UNIT 19 Vectors *NC: Shape and Space*

	St	Ac	Ex	Sp
TOPICS (Text and Practice Book)				
19.1 *Vectors and Scalars*	✗	✗	✓	✓
19.2 *Applications of Vectors*	✗	✗	✓	✓
19.3 *Vectors and Geometry*	✗	✗	✓	✓
19.4 *Further Work with Vectors*	✗	✗	✗	✓
Activities				
19.1 *Vectors*	✗	✗	✓	✓
OH Slides				
19.1 *Equal Vectors*	✗	✗	✓	✓
19.2 *Components*	✗	✗	✓	✓
19.3 *Vector Expressions*	✗	✗	✓	✓
19.4 *Additions and Subraction of Vectors*	✗	✗	✓	✓
19.5 *Vector Geometry*	✗	✗	✓	✓
Revision Test				
19.1	✗	✗	✓	✓

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UNIT 19 *Vectors* Teaching Notes

Background and Preparatory Work

The beginnings of vectors as we now know them started with the greatest of Ireland's men of science, *William Hamilton* (1805 – 1845). He treated complex numbers as an ordered pair of real numbers, i.e. $a + bi$ (when $i = \sqrt{-1}$) as (a, b), and this is the starting point for vectors. In fact, Hamilton tried to generalise the 2-dimensional complex numbers into what could be called hypercomplex numbers. He spent much time developing quaterions, which correspond to 4-dimensional numbers (*a, b, c, d*), and continued to spend much of his life researching quaterions in the hope that they would solve physical problems (as, eventually, vectors did in three dimensions).

On of the insurmountable problems was to define 'multiplication', which was commutative, as is true for real numbers, i.e. $ab = ba$. Eventually, Hamilton realised that for quaterions, it was just not possible to define an algebra keeping multiplication commutative. This is also true for vectors (and matrices) where the vector product is not commutative.

Unfortunately for Hamilton, quaterions were never found to have any particularly important applications, whereas vectors came to be crucial in solving advanced fluid flow problems. The work of the Scottish-American mathematician *Joseph Wedderburn* (1882 – 1948), was particularly significant in developing vectors as we know them, and vector field theory, which is now used to solve a variety of problems in the physical world.

Teaching Points

Introduction

You should stress to students that vectors are a key concept, fundamental to more advanced applications in, for example, fluid flow. So, whilst the applications seen in this unit do not actually *need* vectors (they could all be solved without using vectors), much of the development in continuous mechanics is crucially dependent on a vector approach. It should also be noted that using vectors for applications, for example, in geometry, is a very convenient way of solving problems.

T19.2

T19.3

You should stress the key differences between scalars (which have only magnitude) and vectors (which have both magnitude and direction), and note that we have to develop our algebra for vectors, since they are a new entity.

For example, addition in component form, e.g.

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a + c \\ b + d \end{pmatrix}$$

may seem entirely obvious, yet it must be defined as we are essentially starting with a blank sheet of paper.

Note that, as with addition, we have to define precisely multiplication. In this unit we deal with multiplication of a vector by a scalar, i.e.

$$k \times \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ka \\ kb \end{pmatrix}$$

but we also have definitions for a scalar product and a vector product (when we multiply two vectors to form a scalar in the first case, vector in the second),

i.e.

$$\mathbf{a}.\mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

These are both outside the GCSE syllabus, but they are crucial building blocks for the really important applications of vectors.

Language / Notation

Vector notation always gives rise to problems – particularly for those students who continue on to A-level. It is absolutely vital that pupils always use vector notation (i.e. in written form, with a wavy line underneath, e.g. $\underset{\sim}{a}$, $\underset{\sim}{b}$ – and in typed text with a **bold** typeface, i.e. **a**, **b**; this is probably one of the major sources of confusion!).

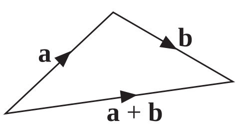
Language needed here includes

scalar, *vector*, *components*, *resultant*, *magnitude*

The zero vector is denoted by **0** (or $\underset{\sim}{0}$, if handwritten).

Key Points

- Vector addition, i.e. $\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a + c \\ b + d \end{pmatrix}$ and geometric interpretation – see diagram.



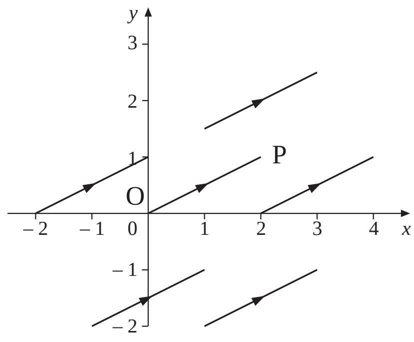
- Multiplication of a vector by a scalar

  i.e. $k \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ka \\ kb \end{pmatrix}$.

- The resultant of two vectors, **a** and **b**, is given by **a** + **b**.

Misconceptions

- Confusion between vectors and scalars, e.g. $2 + \mathbf{a}$ does not make sense.
- 2-D vectors, e.g. $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, are not the same as coordinates – the vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ has *direction* OP where P is the point with coordinates (2, 1), and magnitude OP, but it is not necessarily $\overrightarrow{OP}$. For example, all vectors shown opposite are identical, whereas the point P with coordinates (2, 1) is fixed.
- The vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ has magnitude $\sqrt{2}$ (not 1).



Key Concepts

- For equilibrium, the sum of all vector forces acting on a body is equal to the zero vector, i.e. $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \ldots = \mathbf{0}$