UNIT 14 Loci and Transformations

NC: Shape, Space and Measures 3b, 3c, 3d and 3e

		St	Ac	Ex	Sp
TOPICS (Text and Practice Books)					
14.1	*Drawing and Symmetry*	✓	-	-	-
14.2	*Scale Drawings*	✓	✓	-	-
14.3	*Constructing Triangles and Other Shapes*	✓	✓	-	-
14.4	*Enlargements*	✓	✓	-	-
14.5	*Reflections*	✓	✓	-	-
14.6	*Construction of Loci*	✓	✓	✓	✓
14.7	*Enlargements which Reduce*	✓	✓	✓	✓
14.8	*Further Reflections*	✓	✓	✓	✓
14.9	*Rotations*	✓	✓	✓	✓
14.10	*Translations*	✓	✓	✓	✓
14.11	*Combined Transformations*	✗	✓	✓	✓
14.12	*Congruence*	✗	✓	✓	✓
14.13	*Similarity*	✗	✓	✓	✓
14.14	*Enlargements with Negative Scale Factors*	✗	✓	✓	✓
Activities					
14.1	*Equal Area Polygons*	✓	✓	-	-
14.2	*Reflections*	✓	✓	-	-
14.3	*Repeated Reflections*	✓	✓	✓	✓
14.4	*Transformations*	✓	✓	✓	✓
14.5	*Finding the Centre of Rotation*	✓	✓	✓	✓
OH Slides					
14.1	*Lines of Symmetry*	✓	-	-	-
14.2	*Order of Rotational Symmetry*	✓	✓	-	-
14.3	*Scale Drawings*	✓	✓	-	-
14.4	*Enlargements*	✓	✓	-	-
14.5	*Finding the Centre of Enlargement*	✓	✓	-	-
14.6	*Reflections*	✓	✓	-	-
14.7	*Loci 1*	✓	✓	✓	✓
14.8	*Loci 2*	✓	✓	✓	✓
14.9	*Important Loci*	✓	✓	✓	✓
14.10	*Enlargements which Reduce 1*	✓	✓	✓	✓
14.11	*Enlargements which Reduce 2*	✓	✓	✓	✓
14.12	*Reflections in Oblique Lines*	✓	✓	✓	✓
14.13	*Rotations*	✓	✓	✓	✓
14.14	*Translations*	✓	✓	✓	✓
14.15	*Combined Transformations*	✗	✓	✓	✓
14.16	*Congruence*	✗	✓	✓	✓
14.17	*Similarity*	✗	✓	✓	✓
14.18	*Negative Enlargements 1*	✗	✗	✓	✓
14.19	*Negative Enlargements 2*	✗	✓	✓	✓

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	St	Ac	Ex	Sp
Mental Tests				
14.1	✓	✓	-	-
14.2	✓	✓	-	-
14.3	×	✓	✓	✓
14.4	×	✓	✓	✓
Revision Tests				
14.1	✓	✓	-	-
14.2	✓	✓	✓	✓
14.3	×	✓	✓	✓

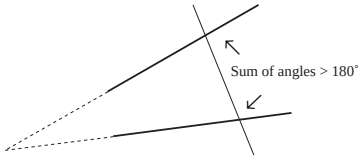
UNIT 14 *Loci and Transformations* Teaching Notes

Background and Preparatory Work

The 'traditional' approach to geometry, which predominated all school geometry up to the 1960s, was based essentially on *Euclid's 'Elements'*, written about 300 BC. Euclid's theorems were based on five assumptions or postulates:

1. For every point P, and for every point Q not equal to P, there exists a unique line, *l*, which passes through P and Q.
2. For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE.
3. For every point O and every point A not equal to O, there exists a circle with Centre O and radius OA.
4. All right angles are equal to each other.
5. If a straight line falls on two other straight lines to make the interior angles on the same side less than two right angles, then the two lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles.

Euclidean and Non-Euclidean Geometries
by M. J. Greenberg
(Freeman)
ISBN: 0 7167 1103 6



The basic tools were congruent triangles and parallel lines and the subject matter mainly concerned

triangles, parallelograms and circles

and their properties.

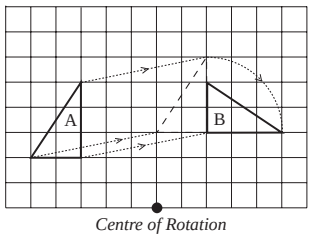
However, in 1972, the German mathematician, *Felix Klein* (1849–1925), in his inaugural lecture as Professor of Mathematics at the University of Enlanger, gave a description of geometry as:

> *those properties of figures in space which remain unchanged under some fixed group of transformations.*

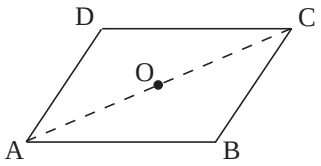
The Mathematics Curriculum: Geometry
by W.W. Willson
(Blackie)
ISBN: 0 216 90337 8

This influential address led directly to the *Erlanger Programme*, which has changed radically the style of geometry taught in schools today. It led to a shift in emphasis away from congruence as the fundamental idea.

