UNIT 3 Angle Geometry

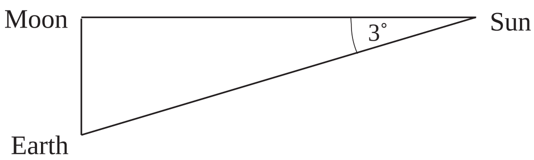
NC: Shape, Space and Measures 2 b,c,d

		St	Ac	Ex	Sp
TOPICS (Text and Practice Books)					
3.1	*Measuring Angles*	✓	-	-	-
3.2	*Line and Rotational Symmetry*	✓	-	-	-
3.3	*Angle Geometry*	✓	✓	-	-
3.4	*Parallel and Intersecting LInes*	✓	✓	-	-
3.5	*Angle Symmetry*	✓	✓	✓	✓
3.6	*Symmetry Properties*	✓	✓	✓	✓
3.7	*Compass Bearings*	✗	✓	✓	✓
3.8	*Angles and Circles 1*	✗	✗	✓	✓
3.9	*Angles and Circles 2*	✗	✗	✓	✓
3.10	*Circles and Tangents*	✗	✗	✓	✓
Activities (* particularly suitable for coursework tasks)					
3.1	*Rotational and Line Symmetry*	✓	-	-	-
3.2	*Symmetry of Regular Polygons*	✓	-	-	-
3.3	*Special Quadrilaterals*	✓	-	-	-
3.4	*Sam Loyd's Dissection*	✓	✓	-	-
3.5*	*Overlapping Squares*	✓	✓	✓	✓
3.6*	*Line Segments*	✓	✓	✓	✓
3.7	*Interior Angles in Polygons*	✓	✓	✓	✓
3.8*	*Lines of Symmetry*	✗	✓	✓	✓
OH Slides					
3.1	*Triangles*	✓	-	-	-
3.2	*Special Quadrilaterals*	✓	-	-	-
3.3	*Angles and Parallel Lines*	✓	✓	✓	-
3.4	*Geometry Properties of Circles*	✗	✓	✓	✓
3.5	*Tangent-Circle Properties*	✗	✗	✓	✓
3.6	*3D Shapes*	✓	✓	✓	✓
3.7	*Compass Directions*	✓	✓	✓	✓
Mental Tests					
3.1, 3.2, 3.3, 3.4		✓	✓	✓	✓
Revision Tests					
3.1		✓	-	-	-
3.2		✗	✓	-	-
3.3		✗	✗	✓	-
3.4		✗	✗	✗	✓

UNIT 3 *Angle Geometry* Teaching Notes

Background and Preparatory Work

The origins of angle measures are not due to any one person but a variety of developments in different countries. For example, *Aristarchus* (around 260 BC) in his treatise, *On the Sizes and Distances of the Sun and Moon*, made the observation that when the moon is half full, the angle between the lines of sight to the sun and the moon is less than a right angle by one-thirtieth of a quadrant (the systematic use of the $360°$ circle came a little later). In today's language this gave the angle $3°$ in the diagram below.

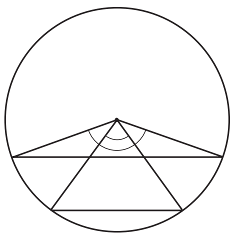


In fact, it should have been about $0° 10°$.

It is not known just when the systematic use of $360°$ was established but it seems likely to be largely due to *Hipparchus* (180–125 BC) who was thought to have produced the first trigonometric table.

It was firmly established by Ptolemy (c 100-178 AD) who used it consistently in his astronomical treatise.

He noted that the ratio of arc to chord reduced as the angle subtended at the centre decreased, with a limit of 1.



He actually produced tables giving values for angles varying from $0°$ to $100°$.

Although the use of $360°$ was adopted by most mathematicians, the idea of using $400°$ for a circle was developed in Scandinavian countries and is in fact still used on a limited basis. (It is even included on most calculators with the 'grad' mode for angles.)

Teaching Points

Introduction

The material and concepts in this unit are probably already familiar to many pupils, so you will need to work on motivation. Angle Geometry is one topic in mathematics where 'proof' is still important and it is an ideal topic for demonstrating that proof is at the heart of mathematical analysis. Some of the theorems and results may *look* obvious but still require a proof to be sure of the result. The difference between a proof, conjecture and verification should be made clear, especially to *Express* and *Special* routes.

It is also an appropriate time to revise the names of familiar shapes (both 2-D and 3-D) and to relate mathematics to more practical problems concerned with compass bearings. You should at least mention that problems dealt with here are all 2-dimensional, although for many applications, the 3-dimensional situation is clearly crucial.

