UNIT 1 *Indices* Activities

Activities

ACTIVITY 1.1 *Multiplication Table*

×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6							
3										
4										
5										
6										
7										
8										
9										
10										

Copy and complete this multiplication table.

ACTIVITY 1.2 *Secret Letter*

Shade the boxes with incorrect answers in the following table. The boxes you shade will form a letter. What letter is it?

×	**5**	**4**	**0**	**9**	**2**	**6**	**1**	**7**	**3**	**8**
3	15	12	0	21	5	81	1	21	9	24
7	35	28	7	63	14	42	7	14	21	56
6	30	24	6	54	12	36	6	42	18	48
1	5	4	1	9	2	6	1	7	3	8
0	0	0	0	9	2	6	1	0	0	0
5	25	20	0	45	10	30	5	30	15	40
2	10	8	0	18	4	12	2	9	6	16
8	40	32	0	72	16	48	8	50	24	64
4	20	16	4	36	8	24	5	28	12	32
9	45	36	0	18	16	45	9	63	27	72

ACTIVITY 1.3 *Last Digit*

Note that the last digit of the value of 7^2 is 9.

$$\boxed{7^2 = 49}$$

The last digit of each of the following squares is also 9.

$$\begin{aligned} 27^2 &= 72\ 9 \\ 87^2 &= 7\ 56\ 9 \\ 137^2 &= 18\ 76\ 9 \\ 407^2 &= 165\ 64\ 9 \end{aligned}$$

Look at the left hand side of each of the above equations. Notice that the last digit of each number to be squared ends in 7.

1. Write down the value of each of the following squares:

   (a) 3^2, 83^2, 173^2, 503^2 (b) 9^2, 19^2, 209^2, 699^2

2. What do you notice about the last digit in your answers to 1.(a) and (b)?
   Is there a rule to help you find the last digit of the value of a square?

3. Investigate with other numbers to check if your rule works.

4. Complete the table below by writing down n^2 and the last digit of n^2.
   (One has been done for you.)

n	0	1	2	3	4	5	6	7	8	9
n^2								49		
Last digit of n^2								9		

Extension

Complete a similar table for n^3, n^4, n^5 and n^6. Study the table and answer the following questions:

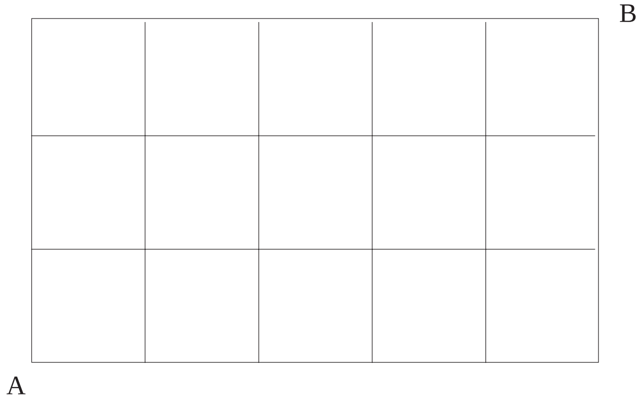
(a) Which power of n has all its last digits the same as the original number n?

(b) Which numbers do **not** appear among the last digits of square numbers?

(c) Identify annd write down other features of the different powers of numbers that you can find.

ACTIVITY 1.4 *Diagonals*

1. In the 5×3 rectangle below, draw a straight line AB to join the two opposite corners.



(a) How many squares does the line AB cross?

(b) How many intersections does the line AB make with the horizontal and vertical lines shown?

