

UNIT 3 *Angle Geometry*

Activities

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- 3.1 Rotational and Line Symmetry
 - 3.2 Symmetry of Regular Polygons
 - 3.3 Special Quadrilaterals
 - 3.4 Sam Loyd's Dissection
 - 3.5 Overlapping Squares
 - 3.6 Line Segments
 - 3.7 Interior Angles in Polygons
 - 3.8 Lines of Symmetry
 - 3.9 Angles in Circles
 - 3.10 Angles in the Same Segment
- Notes and Solutions (1 page)

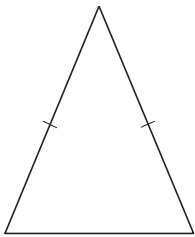
ACTIVITY 3.1

Rotational and Line Symmetry

1. For each polygon below:

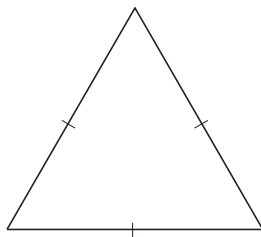
- (a) use dotted lines to show the lines of symmetry, if any;
- (b) check whether it has rotational symmetry and if so, state its order;
- (c) mark the centre of rotational symmetry with a cross (x).

(i)



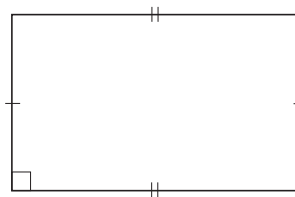
Order _____

(ii)



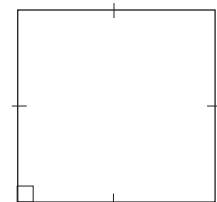
Order _____

(iii)



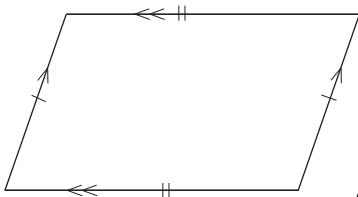
Order _____

(iv)



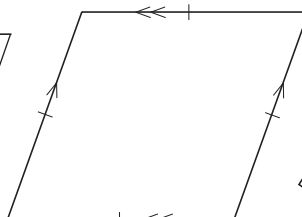
Order _____

(v)



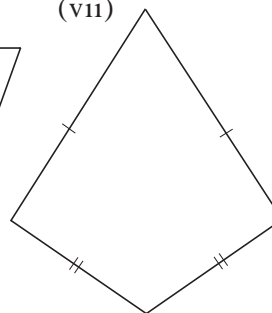
Order _____

(vi)



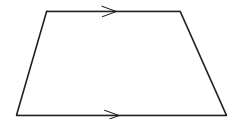
Order _____

(vii)



Order _____

(viii)



Order _____

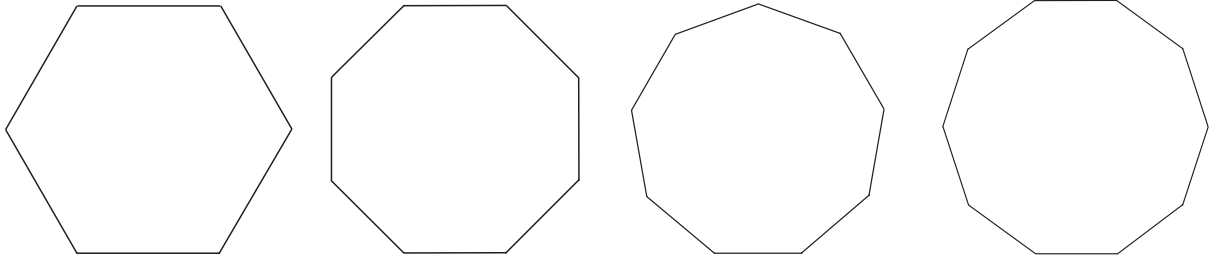
2. Use the results from Question 1 to complete the following table.

	Name of Polygon	Number of Lines of Symmetry	Order of Rotational Symmetry
(i)	<i>Isosceles triangle</i>		
(ii)	<i>Equilateral triangle</i>		
(iii)	<i>Rectangle</i>		
(iv)	<i>Square</i>		
(v)	<i>Parallelogram</i>		
(vi)	<i>Rhombus</i>		
(vii)	<i>Kite</i>		
(viii)	<i>Trapezium</i>		

ACTIVITY 3.2

Symmetry of Regular Polygons

1. For each of the following regular polygons, draw in the lines of symmetry and locate the centre of rotational symmetry.



