$   
 (b) Length of AB  $= \sqrt{20}$ ;      Length of BC  $= \sqrt{68}$   
 Length of CD  $= \sqrt{20}$ ;      Length of DA  $= \sqrt{68}$   
 (c) This type of quadrilateral is a parallelogram.
5. (a) Gradient of PQ  $= \frac{4 - 2}{(-2) - 0} = \frac{2}{-2} = -1$   
 Gradient of RS  $= \frac{4 - 6}{2 - 0} = \frac{-2}{2} = -1$   
 PQ and RS have the same gradient       $\therefore$  PQ is parallel to RS.  
 (b) Gradient of QR  $= \frac{6 - 4}{0 - (-2)} = \frac{2}{2} = 1$   
 Gradient of PS  $= \frac{4 - 2}{2 - 0} = \frac{2}{2} = 1$   
 QR and PS have the same gradient       $\therefore$  QR is parallel to PS.

$$\begin{array}{ll} \text{(c) Length of PQ} = \sqrt{8} & \text{Length of RS} = \sqrt{8} \\ \text{Length of QR} = \sqrt{8} & \text{Length of PS} = \sqrt{8} \end{array}$$

*N.B. The answers to this question mean that PQRS is a rhombus.*

$$\begin{array}{ll} 6. \text{ Gradient of AB} = 3 & \text{Gradient of BC} = 2 \\ \text{Gradient of CD} = 3 & \text{Gradient of DA} = 4\frac{1}{4} \end{array}$$

$\therefore$  AB is parallel to CD.

$$\begin{array}{ll} 7. \text{ (a) Gradient} = \frac{3}{5} & \text{(b) } y = 12 \\ 8. \text{ (a) Gradient} = -2 & \text{(b) } D = (4, -11) \text{ or } D = (-8, 13) \end{array}$$