UNIT 5 Probability

NC: Handling Data 3a - f

		St	Ac	Ex	Sp
TOPICS (Text and Practice Books)					
5.1	*Probability Scale*	✓	-	-	-
5.2	*Simple Probability*	✓	✓	-	-
5.3	*Outcomes of Two Events*	✓	✓	✓	-
5.4	*Finding Probabilities Using Relative Frequency*	✓	✓	✓	✓
5.5	*Determining Probabilities*	✓	✓	✓	✓
5.6	*Probability of Two Events*	✓	✓	✓	✓
5.7	*Tree Diagrams*	✓	✓	✓	✓
5.8	*Multiplication Law for Independent Events*	✗	✓	✓	✓
5.9	*Mutually Exclusive Events*	✗	✓	✓	✓
5.10	*Conditional Probability*	✗	✗	✓	✓
5.11	*Using Venn Diagrams to Find Probabilities*	✗	✗	✗	✓
Activities (* particularly suitable for coursework tasks)					
5.1	*Nearly the Nine O'Clock News*	✓	✓	-	-
5.2	*Evens and Odds*	✓	✓	-	-
5.3	*Experimental Probabilities*	✓	✓	✓	✓
5.4*	*A Russian Fable*	✓	✓	✓	✓
5.5*	*Break the Bank*	✓	✓	✓	✓
5.6*	*Open and Shut Case*	✓	✓	✓	✓
5.7*	*Fruit Machines*	✗	✓	✓	✓
5.8	*Seek and Win*	✗	✗	✓	✓
5.9	*Misconceptions*	✓	✓	✓	✓
5.10*	*Birthdays*	✗	✗	✓	✓
5.11*	*Genetic Fingerprinting*	✗	✗	✓	✓
OH Slides					
5.1	*Outcomes*	✓	✓	✓	✓
5.2	*Probabilities*	✓	✓	✓	✓
5.3	*Tossing Three Coins*	✓	✓	✓	✓
5.4	*Tossing Two Dice*	✓	✓	✓	✓
Mental Tests					
5.1		✓	✓	✓	✓
5.2		✓	✓	✓	✓
5.3		✗	✗	✓	✓
5.4		✗	✗	✓	✓
Revision Tests					
5.1		✓	-	-	-
5.2		✗	✓	-	-
5.3		✗	✗	✓	✓

UNIT 5 *Probability* Teaching Notes

Background and Preparatory Work

The origins of probability are not entirely clear, but we do know of discussions between Pascal and his friend, the Chevalier de Méré, in which, for example, they considered the problem,

"Are you more likely to obtain one six in 4 tosses of one fair die than to obtain at least one double six in 24 tosses of two fair dice?"

Some of these types of problems were published by Huygan in 1657 in his little tract *'On reasoning in Games of Dice'*. Other famous mathematicians had considered similar problems; for example, Galileo considered the problem

"Are you more likely to obtain a total of 9 when three fair dice are tossed than a total of 10?"

and Pepys asked Norton

"Which is more likely – one 6 when six dice are tossed or two 6s when 12 dice are tossed?"

These types of problems led to the Binomial distribution for probabilities, which includes the well known Pascal's triangle of coefficients, but this is beyond the scope of this Unit.

For preparatory work, make sure that you have an adequate supply of dice, spinners and coins – specialist suppliers can provide biased dice which can be used in experiments to find actual probabilities.

Teaching Points

Introduction

Probability is a remarkably interesting area of mathematics, used by many people to solve real problems. You see it for example, in airline safety - figures such as "1 in 100 million" are given for the probability of an engine failure. Nearer to home, the probability of winning the jackpot on the lottery is about 1 in 14 million and many people in this country regularly (or occasionally) bet on large races such as the Grand National. The concepts of probability are used in these types of games to ensure that the organisers, on average, always win and you, the punter, normally lose!

It is important to relate this topic to its well known applications so that pupils do make the connections and realise that what they are studying is a fundamental building block in a vital area of mathematics.

Some experiments have been suggested in the activities, and we would encourage you to adopt a practical approach when appropriate. What we think is obvious is not always so for your pupils, so experimentation could, and should be an important aspect of this unit.

Language / Notation

- There are some key words that are needed in this unit; these include

  Outcomes - events that can occur after an experiment.

  Probability space - the complete set of outcomes for the experiment.

  Relative frequency - the frequency of an event divided by the total frequency, and is used as an estimate for the probabilities of that event.

