13 Graphs

13.8B Equations of Perpendicular Lines

If a line has equation $y = mx + c$, its gradient is m. The gradient of a perpendicular line will be $\frac{-1}{m}$, provided $m \neq 0$.

For example, the line with equation $y = 4x + 2$ is perpendicular to the line $y = \frac{-1}{4}x + 6$.

Worked Example 1

The point with coordinates (4, 9) lies on the line with equation $y = 2x + 1$. Determine the equation of the perpendicular line that also passes through this point.

Solution

The line $y = 2x + 1$ has gradient 2.

The perpendicular line will have gradient $-\frac{1}{2}$ and so its equation will be of the form

$$y = -\frac{1}{2}x + c$$

As the line passes through (4, 9), we can use $x = 4$ and $y = 9$ to determine the value of c.

$$9 = -\frac{1}{2} \times 4 + c$$

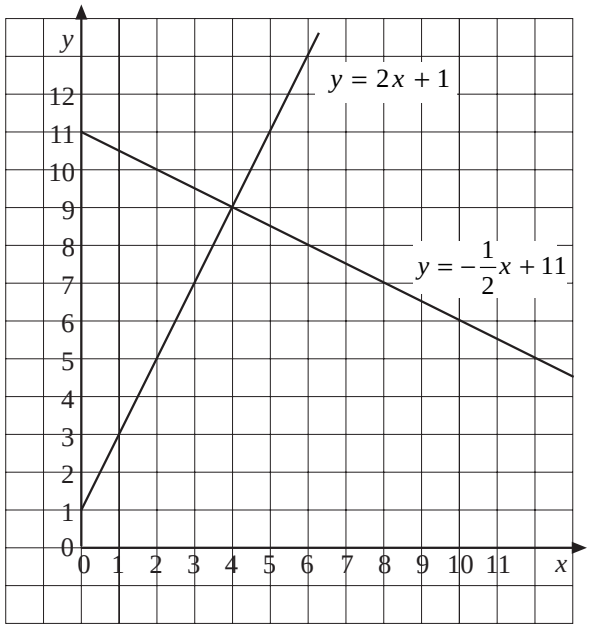
$$9 = -2 + c$$

$$c = 11$$

The equation of the perpendicular line is therefore

$$y = -\frac{1}{2}x + 11$$

The graph shows both lines.



Worked Example 2

Determine the equation of the perpendicular bisector of the line AB, where A has coordinates (4, 8) and B has coordinates (0, 2)

Solution

Gradient of AB $= \dfrac{8-2}{4-0} = \dfrac{6}{4} = \dfrac{3}{2}$

So the gradient of the perpendicular bisector will be $-\dfrac{2}{3}$.

The mid-point of AB has coordinates

$$\left(\frac{4+0}{2}, \frac{8+2}{2}\right) = (2,5)$$

The equation of the perpendicular bisector will be of the form

$$y = -\frac{2}{3}x + c$$

At the point (2, 5), $x = 2$ and $y = 5$, which can be used to determine the value of c.

$$5 = -\frac{2}{3} \times 2 + c$$

$$5 = -\frac{4}{3} + c$$

$$c = 5 + \frac{4}{3}$$

$$= 6\frac{1}{3}$$

The equation of the perpendicular bisector is therefore

$$y = -\frac{2}{3}x + 6\frac{1}{3}$$

Exercises

1. The point with coordinates (4, 8) lies on the line with equation $y = 2x$. Determine the equation of the perpendicular line which also passes through this point.

2. Determine the equation of the line that passes through the point (4, 2) and is perpendicular to the line $y = 3x - 3$.

3. Determine the equation of the line that is perpendicular to $y = 4x - 8$ and that passes through the origin.

4. Determine the equation of the perpendicular bisector of the line AB if

   (a) A is (2, 6) and B is (3, 7)

   (b) A is (3, 2) and B is (–6, –4)

   (c) A is (–2, –7) and B is (–1, –9)

   (d) A is (4, –6) and B is (–3, 8).

5. Determine the equation of the line that passes through the point (4, –7) and is perpendicular to $y = x$.

6. A line is drawn from the point (2, –4) so that it is perpendicular to the line with equation $y = 8 - \frac{1}{2}x$. Determine the equation of this line.

7. A rectangle has vertices at the points with coordinates (0, –4), (–4, 4), (0, 6) and (4, –2).

   (a) Determine the equations of the two lines of symmetry of the rectangle.

   (b) Determine the coordinates of the point where the two lines of symmetry intersect.

8. A parallelogram has vertices at the point A (2, 0), B (6, 4), C (1, 3) and D (–3, –1).

   Determine the equations of the perpendicular bisectors of each side.

Answers

13.8B Equations of Perpendicular Lines

1. $y = -\frac{1}{2}x + 10$

2. $y = -\frac{1}{3}x + 3\frac{1}{3}$

3. $y = -\frac{1}{4}x$

4. (a) $y = -x + 9$ (b) $y = -\frac{3}{2}x - 3\frac{1}{4}$

   (c) $y = \frac{1}{2}x - 7\frac{1}{4}$ (d) $y = \frac{1}{2}x + \frac{3}{4}$

5. $y = -x - 3$

6. $y = 2x - 8$

7. (a) $y = \frac{1}{2}x + 1$; $y = -2x + 1$ (b) (0, 1)

8. The perpendicular bisector of AB is $y = -x + 6$

   The perpendicular bisector of BC is $y = -5x + 21$

   The perpendicular bisector of CD is $y = -x$

   The perpendicular bisector of DA is $y = -5x - 3$