UNIT 15 Variation

NC: Number 4b, Algebra 2c, 2d, Further Material 3a

	St	Ac	Ex	Sp
TOPICS (Text and Practice Books)				
15.1 *Simple Ratios*	✓	-	-	-
15.2 *Proportion and Ratio*	✓	✓	-	-
15.3 *Map Scales and Ratio*	✓	✓	-	-
15.4 *Proportional Division*	✓	✓	-	-
15.5 *Direct Proportion*	×	×	✓	✓
15.6 *Inverse Proportion*	×	×	✓	✓
15.7 *Functional and Graphical Representation*	×	×	✓	✓
15.8 *Further Functional Representation*	×	×	✓	✓
Activities				
15.1 *How Far Away?*	✓	✓	-	-
15.2 *Fishing Competition*	×	×	✓	✓
15.3 *Oscillations*	×	×	✓	✓
15.4 *Leaking Bottles*	×	×	✓	✓
15.5 *Fitting the Graph*	×	×	✓	✓
OH Slides				
15.1 *Map Scales*	✓	✓	-	-
15.2 *Proportional Division*	✓	✓	-	-
15.3 *Direct Proportion*	×	×	✓	✓
15.4 *Inverse Proportion*	×	×	✓	✓
15.5 *Common Graphs*	×	×	✓	✓
15.6 *Relationships*	×	×	✓	✓
15.7 *Filling Bottles*	×	×	✓	✓
Mental Tests				
15.1	✓	✓	-	-
15.2	✓	✓	-	-
Revision Tests				
15.1	✓	✓	-	-
15.2	×	×	✓	✓

UNIT 15 *Variation* Teaching Notes

Background and Preparatory Work

This Unit brings together a number of themes, so much of the historical background has been covered in earlier Units. The concept of a *function* is an important aspect of the latter part of this Unit. Our modern day definition of a function owes much to the Swiss Mathematician *Euler* (1707-1783) who introduced the $f(x)$ notation and even more to the mathematician *Lejuenne Dirichlet* (1805-1859). Various definitions, given in historical order, are repeated below.

- A quantity composed in any manner of a variable and any constants. (*Jean Bernoulli*, 1718)
- Any analytic expression whatsoever made up from that variable quantity and from numbers or constant quantities (*Euler*, 1748)
- Quantities dependent on others, such that as the second change, so do the first, are said to be functions. (*Euler*)
- If a variable *y* is related to a variable *x*, so that whenever a numerical value is assigned to *x* there is a rule according to which a *unique* value of *y* is determined, then *y* is said to be a function of the independent variable *x*. (*Dirichlet*, 1837)

See web site
http://www-groups.dcs.st-and.ac.uk:80/~history/
for History of Maths

These early definitions tried to express in words the idea of the dependence of one quantity on another, and also the concept that the second quantity is uniquely determined from the first by some rule.

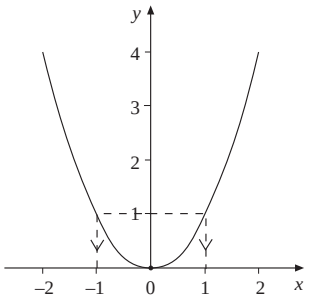
So, for example, if *y* is proportional to the square of *x*, in mathematical terms,

$$y = k x^2$$

for some constant *k*. Then, given a value of *x*, *y* is uniquely determined, e.g. when $k = 1$, $y = x^2$ and $x = 1$ given $y = 1$. The reverse, though, is not necessarily true: in the example above, $y = 1$ gives

$$1 = x^2 \Rightarrow x^2 = 1 \Rightarrow x \pm 1$$

– so *x* is not uniquely determined – it can in fact have two values, namely ± 1.



This can be readily seen by reference to the graph opposite – any value of *x* will uniquely determine the value of *y* – but any positive value of *y* will determine the value of *x*. This leads on to the concept of

one to one, *one to many*, *many to one*

mappings, but beyond the scope of this course, and also on to the concept of

domain and *range*

– the *domain* is the set of values of *x* for which the function is defined and the *range* is the values of *y* which result from the specified domain. Again, these topics go beyond what is required for GCSE, but are of prime importance for A-level work which carries on from

here. It may be helpful to at least expose *Special* students to these ideas.

One more historical subject which might be of interest and of particular relevance to the section on map scales is that of the development of Ordnance Survey maps. The usual scale for the Main Series of UK maps is

$$1 : 50\,000$$

and the origin for the coordinate system used is near the Scilly Isles. So, for example, the grid reference (363242) means

36.3 km east of the origin

and

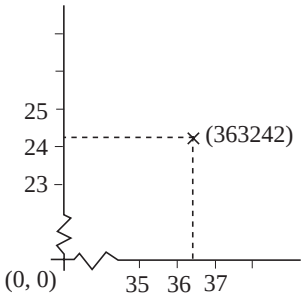
24.2 km north of the origin.

Note also that the 'hundreds' of km are left off the grid reference so the point with coordinates

$$(236.3,\ 124.2)$$

would also have the grid reference 363242.

It should also be noted that the scale 1 : 50 000 results in an exact scale of 2 cm to 1 km, but only an approximate $1\frac{1}{4}$ inches to 1 mile. This was decided at a conference held in Paris in December 1913 and is now a world undertaking in modern countries.



Ordnance Survey web site
http://www.ordsvy.gov.uk/

Teaching Points

Introduction

This unit brings together a number of threads, starting with *ratio* and *proportion* and leading on to *proportional division* and then on to *functions that represent direct and inverse proportion.* So the theme of proportion runs through the unit, starting with numbers and finishing with algebra.

Language / Notation

- For ratio and proportion, we use the notation $1 : n$ or $m : 1$ or $m : n$.
- For map scales, the notation 1 in 2000 (for example) is used to mean the ratio 1 : 2000.
- The symbol '$\propto$' is used to mean 'proportional to', e.g. $y \propto x$.

Other important language used includes

proportional division
direct proportion
inverse proportion

Key Points

- The ratios $a : b$, $1 : \frac{b}{a}$, $\frac{a}{b} : 1$ are all equivalent.
- To divide a quantity in the ratio $m : n$, you must calculate the fractions $\frac{m}{(m+n)}$ and $\frac{n}{(m+n)}$ of the quantity.

Misconceptions

- Care must be taken with units on map scales, e.g. 1 : 5000 means that 1 cm on a map represents 5000 cm in reality (or 50 m). T15.3 OH15.1
- Increasing the map scale actually decreases the map length.

Key Concepts

- When y is *directly proportional* to x, then T15.5 OH15.3

$$y \propto x \quad \text{or} \quad y = kx$$

- When y is *inversely proportional* to x, then T15.6 OH15.4

$$y \propto \frac{1}{x} \quad \text{or} \quad y = \frac{k}{x}$$

- When y is *proportional* to the square of x, then T15.8

$$y \propto x^2 \quad \text{or} \quad y = kx^2$$