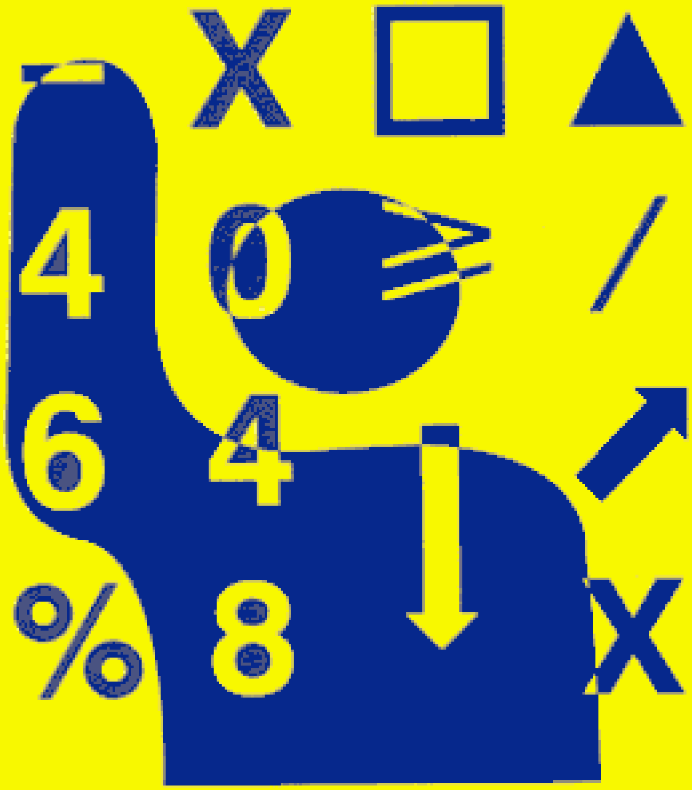
Mathematics Enhancement Programme

Primary Demonstration Project

5A Geometry

Help Booklet



Support for Primary Teachers in Mathematics

Primary Project funded by Pricewaterhouse Coopers

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Mathematics Enhancement Programme

Help Module 5

GEOMETRY

Part A

Contents of Part A

Contents of Part B

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PREFACE

This is one of a series of *Help Modules* designed to help you gain confidence in mathematics. It has been developed particularly for primary teachers (or student teachers) but it might also be helpful for non-specialists who teach mathematics in the lower secondary years. It is based on material which is already being used in the *Mathematics Enhancement Programme: Secondary Demonstration Project*.

The complete module list comprises:

1.	ALGEBRA	6.	HANDLING DATA
2.	DECIMALS	7.	MENSURATION
3.	EQUATIONS	8.	NUMBERS IN CONTEXT
4.	FRACTIONS	9.	PERCENTAGES
5.	GEOMETRY	10.	PROBABILITY

Notes for overall guidance:

- Each of the 10 modules listed above is divided into 2 parts. This is simply to help in the downloading and handling of the material.
- Though referred to as 'modules' it may not be necessary to study (or print out) each one in its entirely. As with any self-study material you must be aware of your own needs and assess each section to see whether it is relevant to those needs.
- The difficulty of the material in **Part A** varies quite widely: if you have problems with a particular section do try the one following, and then the next, as the content is not necessarily arranged in order of difficulty. Learning is not a simple linear process, and later studies can often illuminate and make clear something which seemed impenetrable at an earlier attempt.
- In **Part B, Activities** are offered as backup, reinforcement and extension to the work covered in Part A. **Tests** are also provided, and you are strongly urged to take these (at the end of your studies) as a check on your understanding of the topic.
- The marking scheme for the revision test includes B, M and A marks. Note that:

  M *marks* are for method;

  A *marks* are for accuracy (awarded only following a correct M mark);

  B *marks* are independent, stand-alone marks.

We hope that you find this module helpful. Comments should be sent to:

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CIMT, School of Education
University of Exeter
EXETER EX1 2LU

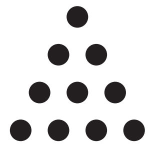
The full range of Help Modules can be found at

www.ex.ac.uk/cimt/help/menu.htm

Help Module 5 *Geometry* Introductory Notes

Historical Background

Very little is known of the life of *Pythagoras*, but he was born on the island of Samos and is credited with the founding of a community at Crotona in Southern Italy by about 530 BC. The community had religious and political purposes, but also dealt with mathematics, especially the properties of whole numbers or positive integers. Mystical attributes, such as that odd numbers were male and even numbers female, were ascribed to numbers. In addition descriptions of arithmetical properties of integers were found.



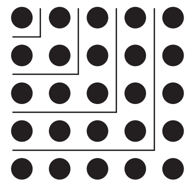
The 4th triangle number or 'Holy tetractys' had mystical significance for the Pythagoreans

The diagram on the right shows that

$$1 = 1^2$$

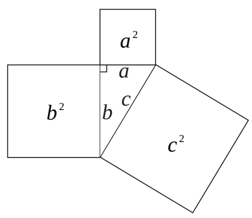
$$1 + 3 = 2^2$$

$$1 + 3 + 5 = 3^2$$



The sum of consecutive odd numbers, starting at 1, is a square number

The Pythagoreans also formulated the idea of proportions in relation to harmonics on stringed instruments. The theorem with which Pythagoras' name is associated was probably only proved later. Specific instances of it were certainly known to the Babylonians. The 'Harpedonaptai', Egyptian rope stretchers, are said to have used the 3, 4, 5 triangle to obtain right angles from equally spaced knots on cords. The ancient Chinese also knew that the 3, 4, 5 triangle was right angled.



Pythagoras' Theorem:

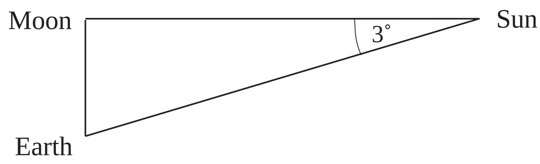
$a^2 + b^2 = c^2$

The Greeks used 'chord' tables rather than tables of trigonometric functions, and the development of trigonometric tables took place around 500 AD, through the work of Hindu mathematicians. In fact, tables of sines for angles up to $90°$ were given for 24 equal intervals of $3\frac{3}{4}°$ each. The value of $\sqrt{10}$ was used for π at that time. Further work a century later, particularly by the Indian mathematician *Brahmagupta* (in 628), led to the sine rule as we know it today.

A useful course book for the historical introduction of these topics is *'Ascent of Man'* by J. Bronowski (BBC publication).

The origins of angle measures are not due to any one person but a variety of developments in different countries. For example, *Aristarchus* (around 260 BC) in his treatise, *On the Sizes and Distances of the Sun and Moon*, made the observation that when the moon is half full, the angle between the lines of sight to the sun and the moon is less than a right angle by one-thirtieth of a quadrant (the systematic use of the $360°$ circle came a little later).

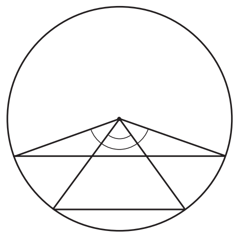
In today's language this gave the angle $3°$ in the diagram below.



In fact, it should have been about $0° \; 10°$.

It is not known just when the systematic use of $360°$ was established but it seems likely to be largely due to *Hipparchus* (180–125 BC) who was thought to have produced the first trigonometric table.

It was firmly established by Ptolemy (c 100-178 AD) who used it consistently in his astronomical treatise.



He noted that the ratio of arc to chord reduced as the angle subtended at the centre decreased, with a limit of 1.
He actually produced tables giving values for angles varying from $0°$ to $100°$.

Although the use of $360°$ was adopted by most mathematicians, the idea of using $400°$ for a circle was developed in Scandinavian countries and is in fact still used on a limited basis. (It is even included on most calculators with the 'grad' mode for angles.)

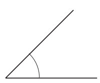
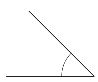
Key Issues

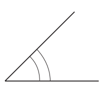
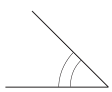
Introduction

Much of the material and concepts in this module are probably already familiar to you. The first part of the module is centred on angle geometry, which is one key area of mathematics where proof is at the heart of analysis. The second part is based on trigonometry, in which *Pythagoras' Theorem* is the key building block. This area of mathematics has obvious applications in, for example, surveying, geography and architecture. Finally, we touch on the topics of scale drawings, construction and loci, all of which are important application areas.

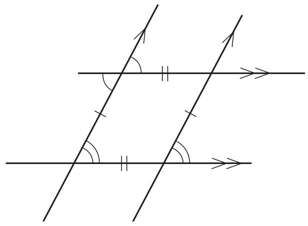
Language / Notation

- It is important always to use the degrees sign, e.g. $60°$, $40°$
- It is convention to label equal angles with the same notation, e.g.

using  and  for the 1st pair of equal angles ,

using  and  for the 2nd pair of equal angles, etc.

