UNIT 4 Trigonometry

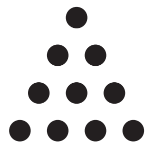
NC: Shape, Space and Measures 2 e,f

		St	Ac	Ex	Sp
TOPICS (Text and Practice Books)					
4.1	*Triangles and Squares*	✓	-	-	-
4.2	*Pythagoras' Theorem*	✓	✓	-	-
4.3	*Extending Pythagoras' Theorem*	✓	✓	-	-
4.4	*Sine, Cosine and Tangent*	✓	✓	✓	-
4.5	*Finding Lengths in Right Angled Triangles*	✓	✓	✓	✓
4.6	*Finding Angles in Right Angled Triangles*	✓	✓	✓	✓
4.7	*Mixed Problems*	✗	✓	✓	✓
4.8	*Sine and Cosine Rules*	✗	✗	✓	✓
4.9	*Angles Larger than* $90°$	✗	✗	✓	✓
Activities (* particularly suitable for coursework tasks)					
4.1	*Pythagoras' Theorem*	✓	✓	-	-
4.2*	*Spirals*	✓	✓	✓	✓
4.3	*Clinometers*	✓	✓	✓	✓
4.4	*Radar*	✗	✓	✓	✓
4.5*	*Posting Parcels*	✓	✓	✓	✓
4.6*	*Interlocking Pipes*	✗	✓	✓	✓
4.7	*Sine Rule*	✗	✗	✓	✓
OH Slides					
4.1	*Trigonometric Relationships*	✓	✓	✓	-
4.2	*Trigonometric Puzzle*	✓	✓	✓	-
4.3	*Sine and Cosine Functions*	✗	✗	✓	✓
4.4	*Tangent Functions*	✗	✗	✓	✓
Revision Tests					
4.1		✓	-	-	-
4.2		✗	✓	-	-
4.3		✗	✗	✓	✓

UNIT 4 *Trigonometry* Teaching Notes

Background and Preparatory Work

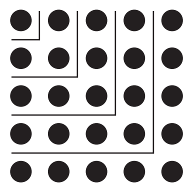
Very little is known of the life of *Pythagoras*, but he was born on the island of Samos and is credited with the founding of a community at Crotona in Southern Italy by about 530 BC. The community had religious and political purposes, but also dealt with mathematics, especially the properties of whole numbers or positive integers. Mystical attributes, such as that odd numbers were male and even numbers female, were ascribed to numbers. In addition descriptions of arithmetical properties of integers were found.



The 4th triangle number or 'Holy tetractys' had mystical significance for the Pythagoreans

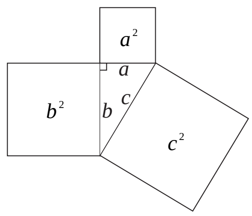
The diagram on the right shows that

$$1 = 1^2$$
$$1 + 3 = 2^2$$
$$1 + 3 + 5 = 3^2$$



The sum of consecutive odd numbers, starting at 1, is a square number

The Pythagoreans also formulated the idea of proportions in relation to harmonics on stringed instruments. The theorem with which Pythagoras' name is associated was probably only proved later. Specific instances of it were certainly known to the Babylonians. The 'Harpedonaptai', Egyptian rope stretchers, are said to have used the 3, 4, 5 triangle to obtain right angles from equally spaced knots on cords. The ancient Chinese also knew that the 3, 4, 5 triangle was right angles.



Pythagoras' Theorem:
$a^2 + b^2 = c^2$

The Greeks used 'chord' tables rather than tables of trigonometric functions, and the development of trigonometric tables took place around 500 AD, through the work of Hindu mathematicians. In fact, tables of sines for angles up to $90°$ were given for 24 equal intervals of $3\frac{3}{4}°$ each. The value of $\sqrt{10}$ was used for π at that time. Further work a century later, particularly by the Indian mathematician *Brahmagupta* (in 628), led to the sine rule as we know it today.

A useful course book for the historical introduction of these topics is '*Ascent of Man*' by J. Bronowski (BBC publication).

Teaching Points

Introduction

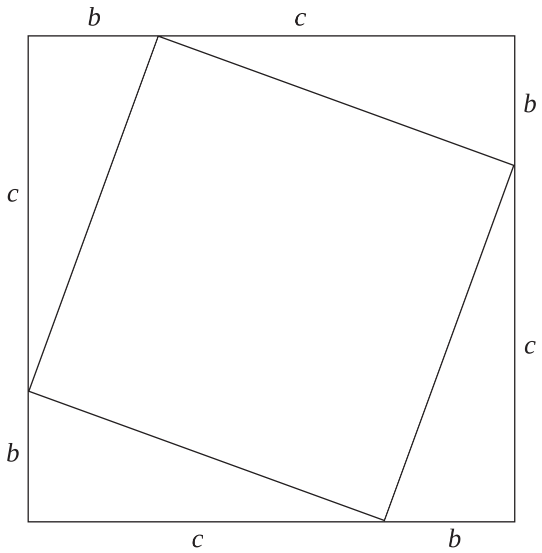
This is a topic that has obvious applications (in surveying, geography, architecture, etc.) and so the motivation for it should not prove too difficult. It is also a topic which UK students do particularly well at in comparison to many other countries!

