16 Inequalities

16.1 Inequalities on a Number Line

An *inequality* involves one of the four symbols

$$>, \quad \geq, \quad < \quad \text{or} \quad \leq .$$

The following statements illustrate the meaning of each of them.

$x > 1$: x is greater than 1.

$x \geq -2$: x is greater than or equal to -2.

$x < 10$: x is less than 10.

$x \leq 12$: x is less than or equal to 12.

Inequalities can be represented on a number line, as shown in the following worked examples.

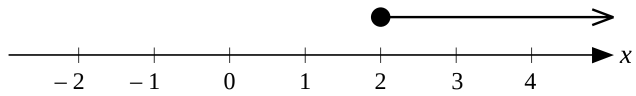
Worked Example 1

Represent the following inequalities on a number line.

(a) $x \geq 2$ (b) $x < -1$ (c) $-2 < x \leq 4$

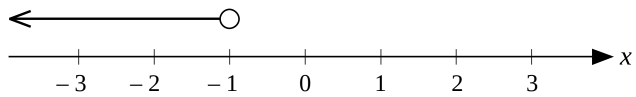
Solution

(a) The inequality, $x \geq 2$, states that x must be greater than or equal to 2. This is represented as shown.



Note that solid mark, •, is used at 2 to show that this value is included.

(b) The inequality $x < -1$ states that x must be less than -1. This is represented as shown.



Note that a hollow mark, ∘, is used at -1 to show that this value is *not* included.

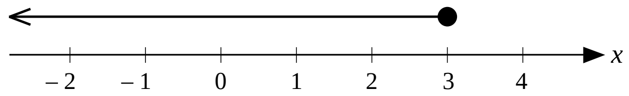
(c) The inequality $-2 < x \leq 4$ states that x is greater than -2 and less than or equal to 4. This is represented as shown.

Note that ∘ is used at -2 because this value is *not* included and • is used at 4 because this value *is* included.

Worked Example 2

Write an inequality to describe the region represented on each number line below.

(a)



(b)

Solution

(a) The diagram indicates that the value of x must be less than or equal to 3, which would be written as

$$x \le 3.$$

(b) The diagram indicates that x must be greater than or equal to -1 and less than 2. This is written as

$$-1 \le x < 2.$$

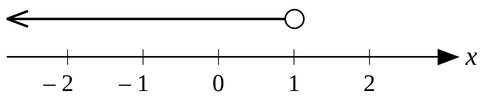
Exercises

1. Represent each of the inequalities below on a number line.

(a)	$x > 3$	(b)	$x < 4$	(c)	$x > -1$	(d)	$x < 2$
(e)	$x \ge 6$	(f)	$x \ge -4$	(g)	$x \le 3$	(h)	$x \le 1$
(i)	$2 \le x \le 4$	(j)	$-1 < x \le 2$	(k)	$-2 < x < 2$	(l)	$1 \le x \le 3$

2. Write down the inequality which describes the region shown in each diagram.

(a)



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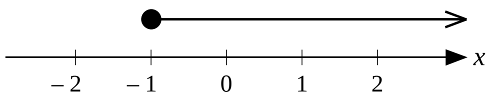
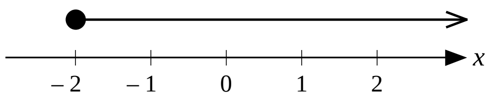


(c)



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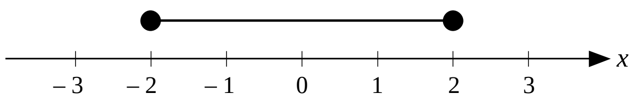
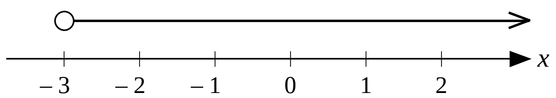
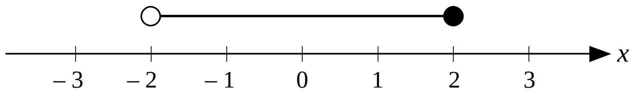
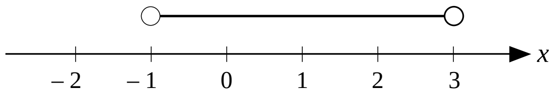
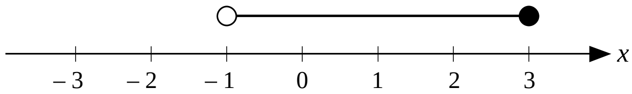


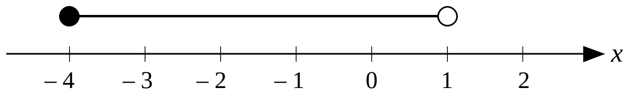
(e)



(f) 

(g) 

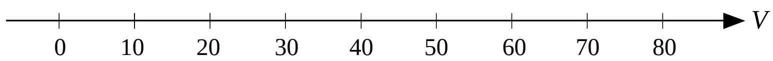
(h) 

(i) 

(j) 

3. On a motorway there is a minimum speed of 30 m.p.h. and a maximum speed of 70 m.p.h.

   (a) Copy the diagram below and represent this information on it. The letter, V, is used to represent the speed.

   (b) Write down an inequality to describe your diagram.

4. Frozen chickens will be sold by a major chain of supermarkets only if their weight is at least 1.2 kg and not more than 3.4 kg.

   (a) Represent this information on a number line.

   (b) Write an inequality to describe the region which you have marked.

5. List all the whole numbers which satisfy the inequalities below.

   (a) $1 \le x \le 8$ (b) $3 < x < 7$

   (c) $2 \le x < 5$ (d) $3 < x < 6$

6. List all the integers (positive or negative whole numbers) which satisfy the inequalities below.

   (a) $-1 \le x \le 1$ (b) $-5 \le x \le -2$

   (c) $-2 < x < 1$ (d) $-5 < x \le -3$

7. Write down one fraction which satisfies the inequalities below.

   (a) $\frac{1}{2} < x < 1$ (b) $\frac{1}{4} < x < \frac{1}{2}$

(c) $\frac{1}{3} < x < \frac{1}{2}$ (d) $0 < x < \frac{1}{3}$

8. List all the possible integer values of n such that

$$-3 \le n < 2.$$

(LON)

16.2 Solution of Linear Inequalities

Inequalities such as $6x - 7 \le 5$ can be simplified before solving them. The process is similar to that used to solve equations, except that there should be no multiplication or division by negative numbers.

Worked Example 1

Solve the inequality

$$6x - 7 \le 5$$

and illustrate the result on a number line.

Solution

Begin with the inequality

$$6x - 7 \le 5.$$

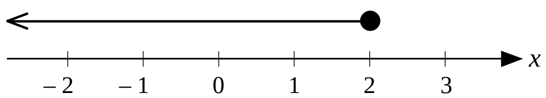
Adding 7 to both sides gives

$$6x \le 12.$$

Dividing both sides by 6 gives

$$x \le 2.$$

This is represented on the number line below.



Worked Example 2

Solve the inequality

$$4(x-2) > 20.$$

Solution

Begin with the inequality

$$4(x-2) > 20.$$

First divide both sides of the inequality by 4 to give

$$x - 2 > 5.$$

Then adding 2 to both sides of the inequality gives $x > 7$.