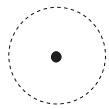
A similar notation is used for equal sides, as shown opposite.



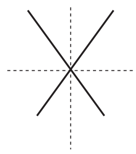
- Parallel lines are marked similarly by open arrows.

Key Points

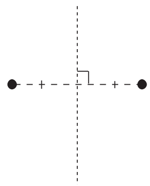
- A right angle is *exactly* $90°$.
- Scale drawings must be drawn accurately.
- Constructions (e.g. triangles, perpendicular bisectors) must be drawn according to the instructions.
- Angles in a triangle sum to $180°$.
- Angles in a quadrilateral sum to $360°$.
- Alternate angles are equal.
- Supplementary angles sum to $180°$.
- Pythagoras' Theorem
- Important loci include: – points equidistant from a fixed point (circle),



  – points equidistant from two fixed lines,

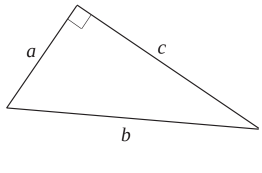


  – points equidistant from two fixed points.

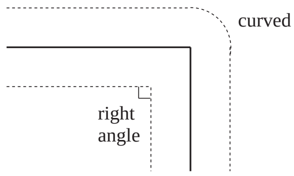


Misconceptions

- The terminology used is inclusive, e.g. a square is a special rectangle; a parallelogram is a special quadrilateral.
- In applying Pythagoras' Theorem, note that it is always the square of the *longest* side which equals the sum of the other two squares, e.g. in the diagram opposite, $b^2 = a^2 + c^2$.



- Note the difference between the outer and inner locus of a point equidistant from two perpendicular walls.



WORKED EXAMPLES and EXERCISES

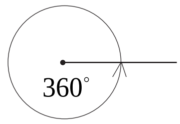
5 Geometry

5.1 Measuring Angles

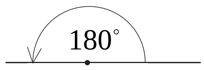
A *protractor* can be used to measure or draw angles.

Note

The angle around a complete circle is is 360°.

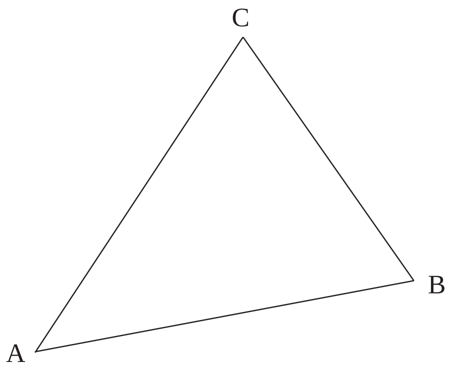


The angle around a point on a straight line is 180°.



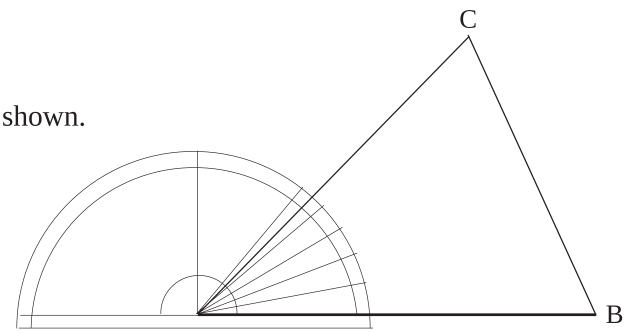
Worked Example 1

Measure the angle CAB in the triangle shown.



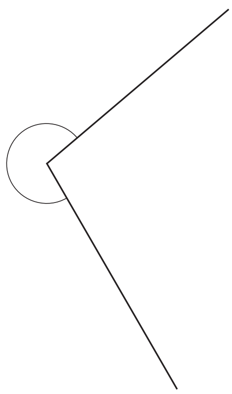
Solution

Place a protractor on the triangle as shown.
The angle is measured as 47°.



Worked Example 2

Measure this angle.



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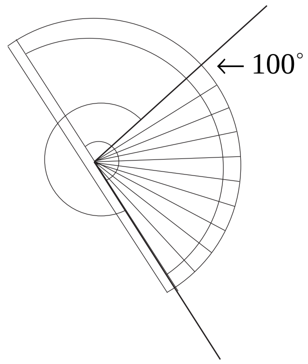
Solution



Using a protractor, the smaller angle is measured as 100°.

So

$$\text{required angle} = 360^\circ - 100^\circ = 260^\circ$$

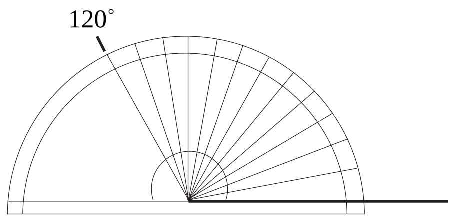
Worked Example 3

Draw angles of

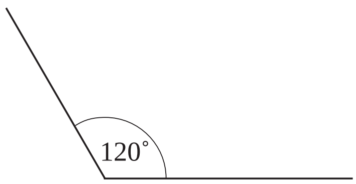
(a) 120° (b) 330°.

Solution

(a) Draw a horizontal line.
Place a protractor on top of the line and draw a mark at 120°.



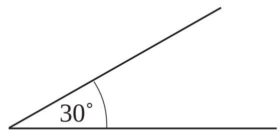
Then remove the protractor and draw the angle.



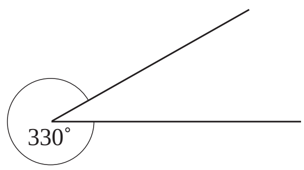
(b) To draw the angle of 330°, first subtract 330° from 360°:

$$360^\circ - 330^\circ = 30^\circ$$

Draw an angle of 30°.

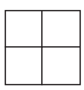


The larger angle will be 330°.



Just for Fun

Arrange 12 toothpicks as shown.
Remove only 2 toothpicks so as to leave only 2 squares.



5.1

Exercises

1. Estimate the size of each angle, then measure it with a protractor.

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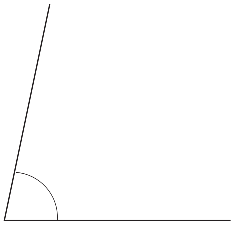
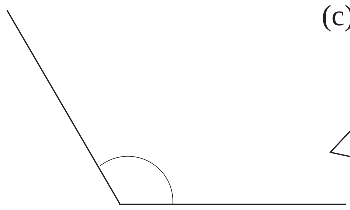


(b)


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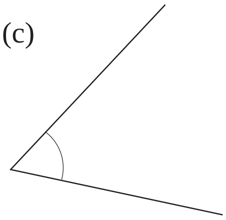

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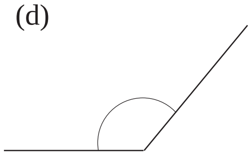
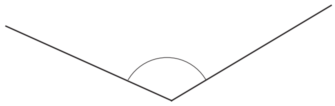


(e)


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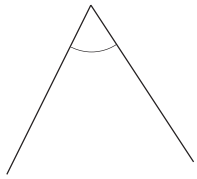
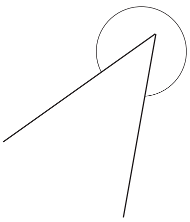


2. Draw angles with the following sizes.

(a)	50°	(b)	70°	(c)	82°
(d)	42°	(e)	80°	(f)	100°
(g)	140°	(h)	175°	(i)	160°

3. Measure these angles.

(a)


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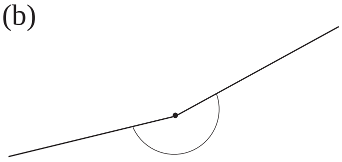

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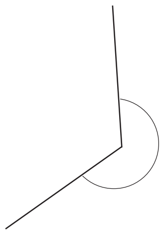

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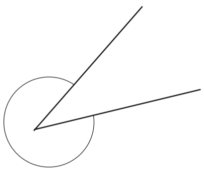

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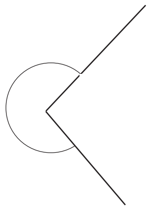
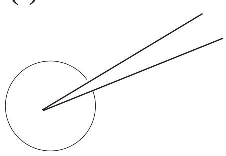
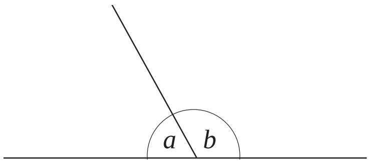


(f)


4. Draw angles with the following sizes.

(a)	320°	(b)	190°	(c)	260°
(d)	210°	(e)	345°	(f)	318°

5. Measure each named (a, b, c) angle below and add up the angles in each diagram. What do you notice?

(a)


