UNIT 2 Formulae

NC: Algebra 3a, b, c

		St	Ac	Ex	Sp
TOPICS (Text and Practice Books)					
2.1	*Using Formulae*	✓	-	-	-
2.1	*Construct and Use Simple Formula*	✓	✓	-	-
2.3	*Revision of Negative Numbers*	✓	-	-	-
2.4	*Substitutions with Formula*	✓	✓	✓	✓
2.5	*More Complex Formula*	×	✓	✓	✓
2.6	*Changing the Subject*	✓	✓	✓	✓
2.7	*Further Change of Subject*	×	✓	✓	✓
2.8	*Expansion of Brackets*	×	✓	✓	✓
2.9	*Factorisation*	×	✓	✓	✓
2.10	*Algebraic Manipulation*	×	×	✓	✓
2.11	*Algebraic Fractions*	×	×	✓	✓
Activities (* particularly suitable for coursework tasks)					
2.1*	*Rectangular grids*	✓	✓	✓	✓
2.2*	*Monumental Towers*	✓	✓	✓	✓
2.3	*Physical Fitness*	✓	✓	✓	-
2.4	*Horseshoes*	✓	✓	✓	✓
2.5	*Hill Walking*	✓	✓	✓	-
2.6	*Spectacles Power*	-	-	✓	✓
2.7	*Bode's Law*	-	✓	✓	✓
2.8*	*Flower Beds*	✓	✓	✓	✓
2.9*	*Pendulums*	✓	✓	✓	✓
2.10	*Chill Factor*	×	-	✓	✓
2.11	*Heptathlon*	×	-	✓	✓
2.12	*Algebraic Manipulation*	×	×	✓	✓
OH Slides					
2.1	*Temperature Differences*	✓	✓	-	-
2.2	*Kinematics*	-	✓	✓	✓
Mental Tests					
2.1		✓	✓	-	-
2.2		✓	✓	-	-
2.3		×	×	✓	✓
2.4		×	×	✓	✓
Revision Tests					
2.1		✓	-	-	-
2.2		×	✓	-	-
2.3		×	×	✓	✓

UNIT 2 *Formulae* Teaching Notes

Background and Preparatory Work

In one sense, people use 'formulae' all the time - often without realising it: a glazier may use tables to work out the cost of panes of glass, and a decorator will have his own 'rule of thumb' for estimating how much paint is needed for the outside of a house. These routines calculate a *required number* (the price of glass or the number of litres of paint needed); but they are often hard to understand because they are not expressed mathematically.

A mathematical formula should take the form of an *equation* which expresses some required quantity in terms of other, easily measured quantities. For example

area of rectangle = length $\times$ breadth.

This is a good beginning, but it is only a beginning. The ancient Babylonians and and Egyptians (c. 2000 BC) used many approximate calculational routines and 'formulae' of this kind, but one cannot do mathematics with words.

To go further we have to replace friendly words by abstract symbols and extend the familiar arithmetic numbers to an 'arithmetic of letters': that is, we need to develop *algebra.*

The word 'algebra' comes from the title of a book

Al - jabr w'al muqabala

written in 830 AD by the Arabic astronomer Al - Khurarizuri. The extract translation of the title is disputed: 'al - jabr' means something like "restoring and balancing" (which refers to the idea of shifting things from one side of an equation to the other), and 'w'al muqabala' means something like "cancelling and simplifying". These two ideas reflect the central art of algebra - namely that of 'rearranging and simplifying expressions'.

Full blooded elementary algebra (in which, for example, the general quadratic equation can be written as

$$ax^2 + bx + c = 0$$

and its solution given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a})$$

was only developed after 1600 AD. The effect was dramatic. After waiting 3500 years for an effective language, the next century saw an explosive growth - with the rise of coordinate geometry (Descartes 1637) and calculus (Newton, Leibniz 1660 - 1684).

Around 1600 Galileo observed that "The Book of Nature is written in the language of mathematics". It soon became clear that the language of all mathematics is algebra!