Worked Example 3

Solve the inequality

$$5 - 6x \geq -19.$$

Solution

Begin with the inequality

$$5 - 6x \geq -19.$$

In this case, note that the inequality contains a '$-6x$' term. The first step here is to add $6x$ to both sides, giving

$$5 \geq -19 + 6x.$$

Now 19 can be added to both sides to give

$$24 \geq 6x.$$

Then dividing both sides by 6 gives

$$4 \geq x$$

$$\text{or} \qquad x \leq 4.$$

Worked Example 4

Solve the inequality

$$-10 < 6x + 2 \leq 32.$$

Solution

Begin with the inequality

$$-10 < 6x + 2 \leq 32.$$

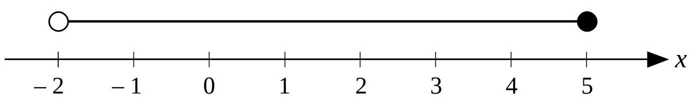
The same operation must be performed on *each part* of the inequality. The first step is to subtract 2, which gives

$$-12 < 6x \leq 30.$$

Then dividing by 6 gives

$$-2 < x \leq 5.$$

The result can then be represented on a number line as shown below.



An alternative approach is to consider the inequality as two separate inequalities:

$$(1) \quad -10 < 6x + 2 \qquad \text{and} \qquad (2) \quad 6x + 2 \leq 32.$$

These can be solved as shown below.

(1) $-10 < 6x + 2 \quad (-2)$

$-12 < 6x \quad (\div 6)$

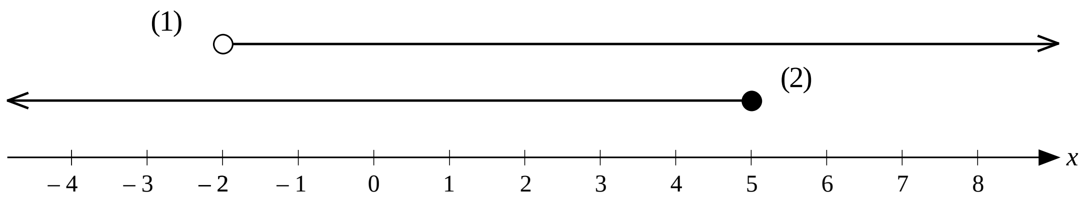
$-2 < x$

(2) $6x + 2 \leq 32 \quad (-2)$

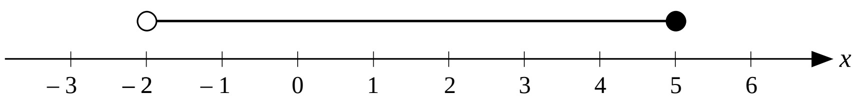
$6x \leq 30 \quad (\div 6)$

$x \leq 5$

Both inequalities can then be displayed as shown below.



Where the two lines overlap gives the solution as



Exercises

1. Solve each inequality below and illustrate the solution on a number line.

(a) $4x + 6 \leq 18$ (b) $5x - 8 > 27$ (c) $6x + 7 < 37$

(d) $5x - 7 \geq -17$ (e) $\dfrac{2x - 6}{4} < 1$ (f) $4(2x - 3) \geq -8$

2. Solve the following inequalities.

(a) $5x - 6 < 29$ (b) $7x + 2 \geq -19$ (c) $5x - 8 \leq 12$

(d) $6x - 3 \geq 1$ (e) $5x + 7 < 11$ (f) $7x - 8 > -2$

(g) $4 - 2x \geq -8$ (h) $5 - 6x < -7$ (i) $4(2 - x) \geq -8$

(j) $7 + 4x \geq 19$ (k) $6 - 5x > -10$ (l) $\dfrac{3x - 4}{2} \geq 3$

3. Solve each of the following inequalities and illustrate them on a number line.

(a) $5 < 3x + 2 \leq 11$ (b) $3 \leq 4x - 5 < 15$

(c) $11 \leq 4x + 7 < 27$ (d) $5 < 6x - 7 < 29$

(e) $-24 \leq 3x - 9 \leq -14$ (f) $-3 \leq 4x + 5 < 17$

4. Solve each of the following inequalities.

(a) $-3 < 5x + 2 \leq 6$ (b) $1 \leq 4x - 7 < 11$

(c) $22 < 3(x + 7) < 23$ (d) $-8 \leq 4(2x - 3) \leq 3$

(e) $\dfrac{1}{2} \leq \dfrac{4x + 5}{2} \leq 1$ (f) $1 < \dfrac{6x - 1}{5} < 7$

5. Chris runs a barber's shop. It costs him £20 per day to cover his expenses and he charges £4 for every hair cut.

   (a) Explain why his profit for any day is £ $(4x - 20)$, where x is the number of haircuts in that day.

   He hopes to make at least £50 profit per day, but does not intend to make more than £120 profit.

   (b) Write down an inequality to describe this situation.

   (c) Solve the inequality.

6. The distance that a car can travel on a full tank of petrol varies between 200 and 320 miles.

   (a) If m represents the distance (in miles) travelled on a full tank of petrol, write down an inequality involving m.

   (b) Distances in kilometres, k, are related to distances in miles by

   $$m = \frac{5k}{8}.$$

   Write down an alternative inequality involving k instead of m.

   (c) How many kilometres can the car travel on a full tank of petrol?

7. A man finds that his electricity bill varies between £50 and £90.

   (a) If C represents the size of his bill, write down an inequality involving C.

   The bill is made up of a standing charge of £10 and a cost of 10p per kilowatt hour of electricity.

   (b) If n is the number of kilowatt hours used, write down a formula for C in terms of n.

   (c) Using your formula, write down an inequality involving n and solve this inequality.

8. In an office, the temperature, F (in degrees Fahrenheit), must satisfy the inequality

   $$60 \le F \le 70.$$

   The temperature, F, is related to the temperature, C (in degrees Centigrade), by

   $$F = 32 + \frac{9}{5}C.$$

   Write down an inequality which involves C and solve this inequality.

9. (a) List all the integers which satisfy

   $$-2 < n \le 3.$$

   (b) Ajaz said, "I thought of an integer, multiplied it by 3 then subtracted 2. The answer was between 47 and 62."

   List the integers that Ajaz could have used.

(MEG)

10. (a) x is a whole number such that

$$-4 \le x < 2.$$

(i) Make a list of all the possible values of x.

(ii) What is the largest possible value of x^2?

(b) Every week Rucci has a test in Mathematics. It is marked out of 20. Rucci has always scored at least half the marks available. She has never quite managed to score full marks.

Using x to represent Rucci's marks, write this information in the form of two inequalities.

(NEAB)

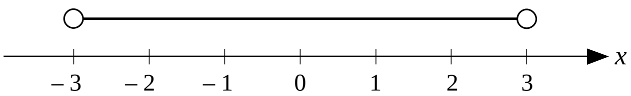
16.3 Inequalities Involving Quadratic Terms

Inequalities involving x^2 rather than x can still be solved. For example, the inequality

$$x^2 < 9$$

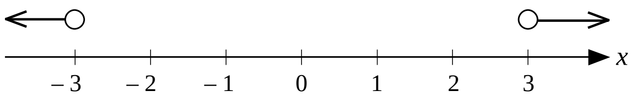
will be satisfied by any number between -3 and 3. So the solution is written as

$$-3 < x < 3.$$



If the inequality had been $x^2 > 9$, then it would be satisfied if x was greater than 3 or if x was less than -3. So the solution will be

$$x > 3 \text{ or } x < -3.$$



The end points of the intervals are defined as $\sqrt{9} = \pm 3$.