2. Use your answers to Question 1 to complete the following table.

Name of Polygon	Number of sides	Number of Lines of Symmetry	Order of Rotational Symmetry
<i>Hexagon</i>			
<i>Octagon</i>			
<i>Nonagon</i>			
<i>Decagon</i>			

3. Use the completed table in Question 2 to find:

- (a) the number of lines of symmetry,
 (b) the order of rotational symmetry,

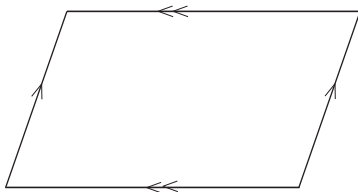
for

- (i) a regular 10-gon (ii) a regular 20-gon (iii) a regular n -gon.

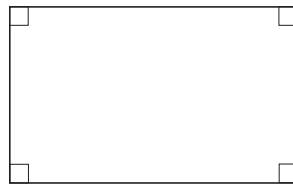
ACTIVITY 3.3

Special Quadrilaterals

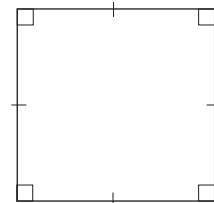
Fill in the table below to identify the properties of these special quadrilaterals.



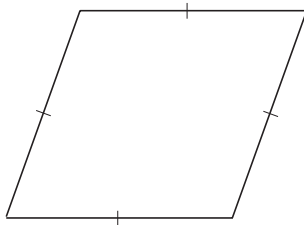
Parallelogram



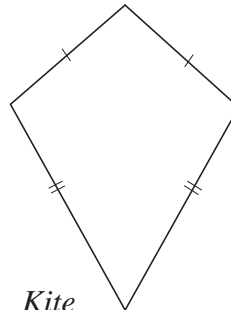
Rectangle



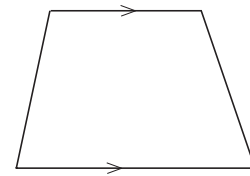
Square



Rhombus



Kite



Trapezium

Property						
All sides equal						
Opposite sides equal						
Opposite sides parallel						
Opposite angles equal						
Diagonals equal						
Diagonals bisect each other						
Diagonals intersect at right angles						
Longer diagonal bisects shorter diagonal						
Two pairs of adjacent sides equal but not all sides equal						
Only one pair of opposite sides parallel						
Only one pair of opposite angles equal						