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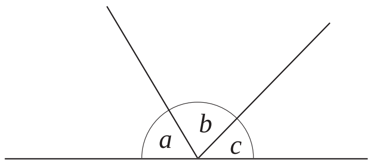

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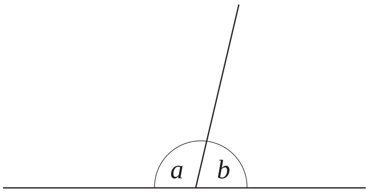
(c)



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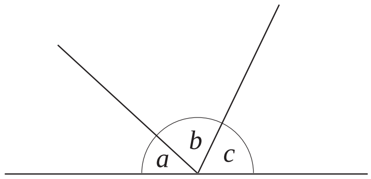
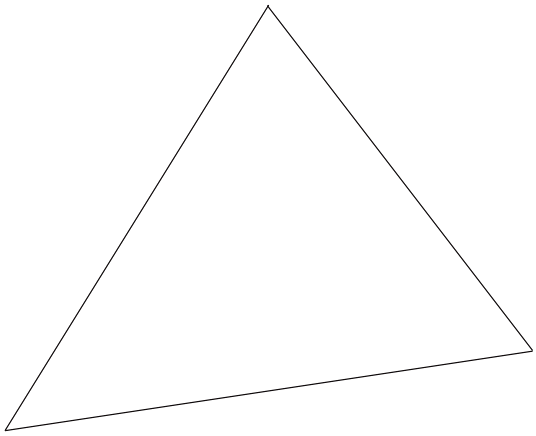
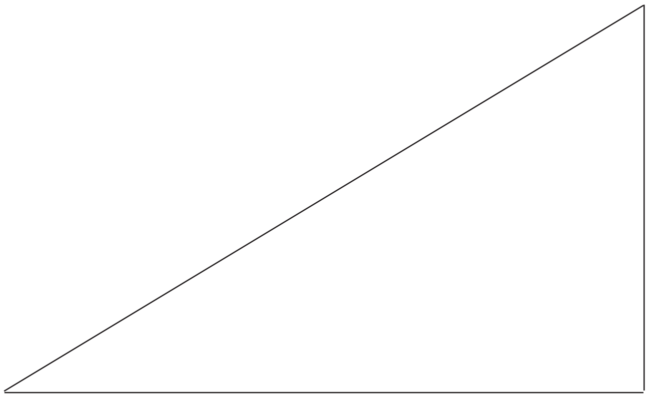


6. For each triangle below, measure each angle and add up the three angles you obtain.

(a)



(b)



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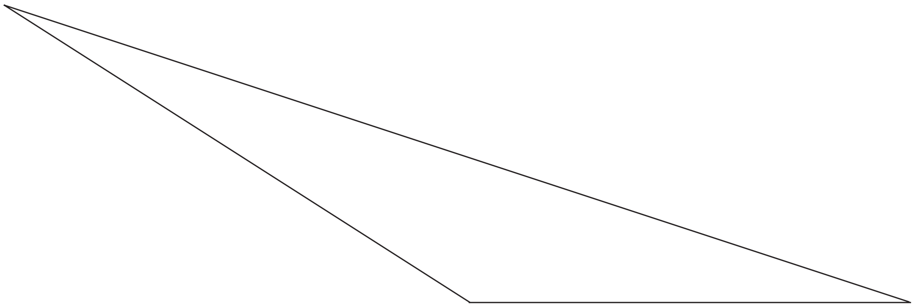
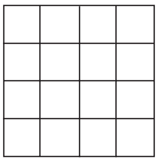


Investigation

How many squares are there in the given figure?



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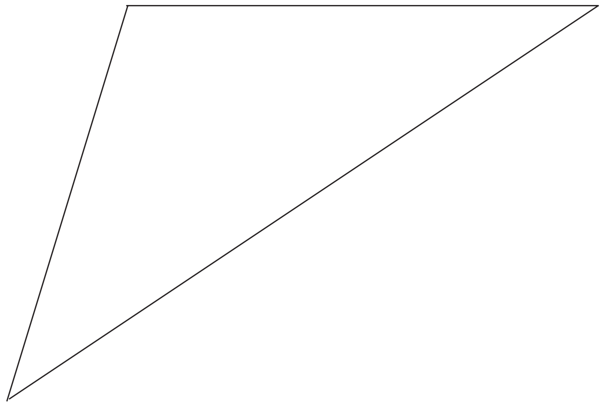
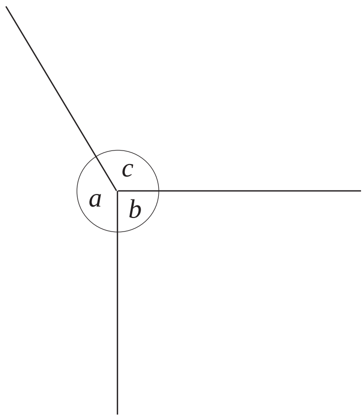
(d)



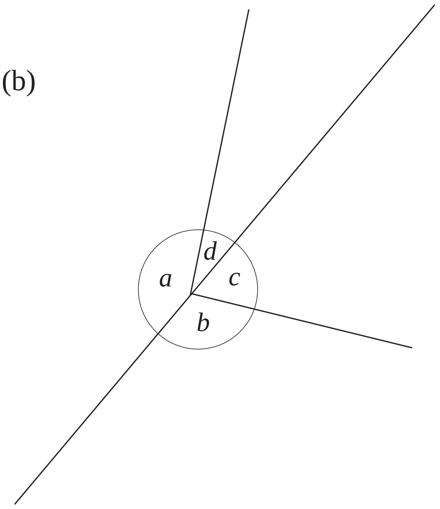
Do you obtain the same final result in each case?

7. In each diagram below, measure the angles marked with letters and find their total.
What do you notice about the totals?

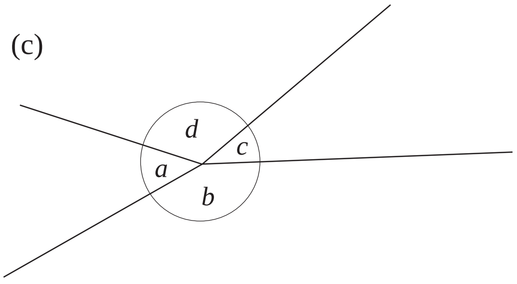
(a)



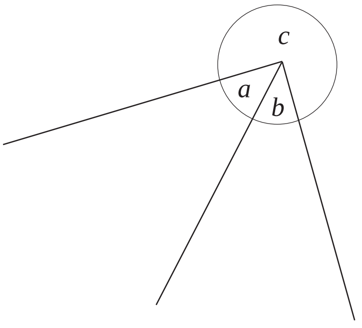
(b)



(c)

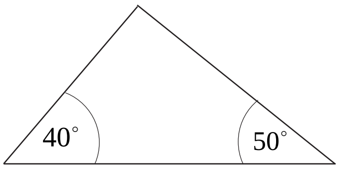


(d)



8. (a) Draw a straight line that is 10 cm long.

(b) Draw angles of 40° and 50° at each end to form the triangle shown in the diagram.



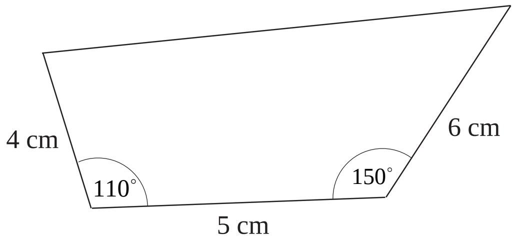
(c) Measure the lengths of the other two sides and the size of the other angle.

5.1

9. The diagram shows a rough sketch of a quadrilateral.

   (a) Draw the quadrilateral accurately.

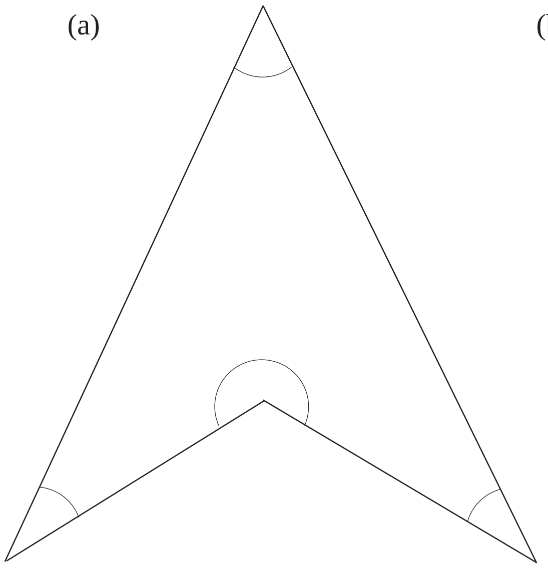
   (b) Measure the length of the fourth side and the size of the other two angles.