Teaching Points

Introduction

This is your chance to motivate the use of algebra, which is central to all subsequent mathematics and to nearly all meaningful applications of mathematics. Pupils need to be equally confident at numerical approaches and algebraic approaches – in fact, the algebraic approach is of much more general applicability, and we must attempt to convince pupils of its importance. Also, in this Unit, you will be using some of the algebraic rules of notation, used earlier but not necessarily understood – these are clear and precise rules, and it is important that pupils learn to abide by them and gradually come to understand why they are correct. You will need to sort out any misconceptions that may already have been picked up.

- For the *Standard* route, you can start with multilink cuboids, asking how many faces are seen, and how many are hidden. This can be generalised.
- For other routes, you can use the *Equations of Motion* OH slide to motivate substitution of numbers, and change of subject.

OS 2.2

- You can use problems like

  I'm thinking of a number;
  double it;
  add 3;
  the answer is 49.
  What is the number I started with?

  to see either the equivalence of the algebraic approach $(2x + 3 = 49)$ or the non-algebraic backwards method, $49 - 3 \Rightarrow \frac{46}{2} = 23$.

Language / Notation

- Note that $3 \times n = 3n$

  $(3n)^2 = (3n) \times (3n) = 9n^2$

Key Points

- Equations must always balance.
- You must always write down clearly the operation which has taken place on each line, e.g. make a the subject of

$$4a + b = c$$
$$\Rightarrow \quad 4a = c - b \quad \text{(add } (-b) \text{ to both sides)}$$
$$\Rightarrow \quad a = \frac{c - b}{4} \quad \text{(divide both sides by 4)}$$

Misconceptions

- $\frac{1}{x} + \frac{1}{y}$ is never equal to $\frac{1}{x + y}$
- The rules of $(2x)^2$ when, for example, $x = 5$, is not $2 \times 25 = 50$.

Activities

2.1 Rectangular Grids*
This is a task that can be used with all abilities, and can be tackled from a practical or algebraic approach. It could be used for a whole class interactive discussion.

2.2 Monumental Towers*
Similar to 2.1 above; this is particularly suited to a practical approach to start with.

2.3 Physical Fitness
This can form the basis of a useful collaboration with the PE department. The final extension (finding the graphical illustration) is only suitable for *Express* and *Special* routes.

2.4 Horseshoes
Interesting applications of mathematics, particularly for those who are interested in horse-riding.

2.5 Hill Walking
Again, this can be a practical problem, and would be a suitable activity to apply to a local hill (if you have one!).

2.6 Spectacles Power
This involves the use of more complex formulae, and so is particularly suitable for the *Express* and *Special* routes. The final extension gives the real practical applications.

2.7 Bode's Law
This is interesting, since it is an empirical law, based only on observation, and (as yet) there is no scientific basis for it. Nevertheless, it provides an excellent motivator for use of indices (2^n). You can use this as a whole class activity, but you will need to draw the graph accurately (on, for example, graph paper on an OHP).

2.8 Flower Beds*
This is a good starter for coursework, since different shapes and configurations can be considered. It can be used with all routes; a practical approach can be adopted.

2.9 Pendulums*
This is an excellent practical activity, which shows the basis of real pendulums. It can be used for coursework, or for group work in the classroom. (You will need though suitable equipment – your science department should be able to help you.)

2.10 Chill Factor
This is an interesting application for mathematically able pupils. The letter problem (3 to 5), in particular, will require pupils to draw graphs accurately.

2.11 Heptathlon
This is the official formula used in the points scoring for the heptathlon. This can be used as a practical application in collaboration with your PE department, but note that the formula requires the use of non-integer powers.

2.12 Algebraic Manipulation

This is a good whole class activity for the *Express* route, with pupils taking turns to find the next cell to move to. You could get pupils to design their own mazes.

Applications

The activities give a good range of applications. The most important applications relate maths to kinematics (see slide 2.2) but you can see from the activities that applications exist in many different disciplines. You need to stress these practical applications of mathematics, as these form the basis of why maths has such a prominent place in education!

OS 2.2