ACTIVITY 3.4

Sam Loyd's Dissection

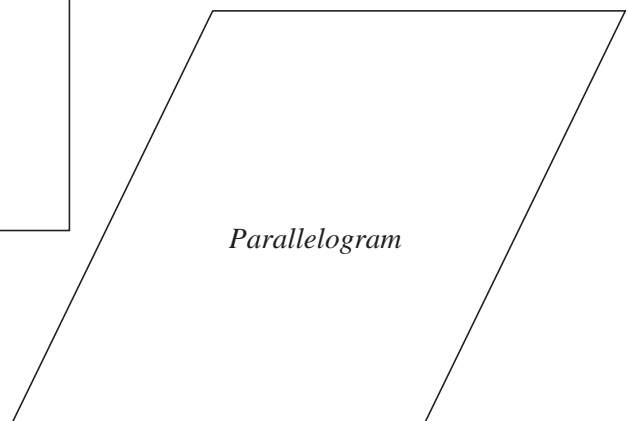
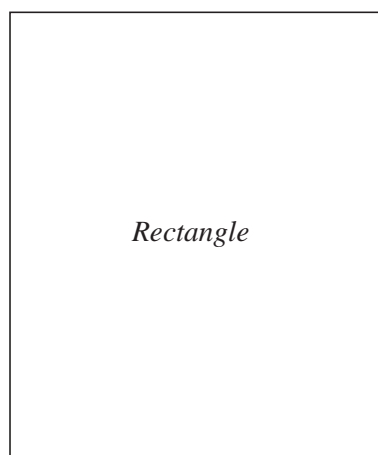
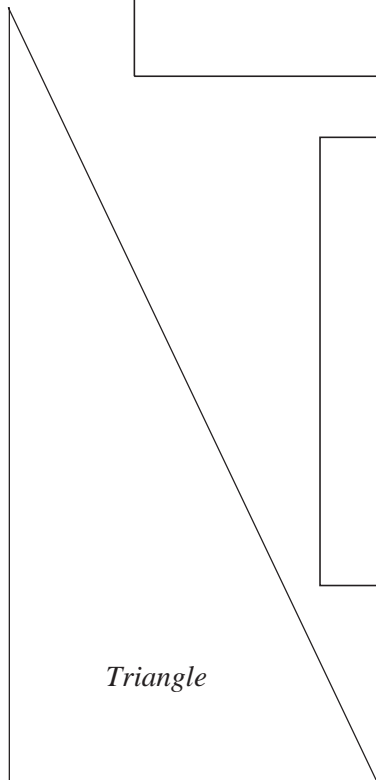
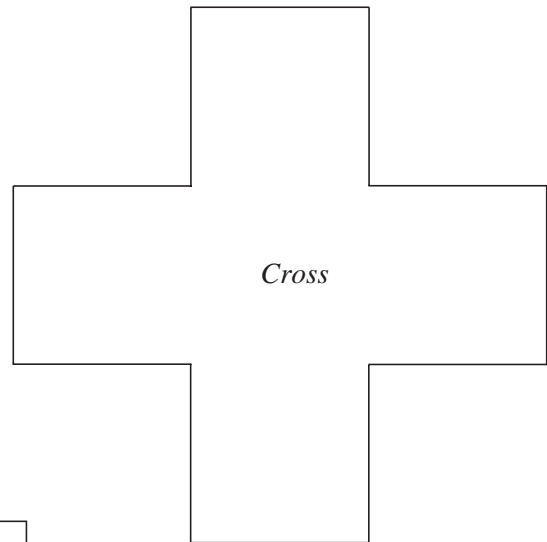
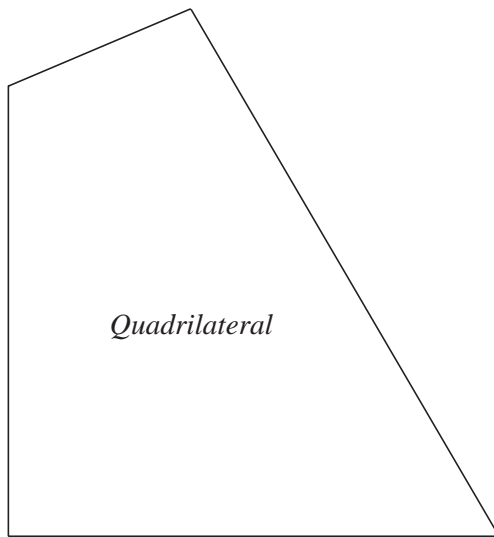
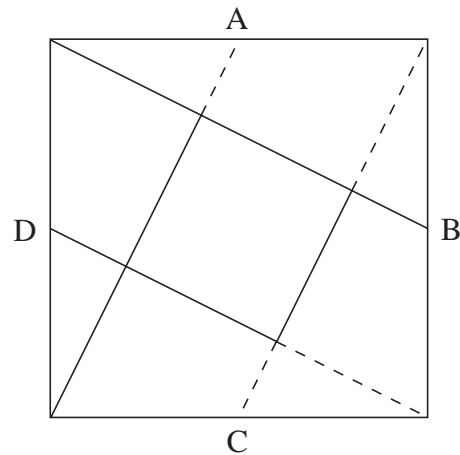
This famous dissection problem was designed by *Sam Loyd* in the 1920s.

Draw a 5 cm square as on the right. Find the mid-points (A, B, C and D in diagram) on each side and join them up.

Using the diagram as a guide, cut your square into 5 pieces along the **bold** lines.