The key building blocks are

- Pythagoras' Theorem A4.1
- trigonometric relationships OS 4.1
- sine and cosine rules. A 4.7

- For *Express/Special* students, the proofs of these results are important; for *Standard/Academic* students, we suggest that you explain that this result *can* be proved in numerous ways.

Here is one particular neat way. Form the square of side length, $b + c$, as shown.



Complete the square ABCD as shown (why is it a square?).

Let the length of a side of the square be a. Thus comparing areas of the original square,

$$(b+c)^2 = a^2 + 4 \times \left(\frac{bc}{2}\right)$$

$$\Rightarrow \quad b^2 + 2bc + c^2 = a^2 + 2bc$$

$$\Rightarrow \quad b^2 + c^2 = a^2, \text{ as required.}$$

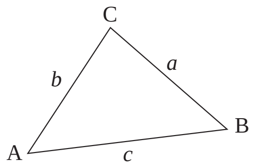
(Of course, some expertise in algebra is needed for this proof!)

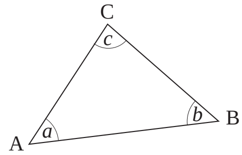
Language / Notation

- The important terms used here are

sine, cosine and tangent	T4.2	T4.2
opposite, adjacent and hypotenuse	T4.2	T4.2
altitude	T4.5	T4.5
angle of elevation	T4.6	T4.6

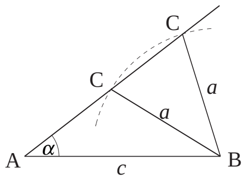
- For a general triangle, ABC, we describe the points at which two sides join as A, B and C; the opposite sides are described as side *a*, side *b* and side *c*, respectively. The angles themselves are described as angle *a*, angle *b* and angle *c*, respectively. Greek lower case letters are also often used for angles, e.g. α, θ.





Key Points

- In a *right* angled triangle, you can use Pythagoras' Theorem and trigonometric relationships to 'solve' the triangle.
- In a *non-right* angled triangle, there may not be a unique solution when two sides and a *non* included angle are given – the diagram opposite shows the case where there are two possible positions for the vertex C.



Misconceptions

- Students are often prone to writing down incorrect or incomplete statements, e.g. '70 tan' rather than '$\tan 70$'.
- The algebraic manipulation required, especially when solving for the length of the hypotenuse in a right angled triangle, can cause problems, e.g. $\tan 50 = \dfrac{20}{x} \Rightarrow x = 20 \tan 50$ is a common mistake.
- Identifying which are actually the 'adj, opp and hyp' in strangely annotated triangles causes problems (see A4.2).

Key Concepts

		St	A	E	Sp
1.	Pythagoras' Theorem	✓	✓	✓	✓
2.	Trigonometric Relationships (sine, cosine, tangent)	✓	✓	✓	-
3.	Sine Rule	✗	✗	✓	✓
4.	Cosine Rule	✗	✗	✓	✓

Activities

4.1 Pythagoras' Theorem

This is a verification of the result, and a useful whole class activity – it should though be stressed that the result does have a proof it is not just an experimental result!

4.2* Spirals

This is a good stretching activity, based on Pythagoras' Theorem. The extension is suitable only for *Express/Special* students, but this topic could provide a useful starter for coursework.

4.3 Clinometers

This is a simple but effective practical apparatus, which students should enjoy constructing and using.

4.4 Radar

This activity is put into a meaningful context, which could be extended and adapted.

4.5* Posting Parcels

There are many similar problems in the Post Office (and also with airline baggage) – the extension is a non-trivial problem, which could be a useful task to be explored for coursework.

4.6* Interlocking Pipes

Although this has been presented as a complete task, it could either be rewritten with less specific instructions, for coursework, or could be extended.

4.7 Sine Rule

This is just a practical verification of the sine rule and, again, you should show that the result can be proved, but that this is *not* a proof.

Applications

The main applications of trigonometry occur in surveying, geography and architecture, but it is also of crucial importance for navigation, warfare, etc. – although this is of course 3-dimensional rather than 2-dimensional.