Note

For this type of inequality it is very easy to find the end points but care must be taken when deciding whether it is the region *between* the points or the region *outside* the points which is required. Testing a point in a region will confirm whether your answer is correct.

For example, for $x^2 > 9$, test $x = 2$, which gives $4 > 9$. This is *not* true, so the region between the points is the wrong region; the region *outside* the points is needed.

Worked Example 1

Show on a number line the solutions to:

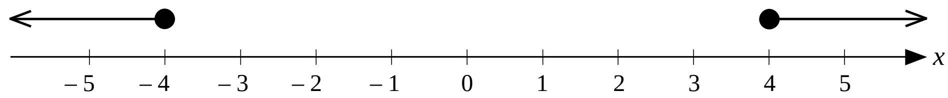
(a) $x^2 \ge 16$ (b) $x^2 < 25$.

Solution

(a) The solution to $x^2 \geq 16$ is

$$x \leq -4 \quad \text{or} \quad x \geq 4,$$

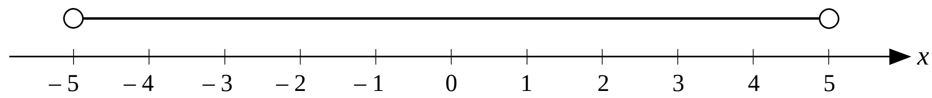
which is shown below.



(b) The solution of $x^2 < 25$ is

$$-5 < x < 5,$$

which is shown below.



Worked Example 2

Find the solutions of the inequalities

(a) $x^2 + 6 > 15$ (b) $3x^2 - 7 \leq 41.$

Solution

(a) By subtracting 6 from both sides, the inequality

$$x^2 + 6 > 15$$

becomes

$$x^2 > 9.$$

Then the solution is

$$x < -3 \quad \text{or} \quad x > 3.$$

(b) Begin with the inequality

$$3x^2 - 7 \leq 41.$$

Adding 7 to both sides gives

$$3x^2 \leq 48.$$

Dividing both sides by 3 gives

$$x^2 \leq 16.$$

Then the solution is

$$-4 \leq x \leq 4.$$

Investigation

Find the number of points (x, y) where x and y are positive integers which lie on the line $3x + 4y = 29$.

Worked Example 3

Solve the inequality

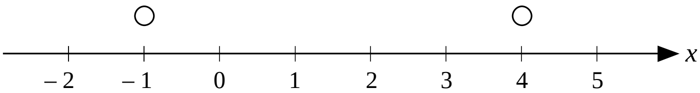
$$x^2 - 3x - 4 > 0.$$

Solution

The left-hand side of the inequality can be factorised to give

$$(x - 4)(x + 1) > 0.$$

The inequality will be equal to 0 when $x = 4$ and $x = -1$. This gives the end points of the region as $x = 4$ and $x = -1$, as shown below.

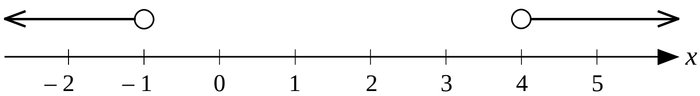


Points in each region can now be tested.

$x = 2$	gives	$-2 \times 3 > 0$	or	$-6 > 0$	*This is **not** true.*
$x = -2$	gives	$-6 \times -1 > 0$	or	$6 > 0$	*This is true.*
$x = 5$	gives	$1 \times 6 > 0$	or	$6 > 0.$	*This is true.*

So the inequality is satisfied for values of x greater than 4, or for values of x less than -1. This gives the solution

$$x < -1 \quad \text{or} \quad x > 4.$$



Exercises

1. Illustrate the solutions to the following inequalities on a number line.

(a) $x^2 \le 1$ (b) $x^2 \ge 4$ (c) $x^2 \ge 25$

(d) $x^2 < 49$ (e) $x^2 > 36$ (f) $x^2 > 4$

(g) $x^2 \ge 6.25$ (h) $x^2 < 0.25$ (i) $x^2 \ge 2.25$

2. Find the solutions of the following inequalities:

(a) $x^2 + 6 \ge 22$ (b) $3x^2 - 4 \ge 8$ (c) $5x^2 - 20 < 105$

(d) $4x^2 < 1$ (e) $9x^2 \ge 4$ (f) $25x^2 - 2 \ge 2$

(g) $36x^2 + 7 \le 11$ (h) $2(x^2 - 5) < 8$ (i) $\dfrac{x^2 + 6}{2} \ge 53$

(j) $10 - x^2 > 6$ (k) $15 - 2x^2 \le -3$ (l) $10 \le 12 - 8x^2$

3. Find the solutions of the following inequalities.

(a) $(x-2)(x+3) \geq 0$ (b) $(x-5)(x-2) \leq 0$

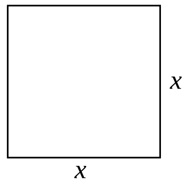
(c) $x(x-5) > 0$ (d) $x^2 - 6x \leq 0$

(e) $x^2 - 7x + 10 < 0$ (f) $x^2 + x - 12 > 0$

(g) $2x^2 - x - 1 \geq 0$ (h) $2x^2 + x - 6 \leq 0$

4. The area, A, of the square shown satisfies the inequality

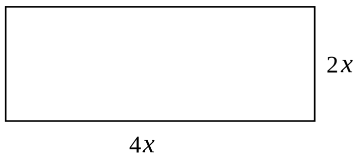
$$9 \leq A \leq 16.$$



(a) Find an inequality which x satisfies and solve it.

(b) What are the possible dimensions of the square?

5. (a) Write down an expression, in terms of x, for the area, A, of the rectangle below.



(b) If the area, A, of the rectangle satisfies the inequality

$$32 \leq A \leq 200,$$

write down an inequality for x and solve it.

(c) What is the maximum length of the rectangle?

(d) What is the minimum width of the rectangle?

6. Solve the following inequalities for x.

(a) $1 + 3x < 7$ (b) $x^2 < 1$

(NEAB)

Just for Fun

Two travellers, one carrying 5 buns and the other 3 buns, met a very rich Arab in a desert.

The Arab was very hungry and, as he had no food, the two men shared their buns and each of the men had an equal share of the 8 buns.

In return for their kindness, the Arab gave them 8 gold coins and told them to share the money fairly.

The second traveller, who had contributed 3 buns, said that he should receive 3 gold coins and the other 5 gold coins should go to the first traveller. However the latter said that he should get more than 5 gold coins as he had given the Arab more of his buns.