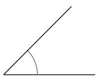
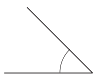
OS3.1 and 3.2
OS 3.6
OS 3.7

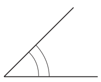
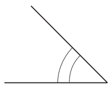
Many of the activities are suitable for whole class investigations and introductions, whilst others are particularly suitable for coursework.

A3.2, 3.5, 3.7, 3.9 and 3.10
A3.5, 3.6, 3.8

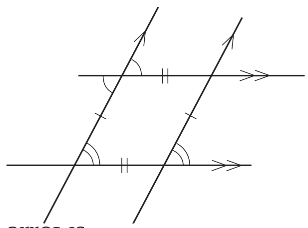
Language / Notation

- It is important always to use the degrees sign, e.g. $60°$, $40°$
- It is convention to label equal angles with the same notation, e.g.

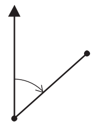
using  and  for the 1st pair of equal angles ,

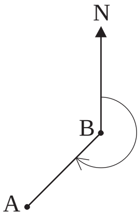
using  and  for the 2nd pair of equal angles, etc.

A similar notation is used for equal sides, as shown opposite.



- Parallel lines are marked similarly by open arrows.
- In finding bearings, say A from B, you must find the angle, measured *clockwise*, made by the N direction with the line from B to A.





Key Points

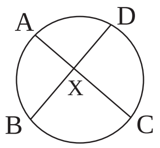
- We now use decimals to measure angles for compass bearings if accuracy is needed, e.g. $30.12°$, which means $30°$ and 0.12 of a degree, **not** $30°$ and $12'$ (minutes). If you use degrees/minutes, then be sure to use the correct notation, e.g. $45° \; 57'$.
- Proofs are important for many of the results in this unit.
- Pupils should be familiar with the usual 2-dimensional and 3-dimensional names of common shapes.
- A right angle is *exactly* $90°$.

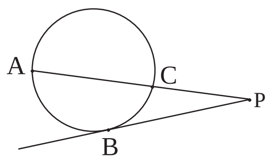
Misconceptions

- The terminology used is inclusive, e.g. a square is a special rectangle; a parallelogram is a special quadrilateral.

Key Concepts

		St	A	E	Sp
1.	Angles in a triangle sum to $180°$.	✓	✓	✓	✓
2.	Angles in a quadrilateral sum to $360°$.	✓	✓	✓	✓
3.	Alternate angles are equal.	✓	✓	✓	✓
4.	Supplementary angles sum to $180°$.	✓	✓	✓	✓
5.	The angle at the centre of a regular n-sided polygon is $\frac{360°}{n}$.	✓	✓	✓	✓
6.	The angle subtended on the circumference of a circle by a diameter is a right angle.	✗	✓	✓	✓
7.	The angles subtended at the circumference on the same side of a circle by a chord are equal.	✗	✓	✓	✓
8.	The angle subtended at the centre of a circle by an arc is twice the angle subtended at a point on the circumference.	✗	✓	✓	✓
9.	Opposite angles of a cyclic quadrilateral sum to $180°$.	✗	✓	✓	✓
10.	The angle subtended between a tangent and a chord equals any angle on the circumference subtended by the chord.	✗	✗	✓	✓
11.	For the circle, $AX.CX = BX.DX$.	✗	✗	✓	✓
12.	For the circle, $BP^2 = AP.CP$.	✗	✗	✓	✓





Activities

3.1 Rotational and Line Symmetry

This is a whole class activity to clarify both the concept of lines of symmetry and rotational symmetry.

3.2 Symmetry of Regular Polygons
This could be used as either a whole class interactive activity, with pupils drawing on the lines of symmetry or as an individual exercises.

3.3 Special Quadrilaterals
An activity to clarify the basic properties of the special quadrilaterals – again very suitable for a whole class activity.

3.4 Sam Loyd's Dissection
This is a good activity to fill in a gap or to take home to share with friends and family.

3.5 Overlapping Squares
This is designed to make pupils think. Part could be done in class and the extension used for coursework.

3.6 Line Segments
This is an investigation, which can result in a general formula being hpypothesised and checked with values. It is a possible coursework activity.

3.7 Interior Angles in Polygons
This is a suitable whole class activity but also includes postulating a general formula. It would give you an opportunity to discuss the differences between a 'proof' and a 'verification'.

3.8 Lines of Symmetry
Although this is a suitable whole class interactive activity, it could also lead, through the extension, to possible coursework.

3.9 Angles in Circles
This is a preliminary investigation to undertake before proving the result. Again it gives you the chance to distinguish between a conjecture, verification, theorem and proof.

3.10 Angles in the Same Segment
Similar comments to A3.9 – very much an introductory activity or homework exercise before proving the result.

Applications

The most important application here is that of bearings in navigation; the other geometric properties in this unit are relevant for planners and designers.