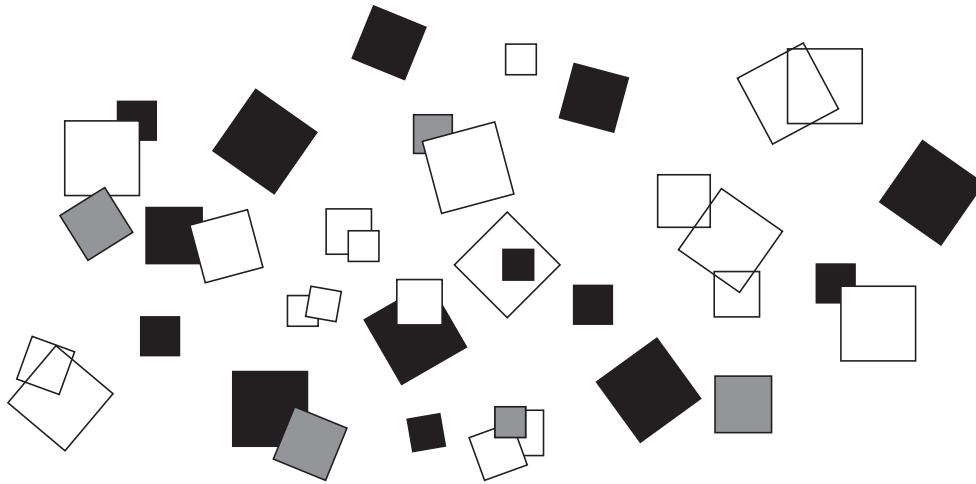
Do not cut along the dotted lines.

With the 5 pieces, try to make all the shapes below.



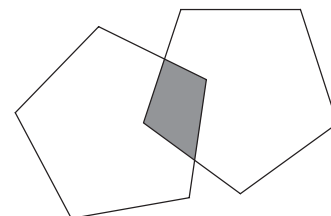
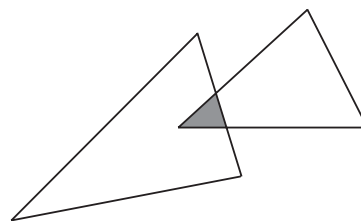
ACTIVITY 3.5

Overlapping Squares



Take two squares and put them down on a surface so that they overlap. The squares can be of any size, not necessarily the same.

1. Which of the following shapes can be formed by the overlap:
 - (a) rectangle
 - (b) square
 - (c) kite
 - (d) rhombus?
2. Can two squares intersect so that a triangle is formed by the overlap?
3. Can two squares intersect so that the overlap forms a polygon of n sides for values of n equal to
 - (a) 5
 - (b) 6
 - (c) 7
 - (d) 8
 - (e) 9
 - (f) 10?
4. What happens when two triangles overlap?



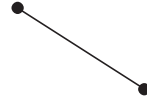
Extensions

1. What happens when two pentagons overlap?
2. What happens when two *different* shapes, e.g. square and triangle, overlap?

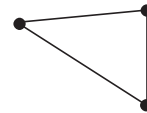
ACTIVITY 3.6

Line Segments

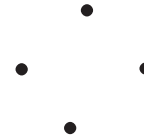
When you join 2 points by a straight line you need **one** line.



When you join 3 points (not on the same straight line) you need **3** line segments.



The situation for 4 points becomes more complex if each point has to be joined to every other point.



1. Show that you need 6 line segments to join each point to every other point when there are 4 *non-collinear* (not on same straight line) points.
2. Repeat this problem for n points when $n = 5, 6, 7$ and 8 .
3. Copy and complete the table below.

No. of points n	No. of lines L
2	1
3	
4	
5	
6	
7	
8	

Extension

1. Study the pattern. What is the formula which connects L and n ?
2. What do you predict is the value for L when $n = 10$? Verify your result.

ACTIVITY 3.7

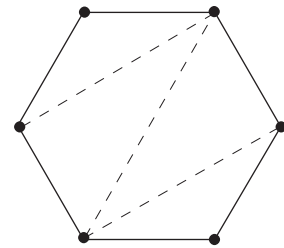
Interior Angles in Polygons

You can find the sum of the interior angles in any polygon by dividing it up into triangles with lines connecting the vertices.

For example, the hexagon shown opposite has been divided into 4 internal triangles.