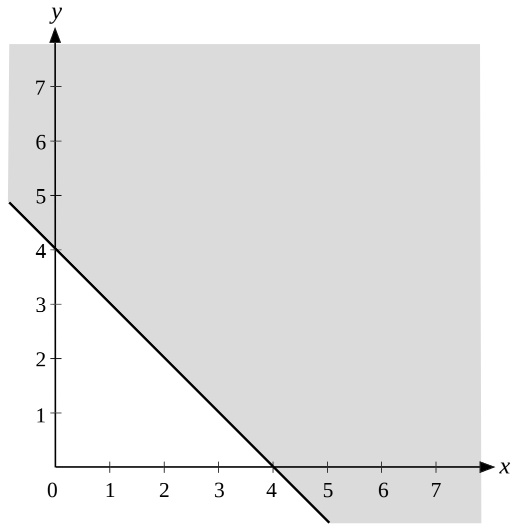
They could not agree and so a fight started. Can you help them to solve their problem?

16.4 Graphical Approach to Inequalities

When an inequality involves two variables, the inequality can be represented by a *region* on a graph. For example, the inequality

$$x + y \geq 4$$

is illustrated on the graph below.

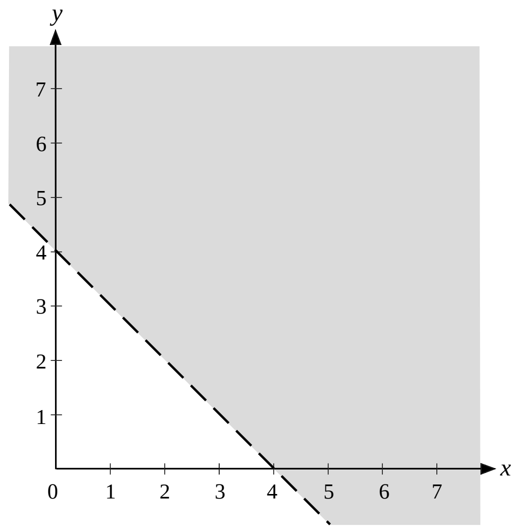


The coordinates of any point in the shaded area satisfy $x + y \geq 4$.

Note

The coordinates of any point on the line satisfy $x + y = 4$.

If the inequality had been $x + y > 4$, then a *dashed* line would have been used to show that points on the line do not satisfy the inequality, as below.



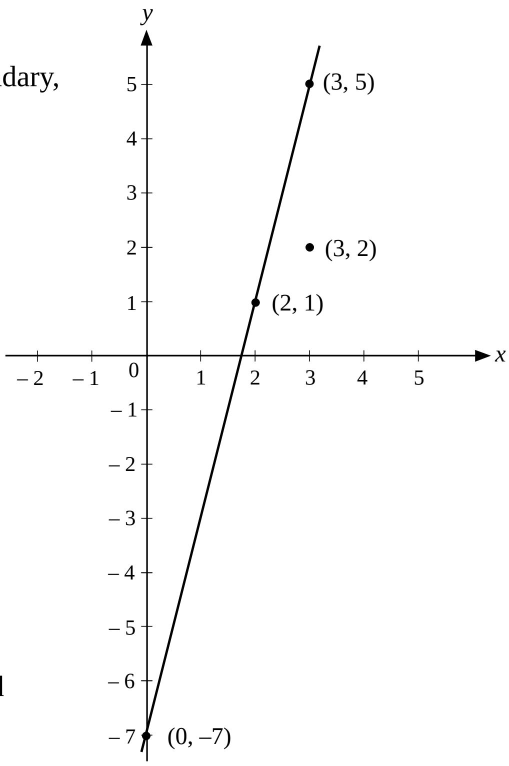
Worked Example 1

Shade the region which satisfies the inequality

$$y \geq 4x - 7.$$

Solution

The region has the line $y = 4x - 7$ as a boundary, so first of all the line $y = 4x - 7$ is drawn.



The coordinates of 3 points on this line are

$$(0, -7), \ (2, 1) \text{ and } (3, 5).$$

These points are plotted and a *solid* line is drawn through them.

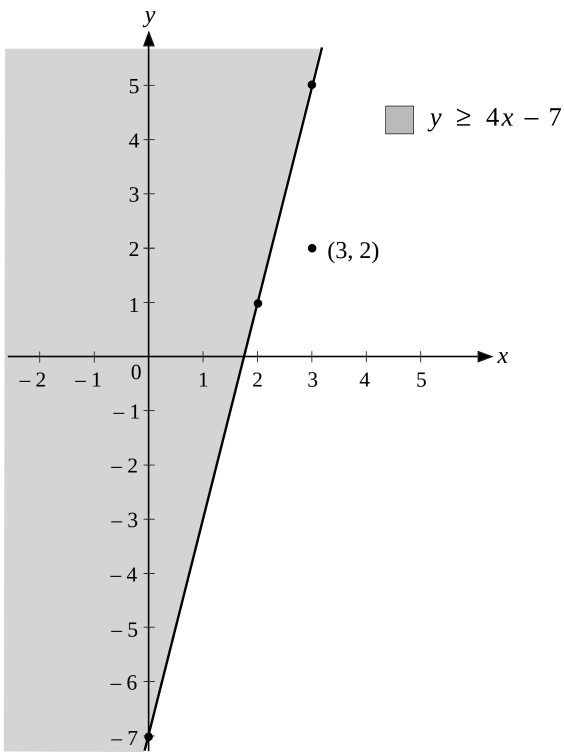
A solid line is drawn as the inequality contains a '≥' sign which means that points on the boundary are included.

Next, select a point such as (3, 2). (It does not matter on which side of the line the point lies.)

If the values, $x = 3$ and $y = 2$, are substituted into the inequality, we obtain

$$2 \geq (4 \times 3) - 7 \quad \text{or} \quad 2 \geq 5.$$

This statement is clearly false and will also be false for any point on that side of the line.



Therefore the *other* side of the line should be shaded, as shown.

Worked Example 2

Shade the region which satisfies the inequality

$$x + 2y < 10.$$

Solution

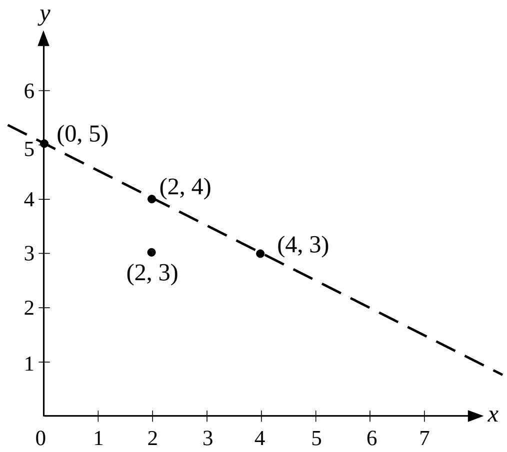
The line $x + 2y = 10$ will form the boundary of the region, but will not itself be included in the region. To show this, the line should be drawn as a *dashed* line.

Before drawing the line, it helps to rearrange the equation as

$$y = \frac{10 - x}{2}.$$

Now 3 points on the line can be calculated, for example $(0, 5)$, $(2, 4)$ and $(4, 3)$.