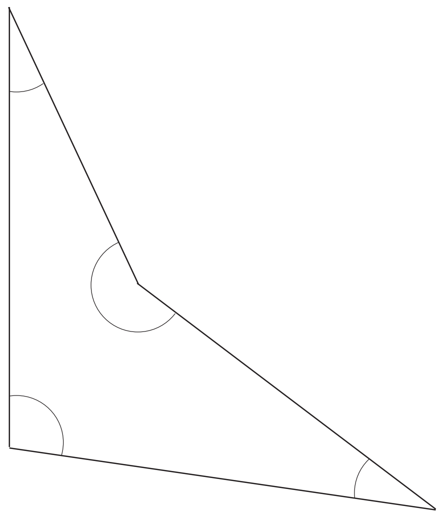
10. Measure the *interior* (inside) angles of these quadrilaterals.

    In each case find the total sum of the angles. What do you notice?

(a)



(b)



11. Draw two different pentagons.

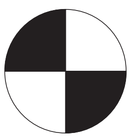
    (a) Measure each of the angles in both pentagons.

    (b) Add up your answers to find the total of the angles in each pentagon.

    (c) Do you think that the angles in a pentagon will always add up to the same number?

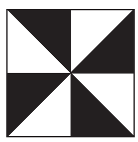
5.2 Line and Rotational Symmetry

An object has *rotational symmetry* if it can be rotated about a point so that it fits on top of itself without completing a full turn. The shapes below have rotational symmetry.



In a complete turn this shape fits on top of itself two times.

It has rotational symmetry of order 2.

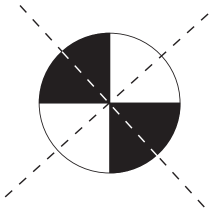


In a complete turn this shape fits on top of itself four times.

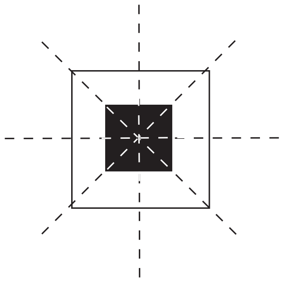
It has rotational symmetry of order 4.

5.2

Shapes have *line symmetry* if a mirror could be placed so that one side is an exact reflection of the other. These imaginary 'mirror lines' are shown by dotted lines in the diagrams below.



This shape has
2 lines of symmetry.



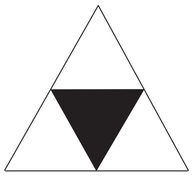
This shape has
4 lines of symmetry.

Worked Example 1

For the shape opposite state:

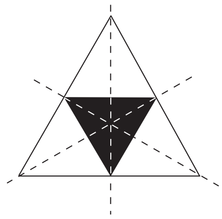
(a) the number of lines of symmetry,

(b) the order of rotational symmetry.

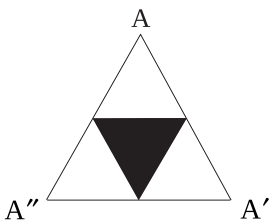


Solution

(a) There are 3 lines of symmetry as shown.



(b) There is rotational symmetry with order 3, because the point marked A could be rotated to A' then to A" and fit exactly over its original shape at each of these points.



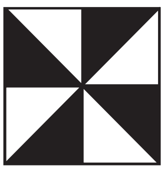
Exercises

1. Which of the shapes below have

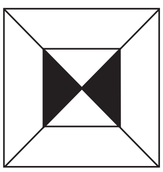
(a) line symmetry (b) rotational symmetry?

For line symmetry, copy the shape and draw in the mirror lines.
For rotational symmetry state the order.

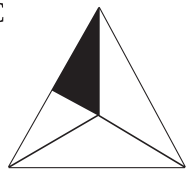
A



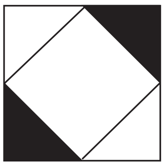
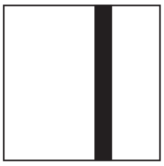
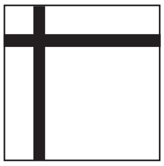
B

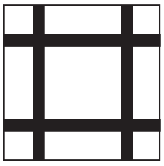
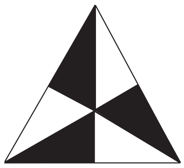
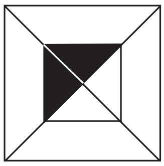


C



5.2

D  E  F 

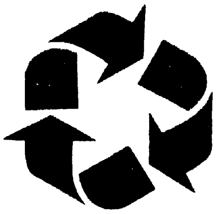
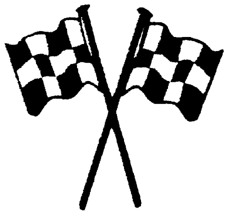
G  H  I 

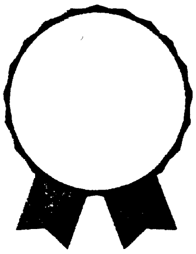
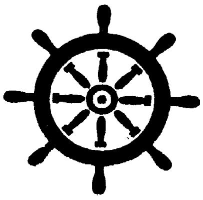
2. For each shape below state:

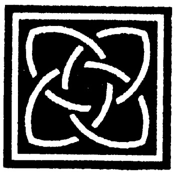
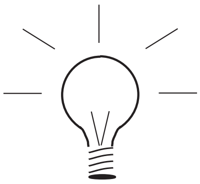
   (a) whether the shape has any symmetry;

   (b) how many lines of symmetry it has;

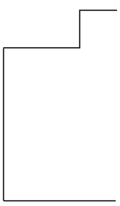
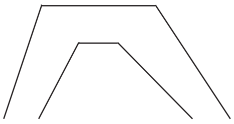
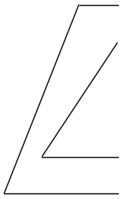
   (c) the order of symmetry if it has rotational symmetry.

A  B  C 

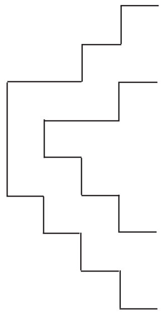
D  E  F 

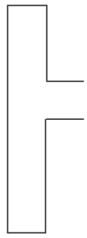
G  H  I 

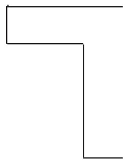
3. Copy and complete each shape below so that they have line symmetry but not rotational symmetry. Mark clearly the lines of symmetry.

(a)  (b)  (c) 

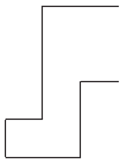
5.2

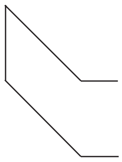
(d) 

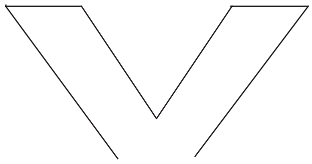
(e) 

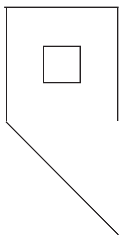
(f) 

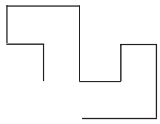
4. Copy and, if possible, complete each shape below, so that they have rotational symmetry, but not line symmetry. In each case state the order of the rotational symmetry.

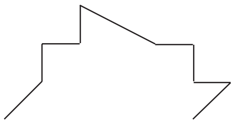
(a) 

(b) 

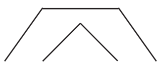
(c) 

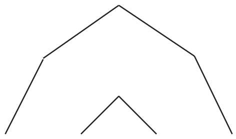
(d) 

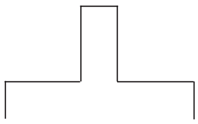
(e) 

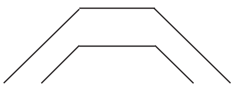
(f) 

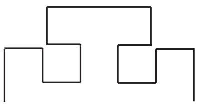
5. Copy and complete each of the following shapes, so that they have both rotational and line symmetry. In each case draw the lines of symmetry and state the order of the rotational symmetry.

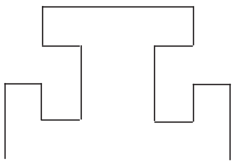
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

6. Draw a square and show all its lines of symmetry.

7. (a) Draw a triangle with:

(i) 1 line of symmetry (ii) 3 lines of symmetry.

(b) Is it possible to draw a triangle with 2 lines of symmetry?

8. Draw a shape which has 4 lines of symmetry.

5.2

9. Draw a shape with rotational symmetry of order:

(a) 2 (b) 3 (c) 4 (d) 5

10. Can you draw:

(a) a pentagon with exactly 2 lines of symmetry,

(b) a hexagon with exactly 2 lines of symmetry,

(c) an octagon with exactly 3 lines of symmetry?

11. These are the initials of the International Association of Whistlers.

I A W

Which of these letters has rotational symmetry?