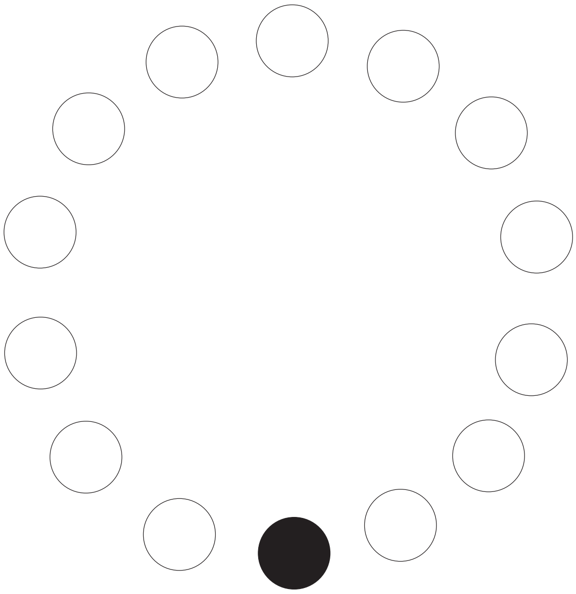
2. Answer the same questions as 1.(a) and (b) for diagonals of **other** dimensions, e.g.

$$2 \times 3, \; 2 \times 4, \; 2 \times 5, \; 3 \times 4, \; 3 \times 6, \; 3 \times 7$$

3. Generalise your results for an $n \times m$ rectangle.

ACTIVITY 1.5 *Stepping Stones*

A ring of stepping stones has 14 stones in it, as shown in the diagram below.



A girl hops around the ring, stopping to change feet every time she has made 3 hops. She notices that when she has been round the ring three times, she has stopped to change feet on each one of the 14 stones.

1. The girl now hops round the ring, stopping to change feet every time she has made 4 hops. Explain why in this case she will **not** stop on each one of the 14 stones, no matter how long she continues hopping round the ring.

2. The girl stops to change feet every time she has made n hops. For which values of n will she stop on each one of the 14 stones to change feet?

3. Find a general rule for stepping on each stone for the values of n hops when the ring contains more (or less) than 14 stones.

ACTIVITY 1.6 *Factors*

Mark out on a large sheet of squared paper a 30 × 30 grid similar to the one below.

Factors

	1	2	3	4	5	6	7	8	9	10	11	12	etc
1	✔												
2	✔	✔											
3	✔		✔										
4	✔	✔		✔									
5													
Numbers 6													
7													
8													
9													
10													
11													
12													
etc													

1. For each column/row, if the number at the top is a factor of the number on the left-hand-side of a row, tick the relevant box (some have been done in the sample grid above).

2. As you fill in your grid you should notice various patterns made by the ticks. Describe and explain these patterns.

ACTIVITY 1.7 *Sieve of Eratosthenes*

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. Cross out all multiples of 2, e.g. 4, 6, etc.

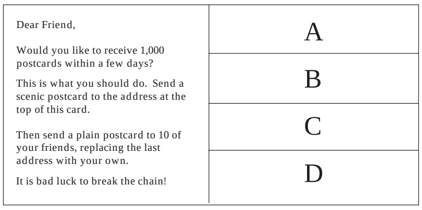
2. Cross out all multiples of 3, e.g. 6, 9, etc.

4. Continue in this way for all other prime numbers less than 10.

5. What can you say about the numbers **not** crossed out?

ACTIVITY 1.8 *Chain Letters*

Have you ever received a postcard as part of a chain?



Usually you are asked to send a scenic postcard to the name at the top, **A**, and then send further plain postcards to, say, **10** friends but this time with the names and addresses

B
C
D
Yours

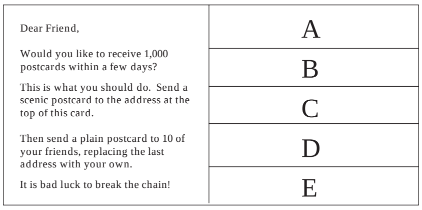
on the right-hand-side. You are then promised that you will receive thousands of scenic postcards in due course. We will analyse the situation and make predictions.

1. (a) Assuming that the chain continues, how many people will put your name in

   (i) 3rd place (ii) 2nd place (iii) 1st place

   on the postcard?

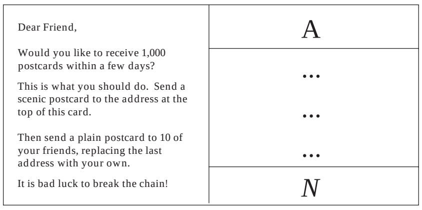
   (b) How many scenic postcards you should receive?



2. Suppose now that there are **5** names on the postcard and you are asked to obtain **4** people for the next stage of the chain.