  Independent event - when the result of one event happening does not affect the probability of the other.

  Mutually exclusive event - when two events cannot happen at the same time.

OS 5.1

- You should also make sure that pupils are happy with the differences between terms such as

  impossible
  unlikely
  possibly
  likely
  certain

- The usual way of writing probabilities is either as a fraction or decimal; e.g. $\frac{1}{4}$ or 0.25. Other notations include, for example, '25%' or '1 in 4', but these are not to be encouraged for the purpose of GCSE assessment.

- We talk about a 'fair' coin or a 'fair' dice to mean that all the outcomes are equally likely. For a 'fair' coin ;

  $$p(\mathrm{H}) = p(\mathrm{T}) = \frac{1}{2} \text{ etc.}$$

  The alternative is that the coin or dice is 'biased'.

Key Points

- The probability of any event p, must satisfy $0 \leq p \leq 1$
- The sum of the probabilities if all outcomes to an experiment must be 1.

- When using tree diagrams, you always multiply along the branches to determine probabilities of combined events.

- Probabilities can either be found by
  symmetry - when all outcomes are the equally likely.
  experiment - when probabilities can be estimated.

- For independent events, A and B,

$$p(\text{A and B}) = p(\text{A}) \times p(\text{B})$$

- For mutually exclusive events, A and B,

$$p(\text{A or B}) = p(\text{A}) + p(\text{B})$$

Misconceptions

- The probability of an event must be ≤ 1. Any probability answer that is > 1 must be incorrect.
- Adding rather than multiplying probabilities (often A level candidates have problems here!) - for example, the probability of getting 3 'sixes' in three throws of a dice is

$$\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \text{ not } 3 \times \frac{1}{6} = \frac{1}{2}$$

(It seems obvious to us but not to many pupils!)

- If you obtain 4 Heads in a row when tossing a fair coin, then the probability of Heads on the fifth throw is still $\frac{1}{2}$. (This is often feels in conflict with the result that over a period of many tosses of the coin, the number of Heads will approximately equate to the number of Tails).

A5.9

Key Concepts

		St	A	E	Sp
1.	Experimental probability (Probability of event = $\frac{\text{frequency of event}}{\text{total frequency}}$)	✓	✓	✓	✓
2.	Theoretical probability (If all outcomes are equally likely Probability of particular outcome = $\frac{\text{no. of ways of obtaining outcome}}{\text{total no. of possible outcomes}}$)	✓	✓	✓	-
3.	Independent events (If two events, A and B, are independent, $p(\text{A and B}) = p(\text{A}) \times p(\text{B})$)	✗	✗	✓	✓
4.	Mutually exclusive events (If two events, A and B, are mutually exclusive, $p(\text{A or B}) = p(\text{A}) + p(\text{B})$)	✗	✗	✓	✓

Activities

5.1 Nearly the Nine O' clock News
Whole class activity to clarify the language and meaning of a probability scale.

5.2 Evens and Odds
This is a straightforward game, suitable for using with *Standard* pupils.

5.3 Experimental Probabilities
For all pupils, this is an experimental approach to assigning probabilities; an alternative is the drawing pin experiment in which you find the probability of a drawing pin landing point up or point down.

5.4 A Russian Fable
This is an interesting non trivial probability activity in which theoretical and experimental results can be compared. The problem can be extended for high level coursework tasks.

5.5 Break the Banks
This is an interesting method of attempting to break the bank without having to lose too much money on the way. Again, this could be useful for coursework.

5.6 Open and Shut Case
This always provokes discussion and it is not at all obvious what the best policy is. The game can be simulated on a computer, which could be useful for coursework.

5.7 Fruit Machine
This is one of many similar simulations, which can be undertaken to ensure that the owner will in time always win. This is particularly suitable for able pupils, and could be extended for coursework.

5.8 Seek and Win
This is an example of the game cards that shops, cafes, newspapers etc. often use. It is probably better done as a whole class interactive activity.

5.9 Misconceptions
These are designed for whole class discussion.

5.10 Birthday
A classical problem that still amazes non statisticians (try it in your staff room!). Only suitable for *Express/ Special* pupils, but easily extended for coursework.

5.11 Genetic Fingerprinting
Very topical but suitable only for *Express/ Special* pupils. Many possible extensions for coursework.

Applications

Many of the activities, and even examples and exercises, clearly show the practical use of probability. Other applications not dealt with here include

- National Lottery
- Horse race gambling
- Life insurance

but all these could be interesting topics to be studied for coursework.