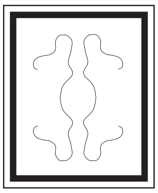
(SEG)

12. Which of the designs below have line symmetry?

(a)

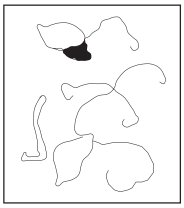
Taj Mahal floor tile

(b)



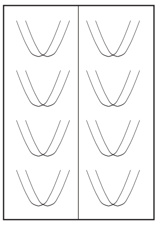
Asian carpet design

(c)



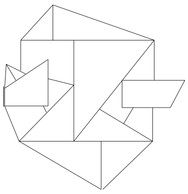
Contemporary art

(d)



Wallpaper pattern

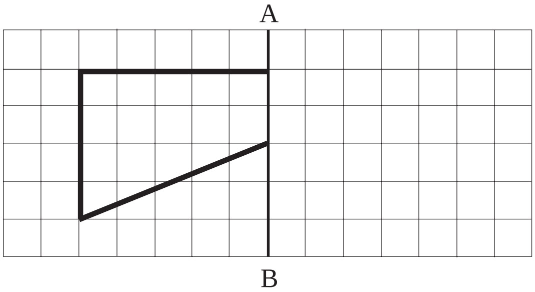
(e)



Tile design

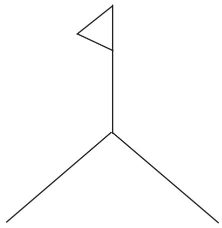
(LON)

13. (a) Copy and draw the reflection of this shape in the mirror line AB.

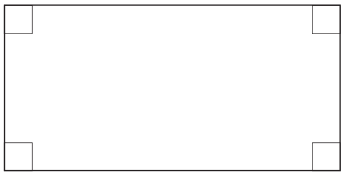


5.2

(b) Copy and complete the diagram so that it has rotational symmetry.



(c) What is the order of rotational symmetry of this shape?



(SEG)

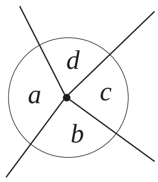
5.3 Angle Geometry

There are a number of important results concerning angles in different shapes, at a point and on a line. In this section the following results will be used.

1. *Angles at a Point*

   The angles at a point will always add up to 360°.

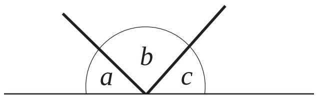
   It does not matter how many angles are formed at the point – their total will always be 360°.



$a + b + c + d = 360°$

2. *Angles on a Line*

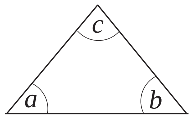
   Any angles that form a straight line add up to 180°.



$a + b + c = 180°$

3. *Angles in a Triangle*

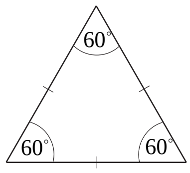
   The angles in any triangle add up to 180°.



$a + b + c = 180°$

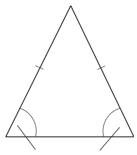
4. *Angles in an Equilateral Triangle*

   In an equilateral triangle all the angles are 60° and all the sides are the same length.



5. *Angles in an Isosceles Triangle*

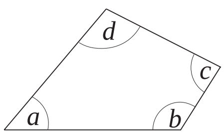
   In an isosceles triangle two sides are the same length and the two base angles are the same size.



equal angles

6. *Angles in a quadrilateral*

   The angles in any quadrilateral add up to 360°.

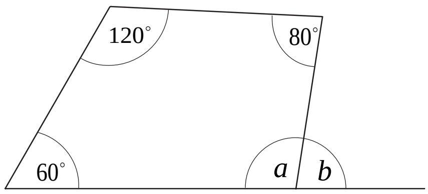


$a + b + c + d = 360°$

5.3

Worked Example 1

Find the sizes of angles a and b in the diagram below.



Solution

First consider the quadrilateral. All the angles of this shape must add up to 360°, so

$$60^\circ + 120^\circ + 80^\circ + a = 360^\circ$$
$$260^\circ + a = 360^\circ$$
$$a = 360^\circ - 260^\circ$$
$$= 100^\circ$$

Then consider the straight line formed by the angles a and b. These two angles must add up to 180° so,

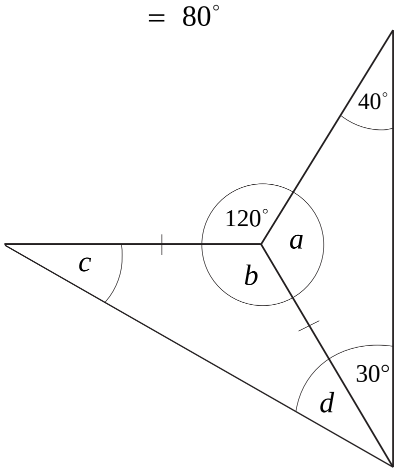
$$a + b = 180^\circ$$

but $a = 100^\circ$, so

$$100^\circ + b = 180^\circ$$
$$b = 180^\circ - 100^\circ$$
$$= 80^\circ$$

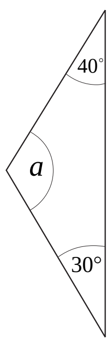
Worked Example 2

Find the angles a, b, c and d in the diagram.



Solution

First consider the triangle shown.
The angles of this triangle must add up to 180°,

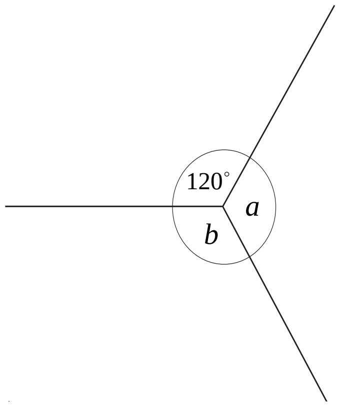


So,

$$40^\circ + 30^\circ + a = 180^\circ$$
$$70^\circ + a = 180^\circ$$
$$a = 110^\circ$$

5.3

Next consider the angles round the point shown.



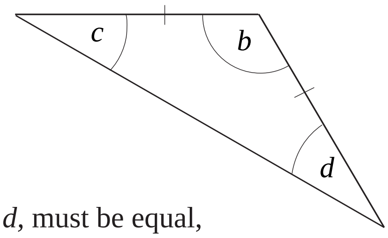
The three angles must add up to 360°, so

$$120^\circ + b + a = 360^\circ$$

but $a = 110^\circ$, so

$$\begin{aligned} 120^\circ + 110^\circ + b &= 360^\circ \\ 230^\circ + b &= 360^\circ \\ b &= 360^\circ - 230^\circ \\ &= 130^\circ. \end{aligned}$$

Finally, consider the second triangle.



The angles must add up to 180°, so

$$c + b + d = 180^\circ$$

As this is an isosceles triangle the two base angles, c and d, must be equal, so using $c = d$ and the fact that $b = 130^\circ$, gives

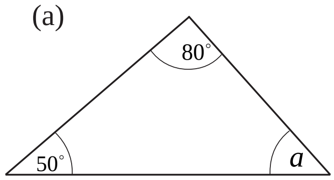
$$\begin{aligned} c + 130^\circ + c &= 180^\circ \\ 2c &= 180^\circ - 130^\circ \\ &= 50^\circ \\ c &= 25^\circ \end{aligned}$$

As $c = 25^\circ$, $d = 25^\circ$.

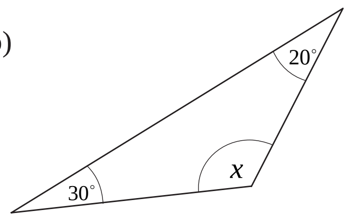
Exercises

1. Find the size of the angles marked with a letter in each diagram.

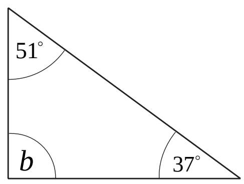
(a)



(b)



(c)



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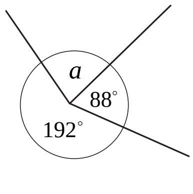

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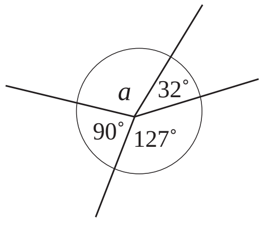
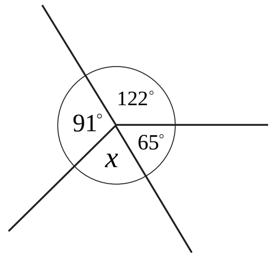
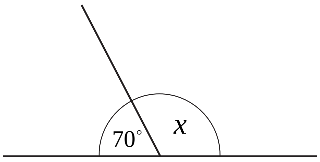


(f)



5.3

(g)