   How many postcards should you now receive?

3. We can now generalise the problem by having n names on the postcard and by introducing m new people at each level.



   (a) How many people will put your name in

   (i) last place (ii) 2nd last last place

   (iii) first place?

   (b) How many scenic postcards should you receive?

Extension

Suppose that you want to receive at least 1,000 postcards. Assuming that no one breaks the chain, investigate possible values for m and n.

ACTIVITY 1.9 *Define*

The object of this whole class activity is to encourage familiarity with different types of numbers.

From a given set of numbers, for example:

2	3	5	9	12	16	17	21
25	27	31	35	36	41	45	48
49	52	64	67	68	72	77	78

you chose one number and ask the class to define it uniquely so that the definition would not fit any of the other numbers in the list. The statements that can be used are:

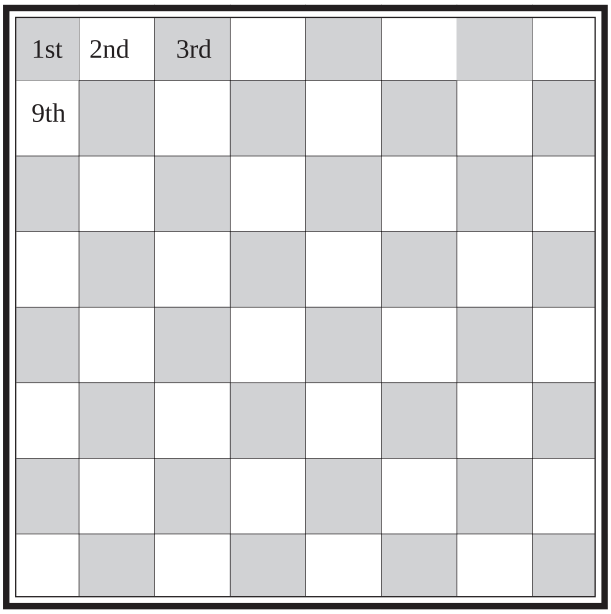
- is a factor of
- is a multiple of
- is a square number
- is a prime number
- is less than
- is greater than
- is not

etc.

(The game becomes more difficult if you do not allow the use of 'is less than' or 'is greater than'.)

ACTIVITY 1.10 *Chess Towers*

A chess board has 8×8 squares, as shown below.



1. If 1 coin is placed on the 1st square
   2 coins are placed on the 2nd square
   4 coins are placed on the 3rd square
   8 coins are placed on the 4th square
   etc.

   how many coins will there be on the 64th square?

2. If all the coins are 1p coins, how much money is on the 64th square?

3. If the thickness of a 1p coin is 1 mm, and the coins are placed one on top of the other, how tall is the tower on the 64th square (assuming it does not collapse!)?

ACTIVITY 1.11 *Standard Index Form*

The object of this game is to calculate numbers in standard index form to see which is the largest or smallest. It is suitable as a whole-class activity.

For each of two numbers, a and b, given in standard form, determine which of

- $a \times b$
- $a \div b$
- $b \div a$

is the largest number and which the smallest number.

For example:

1. $a = 4 \times 10^3$, $b = 2 \times 10^{-4}$
2. $a = 5 \times 10^5$ $b = 2 \times 10^2$
3. $a = 3 \times 10^4$ $b = 8 \times 10^5$

Extension

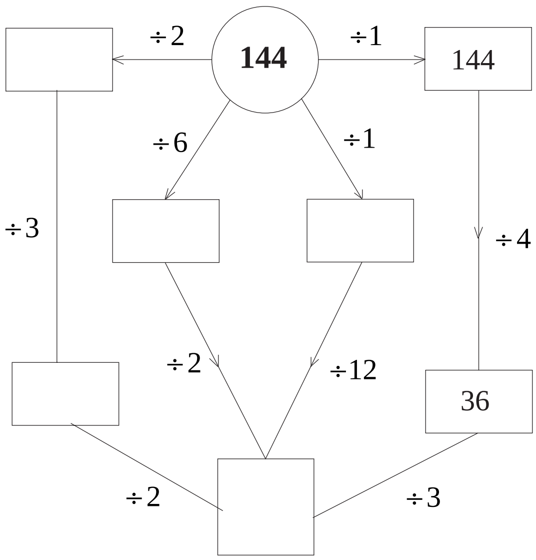
Can you find a rule that determines which of the numbers is the largest?

