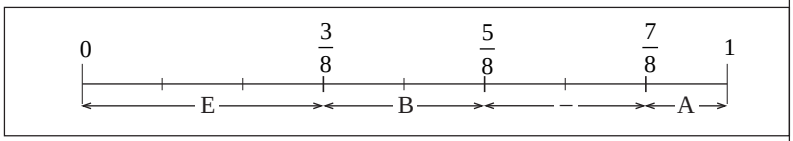
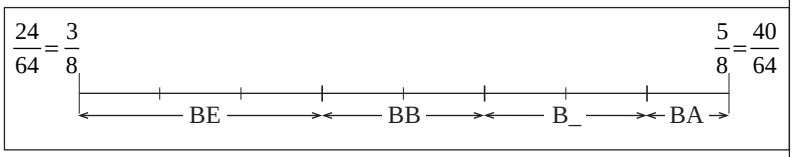
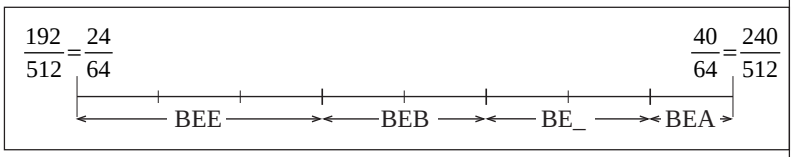
Codes and Ciphers	**UNIT 18** *Arithmetic Coding* Lesson Plan 1	
Activity		***Notes***
		T: Teacher P: Pupil Ex.B: Exercise Book
1	**Introduction**	
	T: We're going to look at one of the most powerful compression techniques – ARITHMETIC CODING.	Discussion regarding internet packages for compression, e.g. STUFFIT, etc. It would also be helpful if Unit 17 on Huffman Codes had been completed.
	We start with the message BE_A_BEE and now we'll design the compression for this message.	
	T: First we find the frequency of letters and characters. Who would like to write this on the board for us?	
	P_1 (on board): E B _ A / 3 2 2 1	Volunteer P writes frequencies on board.
	T: Note that the underscores are included as they are characters in the message.	
	T: Who can tell us the frequency-use proportions?	
	P_2: $\frac{3}{8}$ $\frac{2}{8}$ $\frac{2}{8}$ $\frac{1}{8}$	Another P gives the frequency proportions.
	T: Well done. Now we divide up a number line to show these proportions.	
		T must make sure that Ps understand how the number line is divided up – this is crucial to the method.
	T: As our message starts with a B, we take the B interval and divide it up in the same way.	
		If possible, Ps should find each boundary value and mark it on the number line on board. T clarifies that the segment now being dealt with is the one for $\frac{2}{8}$.
	T: What intervals make the boundaries between ... BE and BB ? $(\frac{3}{8}+\frac{3}{8}\times\frac{2}{8}=\frac{30}{64})$	
	T: ... between BB and B_ ? $(\frac{3}{8}+\frac{5}{8}\times\frac{2}{8}=\frac{34}{64})$	
	T: ... between B_ and BA ? $(\frac{3}{8}+\frac{7}{8}\times\frac{2}{8}=\frac{38}{64})$	
	T: What do you think we do next? *(Take the BE interval and divide it up)*	Ps work these out, working in pairs, volunteering to write their answers on board when they are confident that they are correct.
	T: That's right. Well done.	
		
	T: What are the boundary points now?	
(continued)	T: Between BEE and BEB ? $(\frac{210}{512})$	

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1 *(continued)*	T: ... between BEB and BE_ ? $(\frac{222}{512})$ ...between BE_ and BEA ? $(\frac{234}{512})$ T: We have to continue this process ... how many times? *(5 times)* T: What will be the numerator of the boundary points? $(8^8 = 16\,777\,216)$ T: Here is the final number line. T: So our message can be transmitted by any number in the interval $\left[\frac{7\,763\,888}{16\,777\,216}, \frac{7\,654\,320}{16\,777\,216}\right]$ not, as yet, very helpful!	T shows **OS 18.1** or previously prepared number line.
	20 mins	
2	**Binary representation** T: You are probably familiar with binary representation of numbers Who can explain the system? T: OK. So now try Exercise 1.	Ps volunteer; others listen and help if necessary until T agrees that explanation is correct. It is important that binary representation of whole numbers is well understood by the class. More revision might be needed. Ps are given 4 or 5 minutes to work on this before reviewing answers. T monitors their work; then review and correction of any misconceptions.
	25 mins	
3 *(continued)*	**Binary fractions** T: This might be new to you but it's really just an extension of the process we have just looked at. We can write fractions in binary too. How can we do this? *(?)* T: We write $\frac{1}{2}$ as 0.1 $\frac{1}{4}$ as 0.01 What about $\frac{1}{8}$ and $\frac{1}{16}$? *(0.001 and 0.0001)* T: $\frac{3}{8}$? $(\frac{3}{8} = \frac{1}{4} + \frac{1}{8} \Rightarrow 0.011)$ T: Now it's time for you to try some. See how you get on with Exercise 2.	Ps give their ideas; it might be useful to revise decimals, e.g. $0.12 = \frac{1}{10} + \frac{2}{100}$, etc. T can use **OS 18.2** to help Ps understand. Ps work on Exercise 2 for a few minutes. T monitors their progress and then conducts a review of their work on this exercise.
	35 mins	

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4 *(continued)*	**Finding the message** T: Our problem is how to write $\frac{7\,765\,888}{16\,777\,216}$ in binary. How can we do this? *(Power of 2)* T: We can certainly simplify the fraction: $\frac{7\,653\,888}{16\,777\,216} = \frac{2^9 \times 14\,999}{2^{24}}$ $= ?$ *(Ps give answer)* T: So we now need the binary form of $\frac{14\,949}{2^{15}}$ How do we do this? *(Put* 14949 *into binary)* T: And how do we do this? *(Know the bases?)* T: Yes – let's write out base 2 in full. Ps at board: $2^0 = 1$ $2^1 = 2$ $2^2 = 4$ $2^3 = 8$ $2^4 = 16$ $2^5 = 32$ $2^6 = 64$ $2^7 = 128$ $2^8 = 256$ $2^9 = 512$ $2^{10} = 1024$ $2^{11} = 2048$ $2^{12} = 4096$ $2^{13} = 8192$ T: Is that far enough? *(Yes)* T: What next? P_1: $14\,949 - 8192 = 6757$ *(2^{13} √)* and we need to find 6757 in binary. P_2: $6757 - 4096 = 2661$ *(2^{12} √)* P_3: $2661 - 2048 = 613$ *(2^{11} √)* P_4: $613 > 1024$ *(2^{10} ×)* P_5: $613 - 512 = 101$, etc. *(2^9 √)*	T should involve Ps as much as possible in this, encouraging them to find the method and 'steering' them in the right direction if necessary. Either use the algorithm on **OS 18.3** or keep working logically to obtain the list. T continues asking questions and keeping Ps on task by asking them individually for answers.
(continued)		

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4 *(continued)*	T: This gives $14\,949 \Rightarrow 11101001100101$ What now? P: Divide by 2^{15}. T: How do we do this? *(Count 15 digits from RHS and then put in the 'point')* P: 0.011101001100101 T: Well done!! Finally, we need the shortest binary fraction between this and $\frac{7\,654\,320}{16\,777\,216}$. This is your homework! *45 mins*	T should involve Ps as much as possible in this, encouraging them to find the method and 'steering' them in the right direction if necessary.
	Homework Exercise 3 or Activity 1.	