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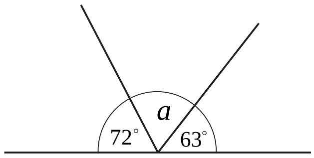
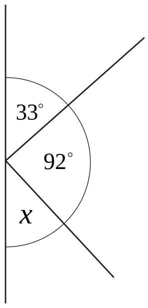
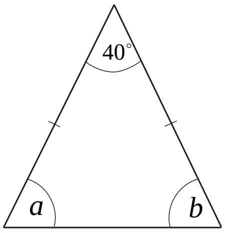


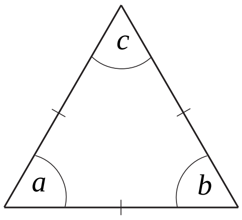
(i)



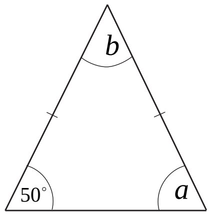
(j)



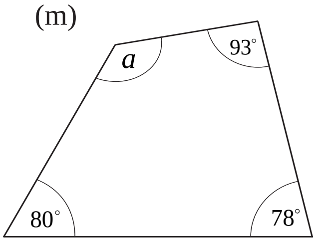
(k)



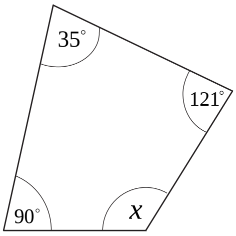
(l)



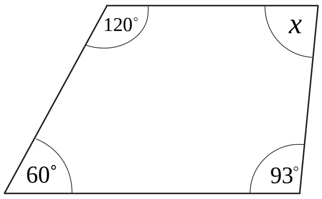
(m)



(n)

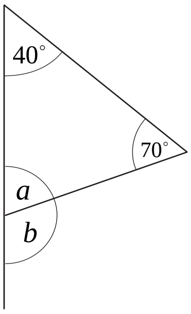


(o)

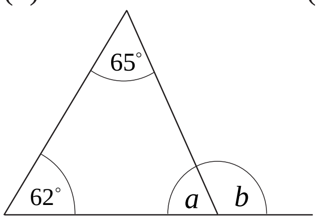


2. (a) For each triangle, find the angles marked *a* and *b* .

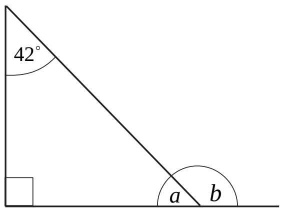
(i)



(ii)



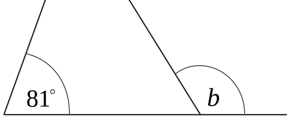
(iii)



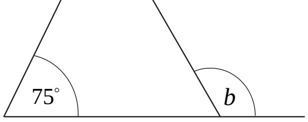
(b) What do you notice about the angle marked *b* and the other two angles given in each problem?

(c) Find the size of the angle *b* in each problem below without working out the size of any other angles.

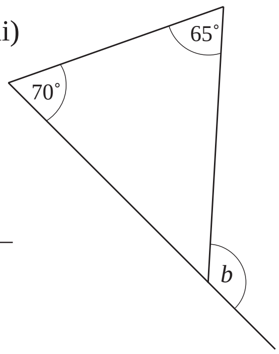
(i)



(ii)

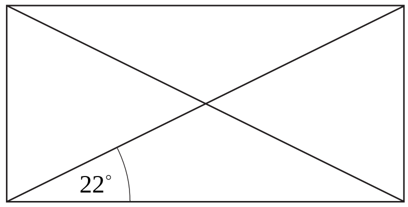


(iii)



5.3

3. The diagram below shows a rectangle with its diagonals drawn in.

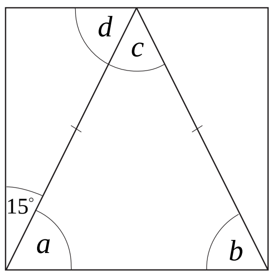


(a) Copy the diagram and mark in all the other angles that are 22°.

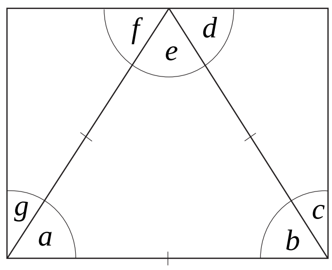
(b) Find the sizes of all the other angles.

4. Find the angles marked with letters in each of the following diagrams.
In each diagram the lines all lie inside a rectangle.

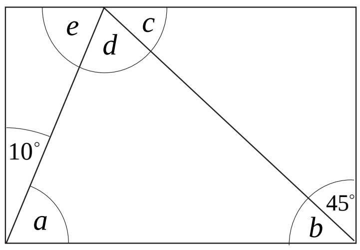
(a)



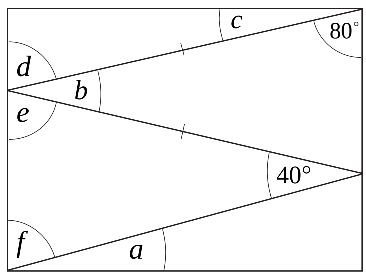
(b)



(c)

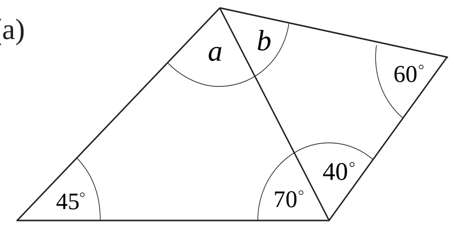


(d)

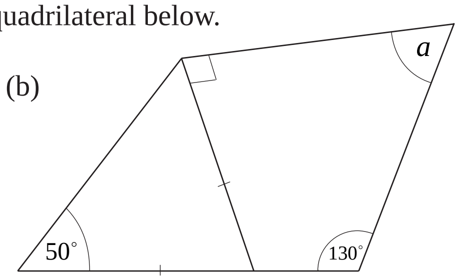


5. Find the angles marked with letters in each quadrilateral below.

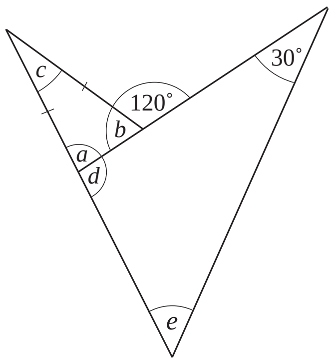
(a)



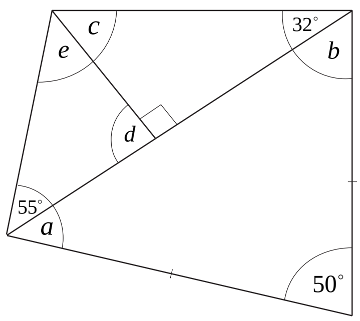
(b)



(c)

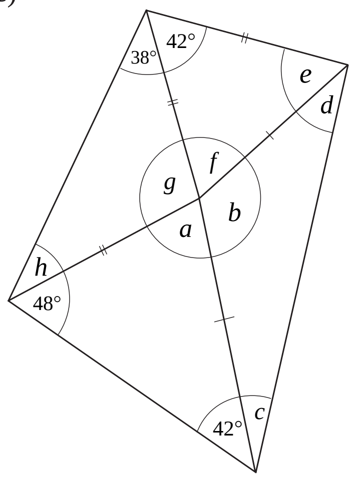


(d)

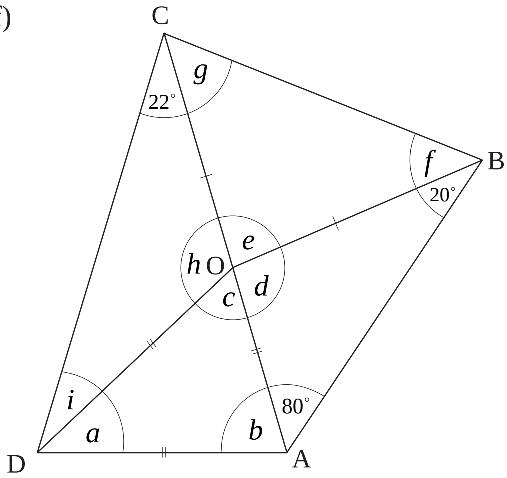


5.3

(e)

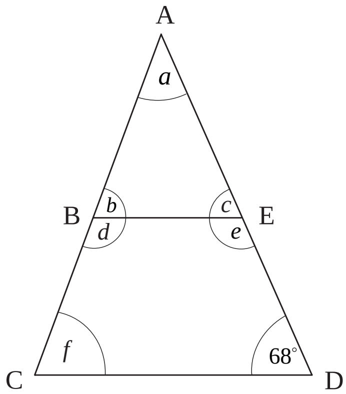


(f)



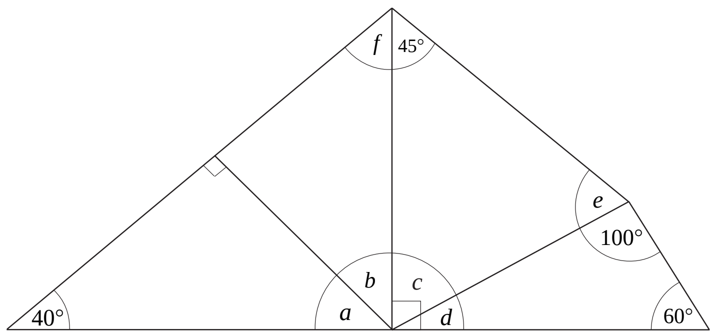
AOC is a straight line.