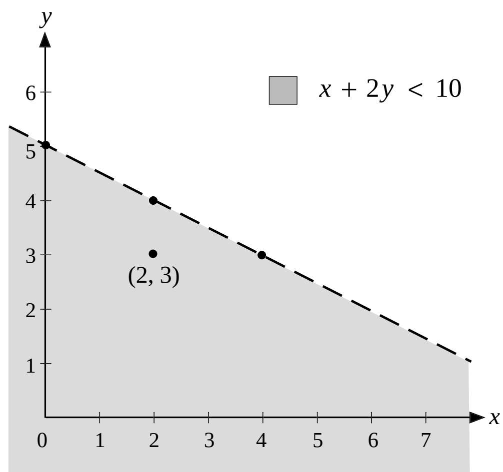
This line is shown below.



Next, a point on one side of the line is selected, for example $(2, 3)$, where $x = 2$ and $y = 3$. Substituting these values for x and y into the inequality gives

$$2 + 2 \times 3 < 10 \quad \text{or} \quad 8 < 10.$$

This is clearly true and so points on this side of the line will satisfy the inequality. This side of the line can now be shaded, as below.



Exercises

1. Use sets of axes with x and y values from -6 to 6 to show the regions which the following inequalities satisfy.

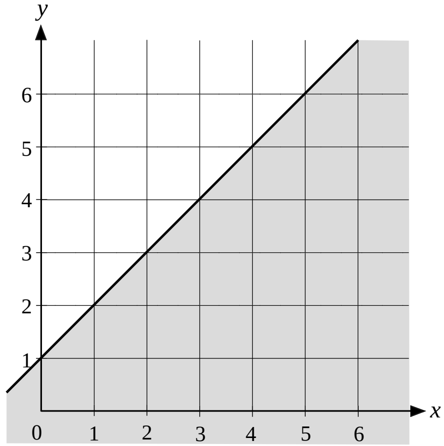
(a)	$y \geq x$	(b)	$y > x + 2$	(c)	$y < x - 1$
(d)	$y > x + 4$	(e)	$y \leq x - 3$	(f)	$y \geq 2x + 1$
(g)	$y \leq 3x - 4$	(h)	$y > 4 - 2x$	(i)	$x + y \geq -2$
(j)	$2x + y \geq 5$	(k)	$4x + y \geq 2$	(l)	$x + 4y < 3$

2. For each region below,

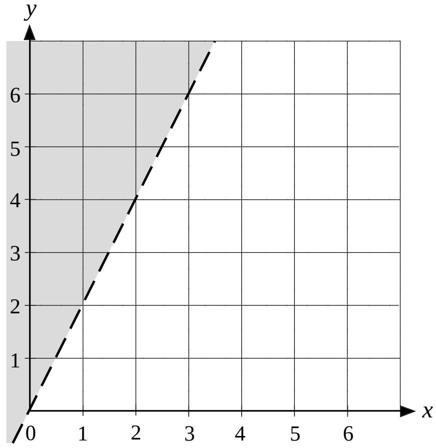
   (i) find the equation of the line which forms the boundary, and

   (ii) find the inequality represented by the region.

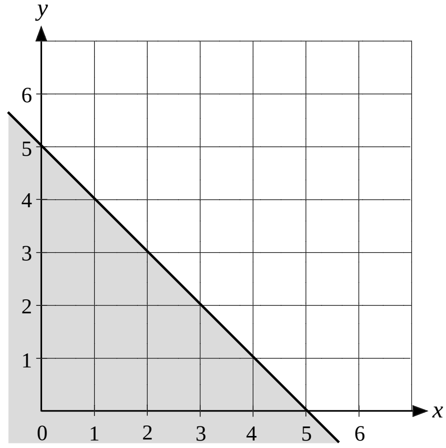
(a)



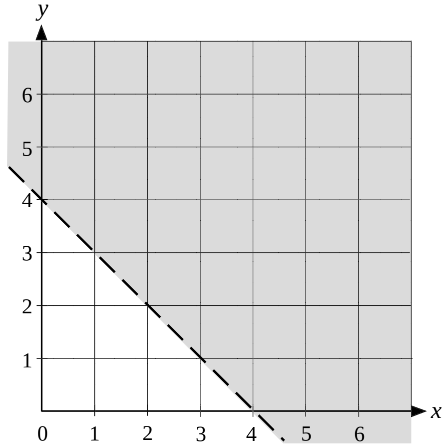
(b)



(c)



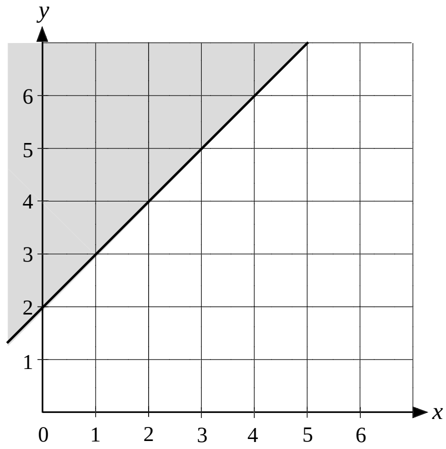
(d)



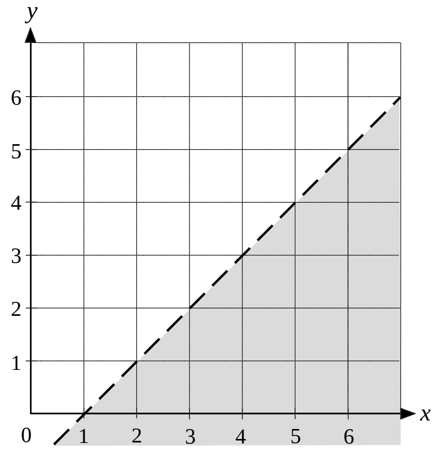
Just for Fun

Without using a calculator or a table, determine which is larger, $\left(\sqrt{10} + \sqrt{29}\right)$ *or* $\sqrt{73}$.

(e)



(f)



3. (a) On the same set of axes, shade the regions which satisfy the inequalities

$$x + y \geq 3 \quad \text{and} \quad x + y \leq 5.$$

Which inequality is satisfied by the region shaded twice?

(b) Shade the region which satisfies the inequality $2 \leq x - y \leq 4$.

4. (a) Draw the graph of $y = x^2$ and shade the region which satisfies the inequality $y \leq x^2$.

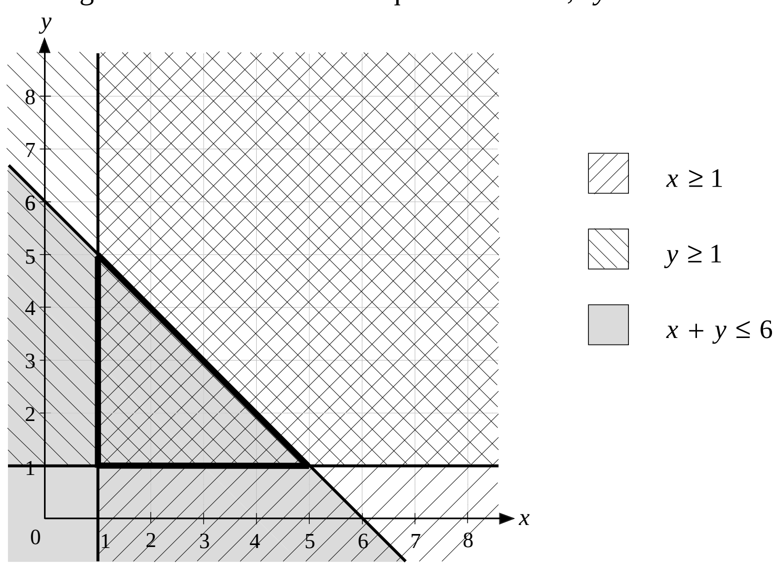
(b) On the same set of axes, draw the graphs of

$$y = x^2 + 1 \quad \text{and} \quad y = x^2 - 1.$$

Shade the region which satisfies the inequality, $x^2 - 1 < y < x^2 + 1$.

