Codes and Ciphers	**UNIT 5** *Binary Codes* Lesson Plan 1	*Error Detection*
Activity		***Notes***
		T: Teacher P: Pupil Ex.B: Exercise Book
1	**Introduction**	
	T: We are used to the word 'noise' meaning sound, often loud sound; in communications it has a different meaning. Read through the first paragraph of section 5.1 on Noise in your text for an explanation.	This is a good opportunity for discussion about electrical transmission, relating it to Ps' knowledge and experiences.
	T: Who can suggest examples of communication systems that might be affected by noise of this type? *(Television picture, telephone calls, signals from satellites)*:	
	T: To see what happens when 'noise' introduces errors, let's look at this very simple code (writes on board): $C = \{(00), (01), (10), (11)\}$	T writes code on board or shows previously prepared OS.
	T (indicating the various components of the code): This code has four *codewords*, each codeword has *length* 2, and all the *digits* are either 0 or 1.	These definitions are straightforward but will be new to Ps, so it might be worth T spending some time on reinforcing the concepts. T could use the codes in the Appendix for this; alternatively, it would be useful for pupils to design their own codes.
	T: Why do we use just 0 and 1 for the digits? *(Electronic signals are either 'off' or 'on, i.e. '0' or '1')*	T points to volunteer Ps for answers.
	T: What is the meant by the 'length' of a codeword? *(The number of digits in the codeword)*	
	T: How can we extend this code so that it can detect errors?	If possible, T should take the ideas from Ps and use them to illustrate the concept of error detection (and correction).
	T: One simple, but inefficient, way is to repeat each codeword; the code becomes $C = \{(0000), (0101), (1010), (1111)\}$	
	Can this detect single errors? *(Yes)*	
	T: Good. Can it correct single errors? *(No)*	T gives Ps time to consider this and then reviews interactively.
	T: That's right.	
	15 mins	
2	**Activity**	
	T: Let's look at the code in Activity 2 of your Text. Can it detect and correct single errors?	T writes code on board, or gives each P a copy. Ps work in pairs for a few minutes and then T stops them for a review.
	T: Who would like to give an answer?	
	P: It can detect single errors but it cannot correct them.	Volunteer P gives answer.
	T: OK. Here is another example; it includes a parity check for the third digit. You get the third digit by adding the first 2 and then using the *remainder* on division by 2. $C = \{(000), (011), (101), (110)\}$	T writes code on board or shows previously prepared OS.
(continued)	T: Can this revised code now detect and count single errors? *(It can detect, but not correct, single errors)*	Division modulo 2, as used here, is another concept that T

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2 *(continued)*	T: It would be better if it could both detect and correct single errors. How can we do this? *(Make longer codewords)* T: Well done.	might want to discuss with Ps in more detail at this stage. This is a good discussion point. T should consider any sensible suggestions from Ps and try to follow through their ideas, giving praise where appropriate.
	25 mins	
3	**Error detection and correction** T (Activity 3 from Text) reads out: Design a code containing 4 codewords, each of length 6, which can detect and correct a single error. T: Who has a solution? Please show us on the board/OS.	T writes code on board or gives each P a copy of it and gives Ps 5 or 6 minutes to work on solutions before stopping them for a review. Several volunteer Ps give their solutions on the board/OS; T praises them for correct solutions.
T:	In what other way could we design a code that detects and corrects single errors? *(Use longer lengths)*	Praising.
	35 mins	
4	**Activity: error correction** T (showing new code on board/OS): What is the length of each codeword here? $C_2 = \{(000000), (111000), (001110), (110011)\}$ *(6)* T (writes on board): (100000) is received; if there is only one error, what should the codeword be?	Interactive, whole class discussion with Ps responding to T's questions.
	P dictates and T writes on board: (000000) T: Why is this? P: The codeword received is only 1 digit different from this one but is 2 or more digits different from any of the other codewords. T: Well done. T: What about (writes on board): (11000) or (010111) ? *(First codeword should be* (11100)*.* *The second codeword is at least 2 digits different from any of the codewords given)*	Ps answer orally; T writes codewords on board. T gives Ps a few minutes to think about these questions, then reviews their answers and makes sure that they have the correct reasoning, giving praise where due. The idea of distance between codewords is being introduced here (although not stated). This is the key concept for error correction.
	45 mins	
	Homework: Activity 5 from Text Consider Code 5 given in the Appendix. Find out how many errors this code can detect and correct by considering, for example, codewords such as (a) (1 1 0 0 0 0 0) (b) (0 1 1 1 1 1 1) (c) (1 0 0 0 2 0 0) which are in error.	