6. A swing is built from two metal frames as shown below.



The lengths of AB and AE are the same and the lengths of AC and AD are the same. Find the sizes of the angles *a*, *b*, *c*, *d*, *e* and *f*.

7. The diagram shows a wooden frame that forms part of the roof of a house.



Find the sizes of the angles *a*, *b*, *c*, *d*, *e* and *f*.

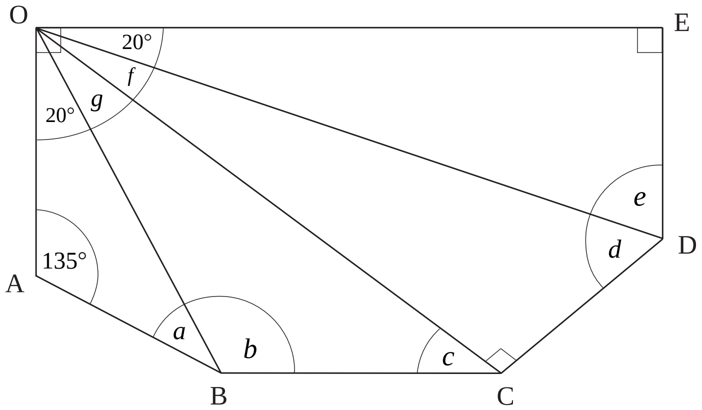
Information

The word 'geometry' is derived from the Greek words, ge (earth) and metrein (to measure). Euclid's masterpiece, 'The Elements', survived as the basic textbook for over 2 000 years. The geometry we are studying in this unit is sometimes referred to as Euclidean geometry.

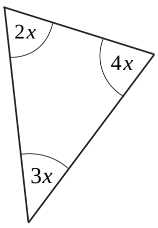
5.3

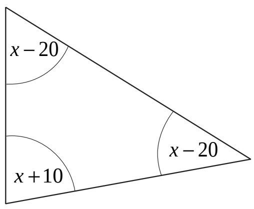
8. The diagram shows the plan for a conservatory. Lines are drawn from the point O to each of the other corners. Find all the angles marked with letters, if

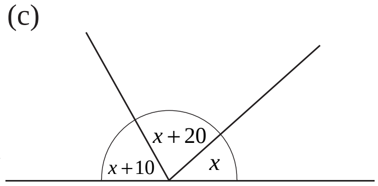
$$A\hat{B}C = B\hat{C}D = C\hat{D}E = 135°.$$

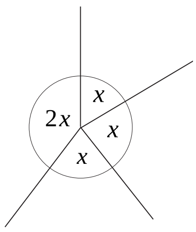


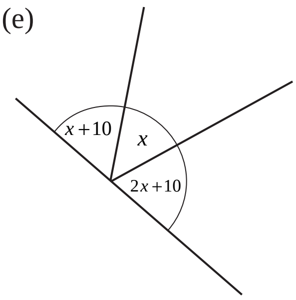
9. Write down an equation and use it to find the value of x in each diagram.

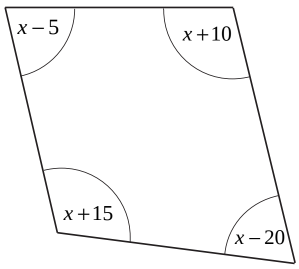
(a)


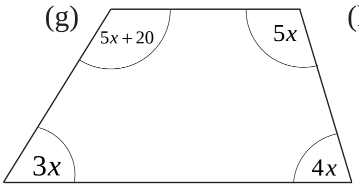
(b)


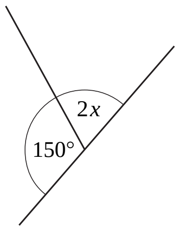
(c)


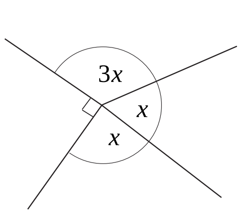
(d)


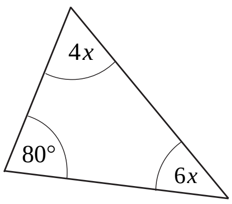
(e)


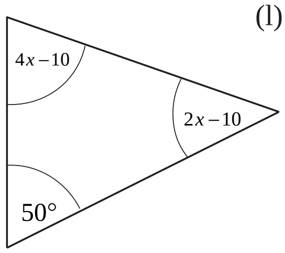
(f)


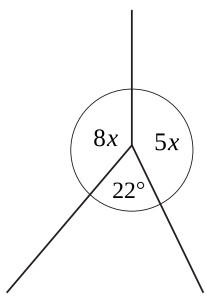
(g)


(h)


(i)


(j)


(k)


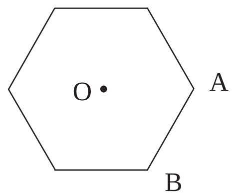
(l)


5.3

10. The diagram shows a regular hexagon.

O is the point at the centre of the hexagon.

A and B are two vertices.



(a) Write down the order of rotational symmetry of the regular hexagon.

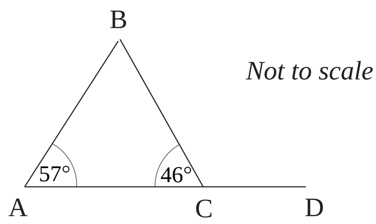
(b) Draw the lines from O to A and from O to B.

(i) Write down the size of angle AOB.

(ii) Write down the mathematical name for triangle AOB.

(LON)

11. Calculate angles BCD and ABC, giving reasons for your answers.



(MEG)

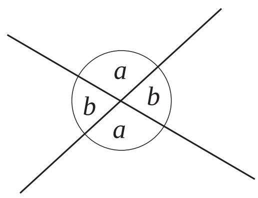
5.4 Angles with Parallel and Intersecting Lines

Opposite Angles

When any two lines cross, two pairs of equal angles are formed.

The two angles marked *a* are a pair of *opposite* equal angles.

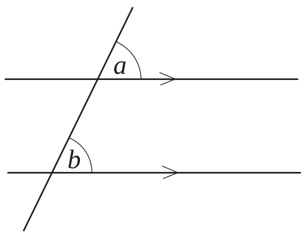
The angles marked *b* are also a pair of opposite equal angles.



Corresponding Angles

When a line crosses a pair of parallel lines, $a = b$.

The angles *a* and *b* are called *corresponding* angles.



Alternate Angles

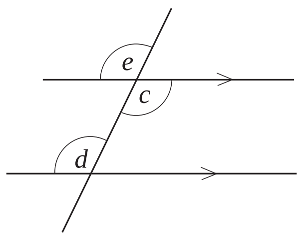
The angles *c* and *d* are equal.

Proof

This result follows since *c* and *e* are opposite angles, so $c = e$, and *e* and *d* are corresponding angles, so $c = d$.

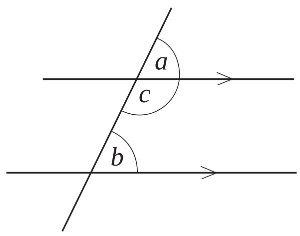
Hence $c = e = d$

The angles *c* and *d* are called *alternate* angles.



Supplementary Angles

The angles *b* and *c* add up to $180°$.



Proof

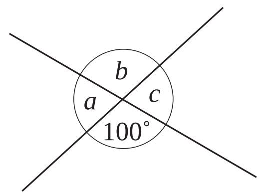
This result follows since $a + c = 180°$ (straight line), and $a = b$ since they are corresponding angles.

Hence $b + c = 180°$.

These angles are called *supplementary* angles.

5.4

Worked Example 1



Find the angles marked a, b and c.

Solution

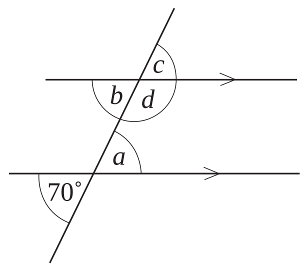
There are two pairs of opposite angles here so:

$$b = 100 \text{ and } a = c.$$

Also a and b form a straight line so

$$\begin{aligned} a + b &= 180° \\ a + 100° &= 180° \\ a &= 80°, \text{ so } c = 80°. \end{aligned}$$

Worked Example 2



Find the sizes of the angles marked a, b, c and d in the diagram.