16.5 Dealing With More Than One Inequality

If more than one inequality has to be satisfied, then the required region will have more than one boundary. The diagram below shows the inequalities $x \geq 1$, $y \geq 1$ and $x + y \leq 6$.



The triangle indicated by **bold** lines has been shaded three times. The points inside this region, including those points on each of the boundaries, satisfy all three inequalities.

Worked Example 1

Find the region which satisfies the inequalities

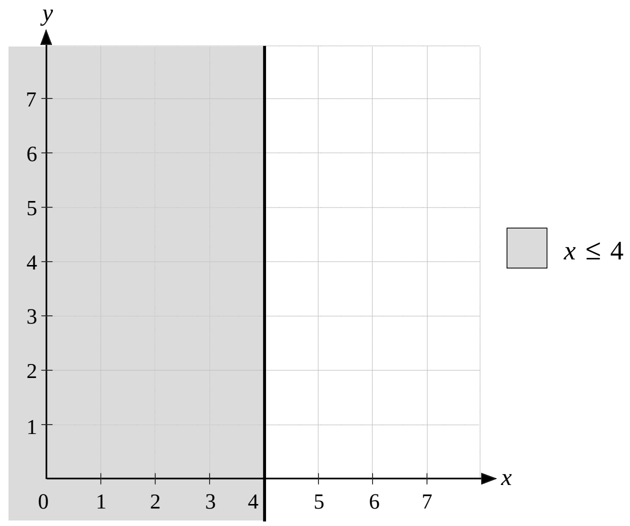
$$x \le 4, \quad y \le 2x, \quad y \ge x + 1.$$

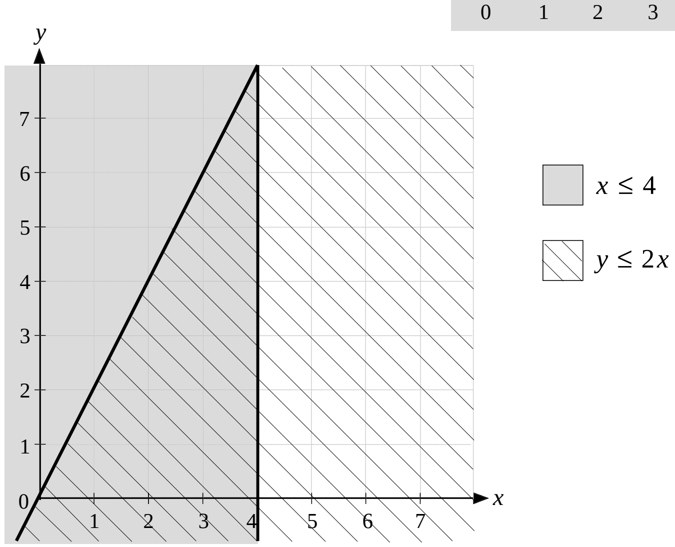
Write down the coordinates of the vertices of this region.

Solution

First shade the region which is satisfied by the inequality

$$x \le 4.$$





Then add the region which satisfies

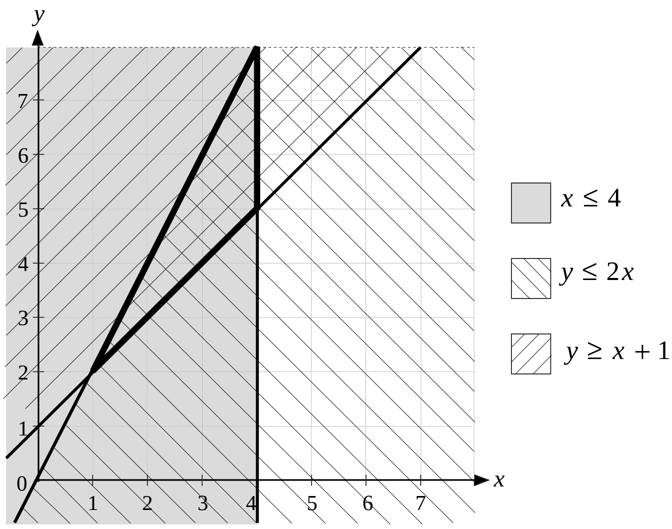
$$y \le 2x$$

using a different type of shading, as shown.

Finally, add the region which is satisfied by

$$y \ge x + 1$$

using a third type of shading.



The region which has been shaded in all three different ways (the triangle outlined in bold) satisfies all three inequalities.

The coordinates of its vertices can be seen from the diagram as $(1, 2)$, $(4, 5)$ and $(4, 8)$.

Note

When a large number of inequalities are involved, and therefore a greater amount of shading, the required region becomes more difficult to see on the graph.

Therefore it is better to shade *out* rather shade *in*, leaving the required region *unshaded*. This method is used in the following example, where 'shadow' shading indicates the side of the line which does *not* satisfy the relevant inequality.

Worked Example 2

A small factory employs people at two rates of pay. The maximum number of people who can be employed is 10. More workers are employed on the lower rate than on the higher rate.

Describe this situation using inequalities, and draw a graph to show the region in which they are satisfied.

Solution

Let x = number employed at the lower rate of pay,

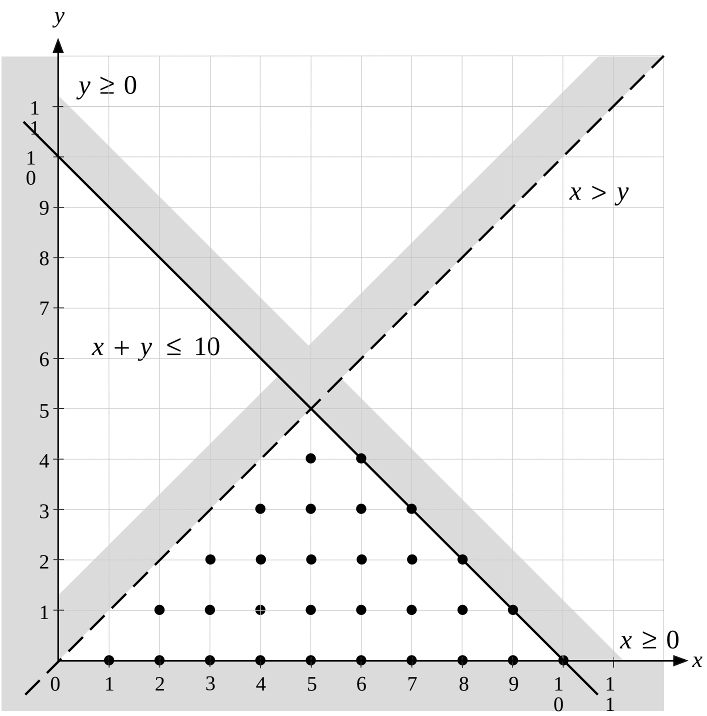
and y = number employed at the higher rate of pay.