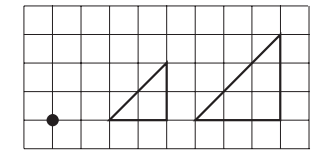
To say that two plane figures are *congruent* means that one can be moved to fit exactly onto the other. Klein's approach would be to view this as a translation and (possible) rotation. For example, A and B as shown opposite are congruent but one shape can be obtained from the other by a translation, followed by a rotation or, indeed, by a single rotation about the centre of rotation as shown.



As another example, consider a parallelogram ABCD, as shown opposite. The triangles ABC and CDA are congruent. You can prove this either by *SAS* or *SSS* in the traditional way. Klein geometry, though, would consider rotating ADC through 180° about the midpoint, O, of the line AD and, in so doing, show that the triangle ADC fits exactly onto ABC. This is essentially the same mathematics but by a very different approach.



Also the traditional work on similarity can, in Klein geometry, be thought of as an enlargement with different scale factors (2 or $\frac{1}{2}$ shown opposite).



Some people may feel that geometry managed very well for more than 2000 years without transformations and that the introduction of transformation geometry is just a fad – but there are strong reasons for the use of transformations in school geometry.

One reason is that rotation, reflection, etc. can be introduced in a practical way and so should be more accessible to some pupils than the more theoretical traditional geometry. Another reason is that this geometry is, in fact, fundamental to future work, when the use of vectors becomes an integral part (they are used in Unit 14.10). It should also be noted that this approach does still provide logical and powerful analysis, although it is rather difficult in nature to that of traditional geometry.

Teaching Points

Introduction

This Unit brings together a number of geometric ideas, including:

scale drawings
constructions
transformations
congruence
similarity.

The ability to revolve, as well as accurately draw and construct, is an important aspect of the work of this Unit.

We would strongly encourage you to use squared paper, particularly in the section on transformations and scale drawing, although it should be noted that some exam questions on transformations come either on plain paper or on graph paper.

A knowledge of lines of symmetry (and order of rotational symmetry) is helpful here and these topics are revised in the first section.

OS 14.1 and 14.2

Language / Notation

There is an extensive language component to the work on transformation; for example:

Reflections	–	mirror line, oblique	OS 14.6 and 14.12
Rotations	–	centre of rotation, angle, direction (clockwise/anticlockwise)	OS 14.13
Translations			OS 14.14
Enlargements	–	scale factor, centre of enlargement.	OS 14.4, 14.5, 14.10, 14.11, 14.18 and 14.19

(Note that the transformations, shears and stretches, are *not* included.)

Other language needed includes scale, loci, congruent and similar.

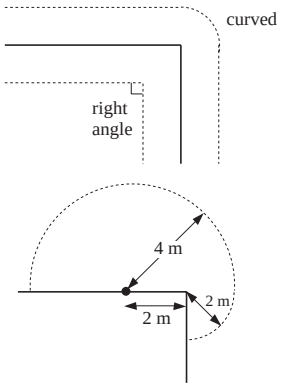
Key Points

- Scale drawings must be drawn accurately.
- Constructions (e.g. triangles, perpendicular bisectors) must be drawn according to the instructions. A 14.1
- For rotations you need: A 14.2 and 14.3
  (a) point of rotation
  (b) angle of rotation
  (c) direction, e.g. clockwise/anticlockwise.
- For enlargements, you need: OS 14.5
  (a) scale factor
  (b) centre of enlargement.
- A single rotation of $180°$ is equivalent to two reflections (in perpendicular lines).

Misconceptions

Note that:

- rotations are not always about the origin;
- clockwise $180°$ rotation is equivalent to anticlockwise $180°$ rotation;
- scale factors for enlargement can take any value but if: OS 14.10 and 14.11 OS 14.18 and 14.19
  $0 <$ scale factor < 1, it is actually a reduction,
  $\alpha < 0$, it rotates the object through $180°$.
- Note the difference between the outer and inner locus of a point equidistant from two perpendicular walls.

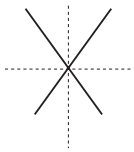


- Beware of goats tethered to walls in L-shaped gardens!

Key Concepts

- A translation moves all points of an object in the same direction and for the same distance. OS 14.4
- Two shapes are congruent if they are identical. OS 14.16
- Two shapes are similar if they have the same shape, but may differ in size. OS 14.17
- Important loci include: – points equidistant from a fixed point (circle), OS 14.9

– points equidistant from two fixed lines,



– points equidistant from two fixed points.