ACTIVITY 1.12 *Division Networks*

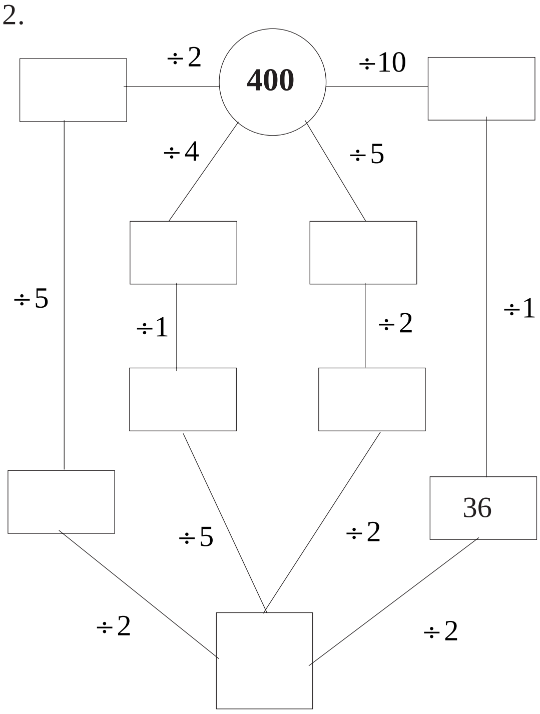
Without using a calculator, complete each diagram as quickly as possible by filling in each box with the correct number.

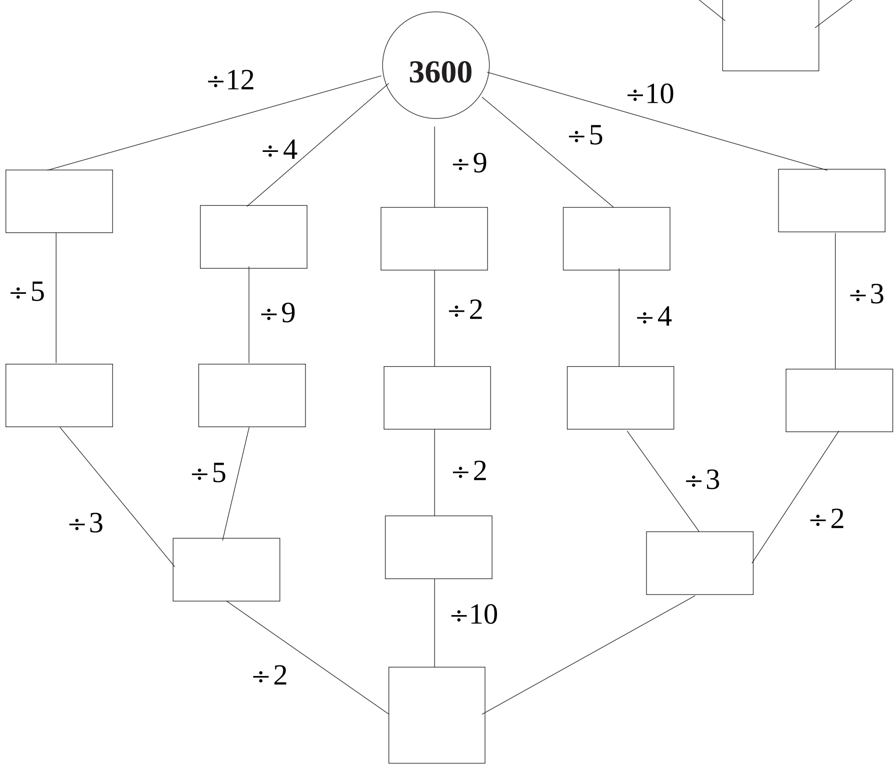
The first part of Problem 1 has been done for you.

1.



2.





Extension

Design your own network and give it to a friend to complete.