The maximum number of people who can be employed is 10, so $x + y \leq 10$.

As more people are employed at the lower rate than the higher rate, then $x > y$.

As neither x nor y can be negative, then $x \geq 0$ and $y \geq 0$.

These inequalities are represented on the graph below.



The triangle formed by the *unshaded* sides of each line is the region where all four inequalities are satisfied. The dots indicate all the possible employment options. Note that only *integer* values inside the region are possible solutions.

Exercises

1. On a suitable set of axes, show by shading the regions which satisfy both the inequalities given below.

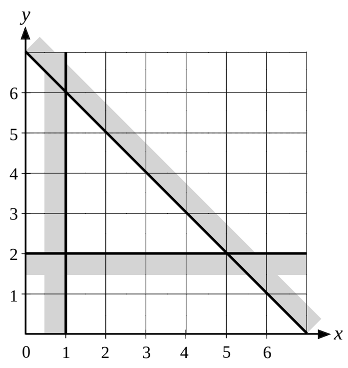
(a) $x \ge 4$, $y < 8$	(b) $x < 7$, $y \ge 1$	(c) $x \ge -2$, $y \ge 4$
(d) $x + y \ge 2$, $y < 6$	(e) $x + y \le 4$, $x + y > 1$	(f) $x \ge y$, $x > 1$
(g) $y \le 2x$, $y \ge x + 2$	(h) $y \ge 2x$, $y \le 3x$	(i) $y \ge x$, $y \le x + 3$

2. For each set of three inequalities, draw graphs to show the regions which they all satisfy. List the coordinates of the points which form the vertices of each region.

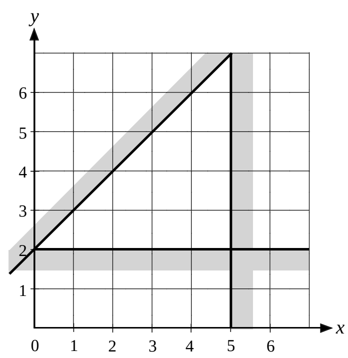
(a) $x \ge 2$, $y \ge x + 1$, $y \le 3x$	(b) $x \ge 0$, $x \le 5$, $y \ge x$	(c) $x > -2$, $y \le 2x + 3$, $y \ge x - 2$
(d) $x + y < 6$, $x > 2$, $y \le 3$	(e) $y \le 2x + 1$, $y \ge x - 1$, $x \ge 2$	(f) $y > x - 1$, $y > 2 - x$, $y \ge 4$

3. Each diagram shows a region which satisfies 3 inequalities. Find the three inequalities in each case.

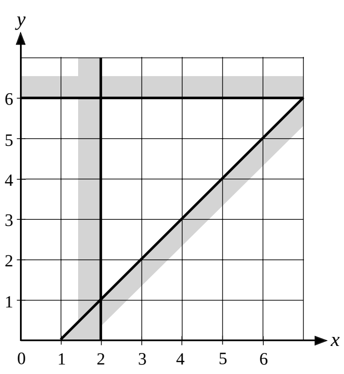
(a)



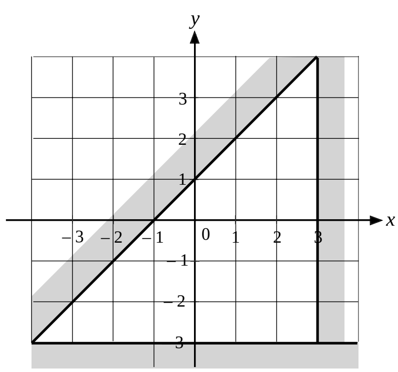
(b)



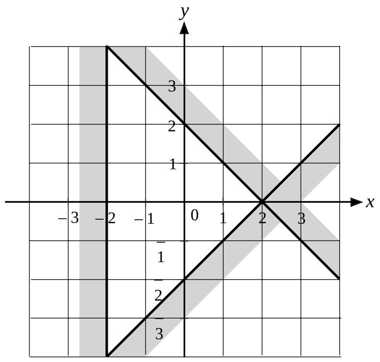
(c)



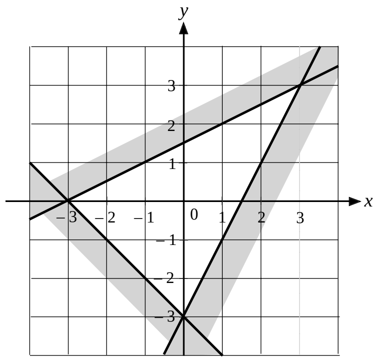
(d)



(e)



(f)



4. At a certain shop, CDs cost £10 and tapes cost £8. Andrew goes into the shop with £40 to spend.

   (a) If $x =$ the number of CDs
   and $y =$ the number of tapes

   which Andrew buys, explain why

$$10x + 8y \le 40.$$

   (b) Explain why $x \ge 0$ and $y \ge 0$.

   (c) Draw a graph to show the region which satisfies all three inequalities.

5. A security firm employs people to work on foot patrol or to patrol areas in cars. Every night a maximum of 12 people are employed, with at least two people on foot patrol and one person patrolling in a car.

   (a) If $x =$ the number of people on foot patrol
   and $y =$ the number of people patrolling in cars,

   complete the inequalities below.

   (i) $x + y \le ?$ (ii) $x \ge ?$ (iii) $y \ge ?$

   (b) Draw a graph to show the region which satisfies these inequalities.

6. In organising the sizes of classes, a head teacher decides that the number of children in each class must never be more than 30, that there must never be more than 20 boys in a class and that there must never be more than 22 girls in a class.

   (a) If $x =$ the number of boys in a class
   and $y =$ the number of girls in a class,

   complete the inequalities below.

   (i) $x + y \le ?$ (ii) $x \le ?$ (iii) $y \le ?$

   (b) The values of x and y can never be negative. Write down two further inequalities.

   (c) Draw a diagram to show the region which satisfies all the inequalities above.

7. Ice cream sundaes are sold for either £1 or £2. Veronica is going to buy sundaes for some of the 6 members of her family, but only has £10 to spend.

Use x = the number of £1 sundaes bought
and y = the number of £2 sundaes bought.