Codes and Ciphers	**UNIT 5** *Binary Codes* Lesson Plan 2	*Error Correction*
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1	**Review homework** T: Who has an answer to the homework question? P: It can detect 2 errors but only correct 1. T: Good. Why is this? *(Ps give example to illustrate)* *5 mins*	Review Activity with Ps giving their answers and, importantly, their reasoning. T praises where appropriate.
2	**Distance between codewords** T: You have already been using this concept but it is now time to be precise and to define DISTANCE between any two codewords in a code. Distance between any two codewords is defined as the sum of the number of actual differences between the digits in the two codewords. For example, $d\{(111),(010)\}=2$ T: What is the distance between these two codewords (writes on board): $d\{(0101),(1011)\}=$? *(3)* T: Now work out the distance between all possible pairs of codewords in this code (writes on board): $C=\{(11000),(00101),(10101),(11111)\}$ T: Another really important definition is: The Hamming distance is defined as the minimum distance between any two codewords in a code. T: What is the Hamming distance for this code? *(1)* T: Why is the Hamming distance important in correcting errors? *15 mins*	Ps copy definition into their Ex.Bs. T must ensure that Ps realise that 'distance' is defined for any two codewords whereas Hamming distance is just for the code itself. T chooses a P to give solution and reasoning. T gives Ps a few minutes to find all the distances. Ps should work systematically (use **OS 5.2**). T introduces the notation δ = Hamming distance, Ps write definition in their Ex.Bs. At this stage Ps should begin to see its significance in deciding if errors can not only be detected, but also corrected.
3	**Activity 6 from Text: Hamming distance** T: Copy the table into your Ex.Bs and then complete it. *25 mins*	T gives each P a copy of **OS 5.1** or refers them to the Appendix in their Text, and then sets the Activity. T monitors their work and after a few minutes, intervenes with a review to make sure that all are making progress. A few more minutes are allowed for them to finish.
4	**Activity 7 from Text: Error detection** T (showing table already drawn on board, or **OS 5.3**): We're going to work together to complete this table. Put up your hand when you can complete the figures for Code 3. Now who can do the same for Code 4? .... Code 5? *35 mins*	Whole class, interactive activity. Ps show when they have the answers and T chooses volunteers (or reluctant other Ps) to write on the board/OS. T praises Ps for correct answers. T reviews progress with class.

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5	**Theorem** T: We are now ready to get a major result. What do you notice about the results in Activity 7 ? *(Different patterns for odd/even Hamming distances)* T: That's right. What happens if $\delta = 2$ *(Detects 1, corrects 0)* $\delta = 3$ *(Detects 1, corrects 1)* $\delta = 4$ *(Detects 2, corrects 1)* $\delta = 5$ *(Detects 2, corrects 2)* T: Why? *(If $\delta = 5$, any codeword with 2 errors is 2 away from the actual codeword but at least $5-2=3$ away from any other codeword)* T: Well done! T: We can now generalise the result! *45 mins*	T will need to judge how prepared the Ps are for this; it might be necessary to give more examples to reinforce the concept. T might need to construct codes to further illustrate this answer. T dictates result.
	Homework Activity 9 from Text, or any questions from Exercise 1.	