The sum of all the interior angles of the hexagon is equal to the sum of all the angles in each triangle; so

$$\text{sum of interior angles} = 4 \times 180^\circ = 720^\circ.$$



1. Repeat the same analysis for the following shapes:

- (a) quadrilateral (b) pentagon (c) heptagon
 (d) octagon (e) nonagon (f) dodecagon.

2. Copy and complete the table.

Name of Polygon	Number of sides	Number of Triangles	Sum of Interior Angles
<i>Triangle</i>	3	1	180°
<i>Quadrilateral</i>			
<i>Pentagon</i>			
<i>Hexagon</i>	6	4	720°
<i>Heptagon</i>			
<i>Octagon</i>			
<i>Nonagon</i>			
<i>Dodecagon</i>			

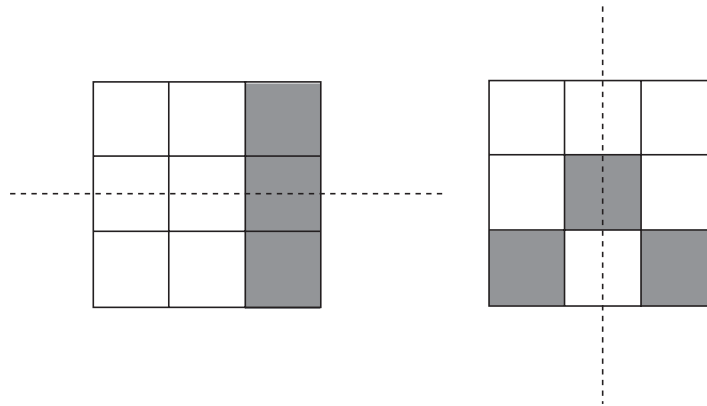
Extension

What is the formula for the sum of the interior angles of an n -gon?

ACTIVITY 3.8

Lines of Symmetry

Each of the 3×3 squares below has **3** shaded squares and **one** line of symmetry.



1. How many more ways can you find to shade 3 squares in a 3×3 square so that there is only one line of symmetry? Record your patterns.
2. (a) In a 3×3 square find a pattern of 3 shaded squares which has 2 lines of symmetry.
(b) Is it the only one? If not, try to find all such patterns.
3. Using a 3×3 square, find all the possible patterns of **4** shaded squares which have
 - (a) one line of symmetry
 - (b) two lines of symmetry
 - (c) three lines of symmetry
 - (d) four lines of symmetry.

Extension

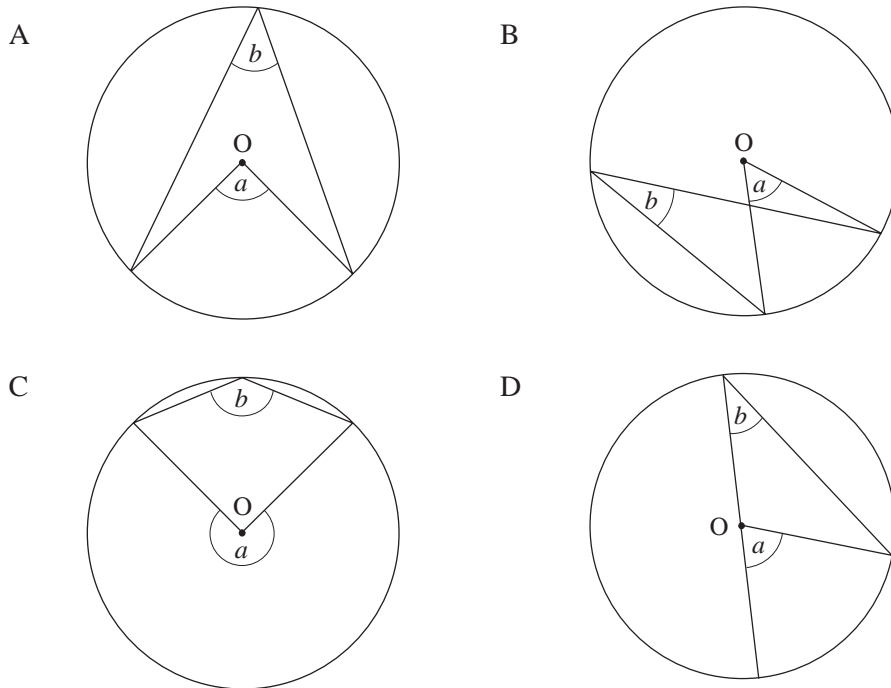
Do a similar study for a 4×4 square with different patterns of

- (a) 3 shaded squares
- (b) 4 shaded squares
- (c) 5 shaded squares
- (d) 6 shaded squares.