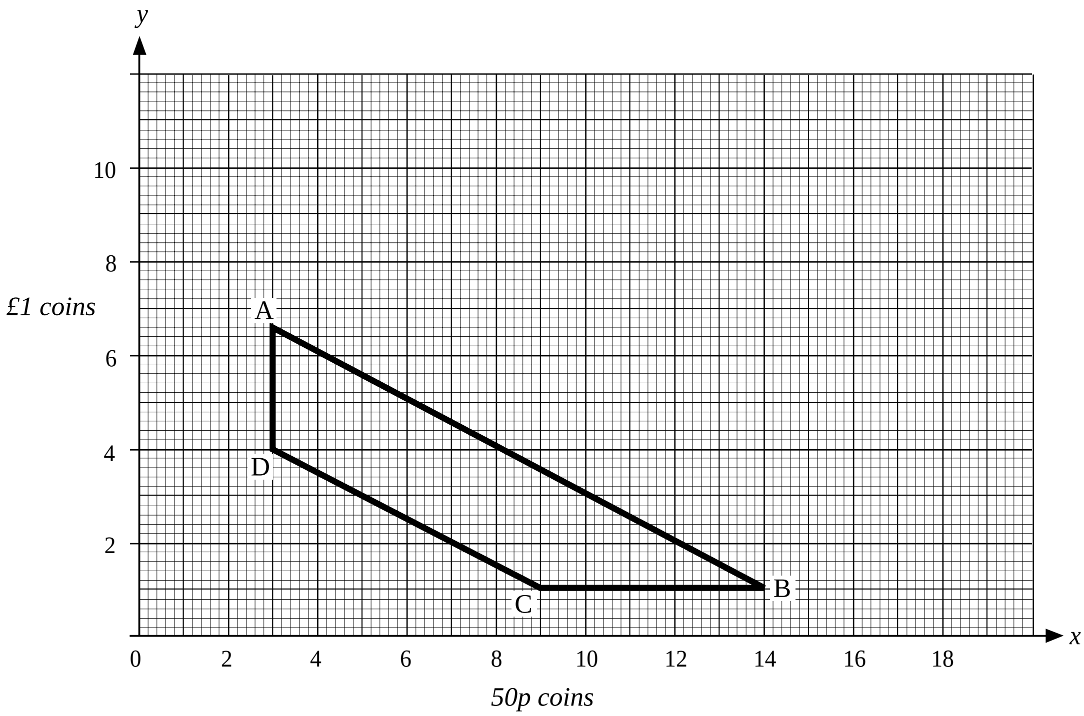
(a) Write down 4 inequalities which describe the situation above.

(b) Draw a diagram to show the region which satisfies all four inequalities.

8. Mrs. Brown's purse contains only fifty pence coins and pound coins.

On the graph below, the region ABCD (including the boundary) contains all the possible combinations (x, y) of 50p coins and £1 coins in Mrs. Brown's purse.

x is the number of 50p coins,
y is the number of £1 coins.



(a) (i) Write down the minimum possible number of 50p coins in the purse.

(ii) Find the maximum possible amount of money in the purse.

(iii) Find the minimum possible amount of money in the purse.

(iv) Use the graph to find the maximum possible number of coins in the purse.

(b) Mrs. Brown now looks in her purse and notices that she has twice as many fifty pence coins as pound coins. How many pound coins could her purse contain?

(MEG)

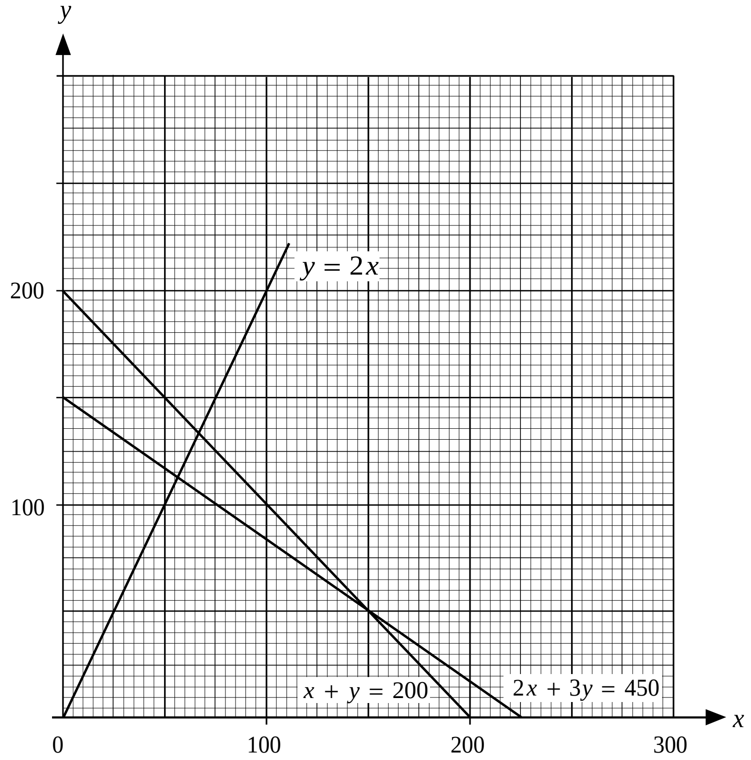
9. The school hall seats a maximum audience of 200 people for performances. Tickets for the Christmas concert cost £2 or £3 each. The school needs to raise at least £450 from this concert. It is decided that the number of £3 tickets must not be greater than twice the number of £2 tickets.

There are x tickets at £2 each and y tickets at £3 each.

(a) Explain why:

(i) $x + y \le 200$ (ii) $2x + 3y \ge 450$ (iii) $y \le 2x$.

The graphs of $x + y = 200$, $2x + 3y = 450$ and $y = 2x$ are drawn on the grid below.



(b) Copy the grid and show by shading the region of the grid which satisfies all three inequalities in (a).

(c) (i) Hence find the number of £2 and £3 tickets which should be sold to obtain the maximum profit.

(ii) State this profit.

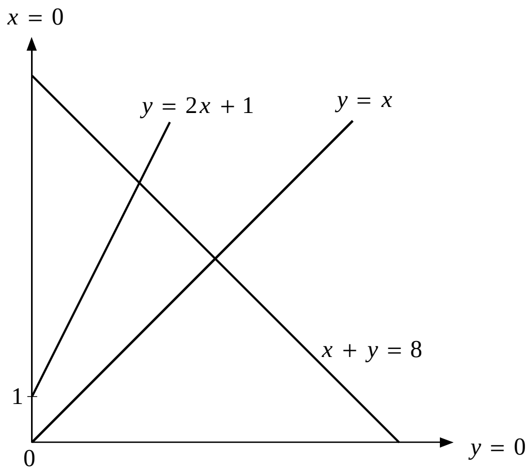
(NEAB)

10. (a) Find all integer values of n which satisfy the inequality

$$1 \le 2n - 5 < 10.$$

(b) Copy the diagram below and label with the letter 'R' the single region which satisfies all the inequalities

$$x \ge 0, \quad y \ge x, \quad y \le 2x + 1, \quad y \le 8 - x.$$



(SEG)

11. At each performance of a school play, the number of people in the audience must satisfy the following conditions.

    (i) The number of children must be less than 250.

    (ii) The maximum size of the audience must be 300.

    (iii) There must be at least twice as many children as adults in the audience.

    On any one evening there are x children and y adults in the audience.

    (a) Write down the three inequalities which x and y must satisfy, other than $x \geq 0$ and $y \geq 0$.

    (b) By drawing straight lines and shading on a suitable grid, indicate the region within which x and y must lie to satisfy all the inequalities.

    Tickets for each performance cost £3 for a child and £4 for an adult.

    (c) Use your diagram to find the maximum possible income from ticket sales for one performance.

    To make a profit, the income from ticket sales must be at least £600.

    (d) Use your diagram to find the least number of children's tickets which must be sold for a performance to make a profit.

(LON)

Just for fun

Bag A contains 2 black balls, Bag B contains 2 white balls and Bag C contains 1 black ball and 1 white ball.

John decides to switch all the three labels so that all three bags now have the wrong labels.

If you are allowed to draw one ball from any one of the three bags only once and look at its colour, which bag would you choose so that you can determine the colours of the balls in each bag?

Give a reason for your choice.