ACTIVITY 3.9

Angles in Circles

O is the centre of each of the circles. The angle at the centre, angle a , and the angle at the circumference, angle b , are subtended by the same arc.



1. For each circle,
 - (i) measure angles a and b
 - (ii) calculate the ratio $a : b$
 - (iii) copy and complete the table below.

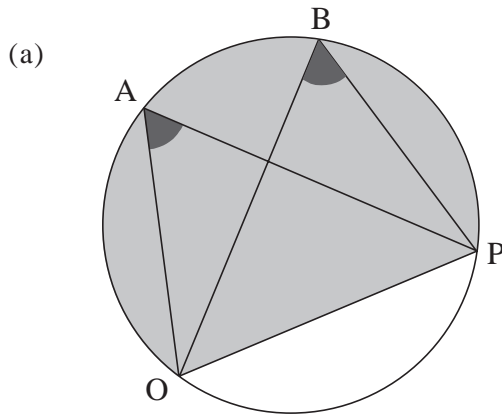
Circle	Angle at centre (angle a)	Angle at circumference (angle b)	Ratio $a : b$
<i>A</i>			
<i>B</i>			
<i>C</i>			
<i>D</i>			

2. What do you conclude about the relationship between the angle at the centre and the angle at the circumference subtended by the same arc?

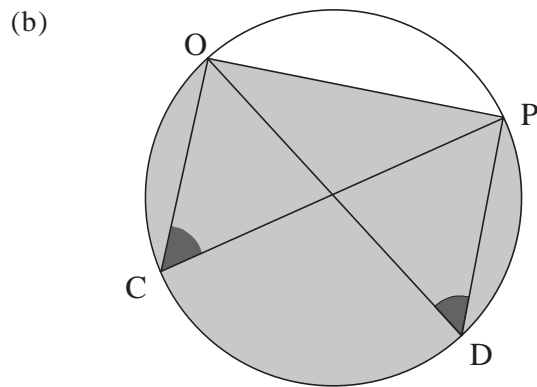
ACTIVITY 3.10

Angles in the Same Segment

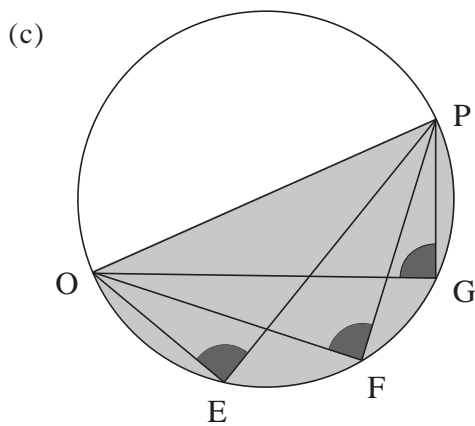
1. For each circle below, measure the *shaded* angles (i.e. angles in the same segment subtended by the chord OP) and record your results in the tables.



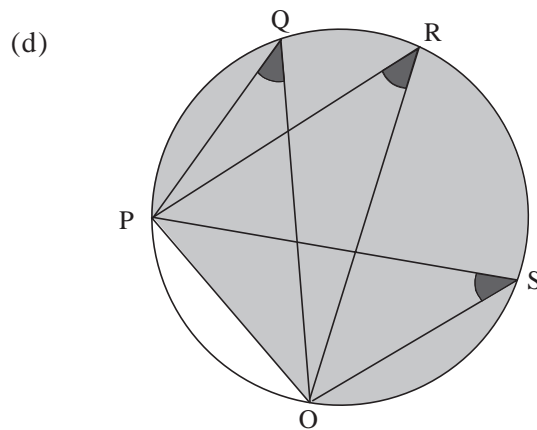
Angle	Size
\widehat{OAP}	
\widehat{OBP}	



Angle	Size
\widehat{OCP}	
\widehat{ODP}	



Angle	Size



Angle	Size

2. From your results, what can you say about angles in the same segment?