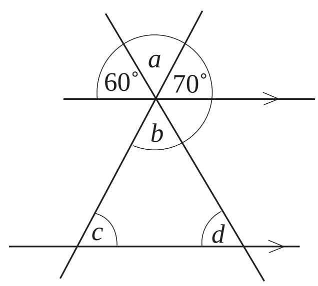
Solution

First note the two parallel lines marked with arrow heads.

Then find a. The angle a and the angle marked $70°$ are *opposite* angles, so $a = 70°$.
The angles a and b are *alternate* angles so $a = b = 70°$.
The angles b and c are *opposite* angles so $b = c = 70°$.
The angles a and d are a pair of *interior* angles, so $a + d = 180°$, but $a = 70°$, so

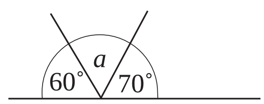
$$\begin{aligned} 70° + d &= 180° \\ d &= 180° - 70° \\ &= 110°. \end{aligned}$$

Worked Example 3



Find the angles marked a, b, c and d in the diagram.

Solution



To find the angle a, consider the three angles that form a straight line. So

$$\begin{aligned} 60° + a + 70° &= 180° \\ a &= 180° - 130° \\ &= 50°. \end{aligned}$$

The angle marked b is opposite the angle a, so $b = a = 50°$.

Now c and d can be found using *corresponding* angles.

The angle c and the $70°$ angle are corresponding angles, so $c = 70°$.

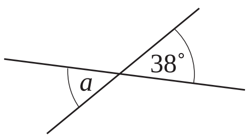
The angle d and the $60°$ angle are corresponding angles, so $d = 60°$.

5.4

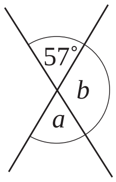
Exercises

1. Find the angles marked in each diagram, *giving reasons for your answers.*

(a)



(b)



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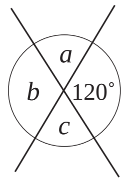

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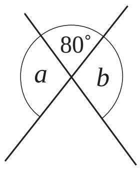

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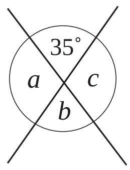

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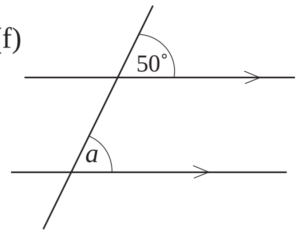

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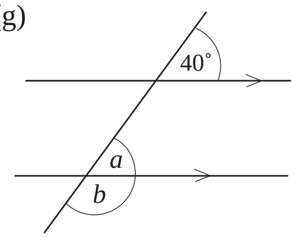

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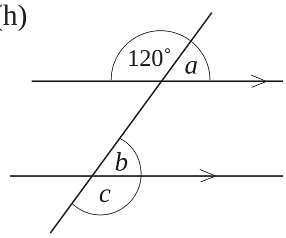

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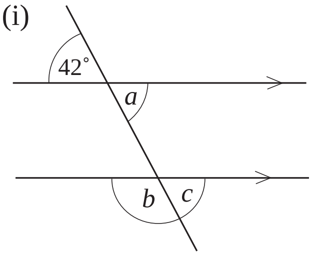

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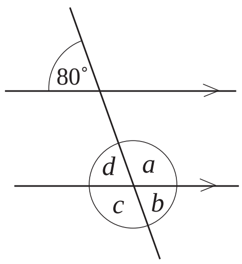

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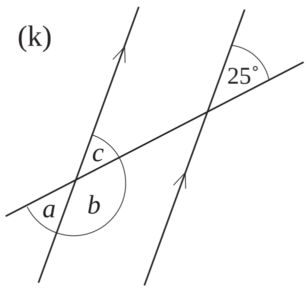

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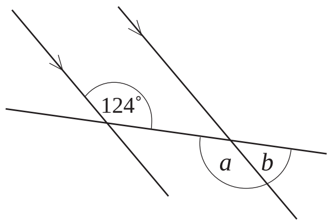

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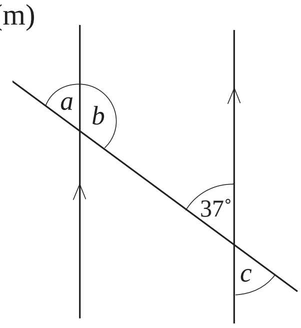
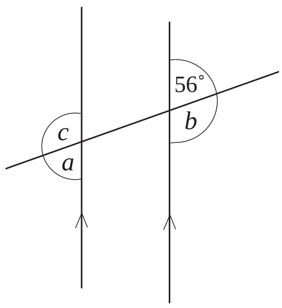


(n)



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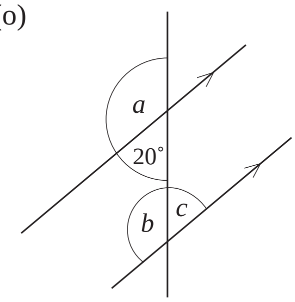
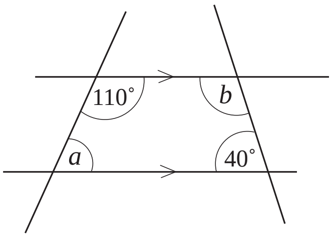


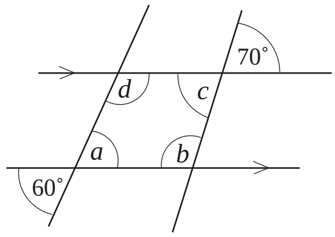
5.4

2. Find the size of the angles marked *a*, *b*, *c*, etc. in each of the diagrams below.

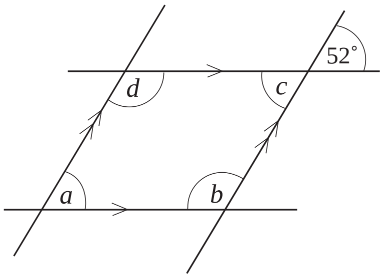
(a)



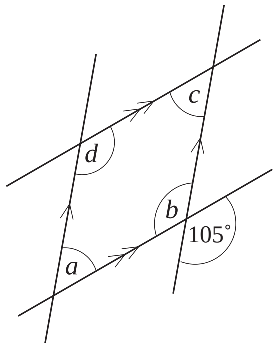
(b)



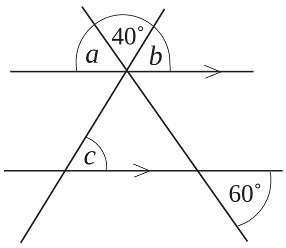
(c)



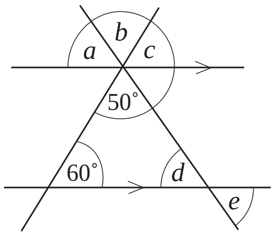
(d)



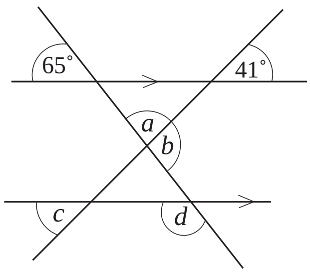
(e)



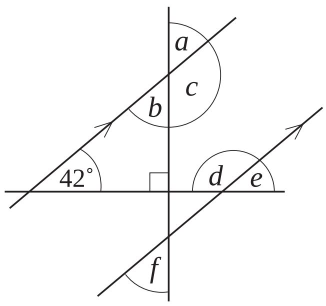
(f)



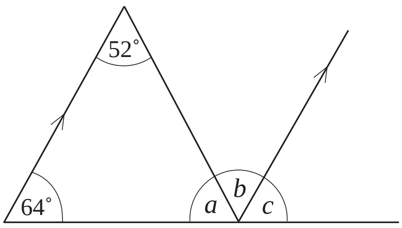
(g)



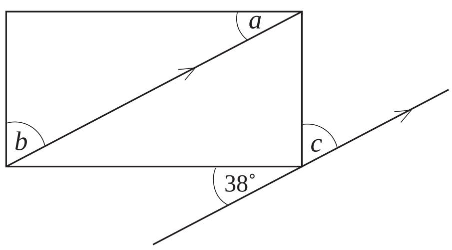
(h)



(i)



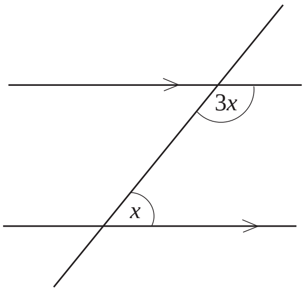
(j)



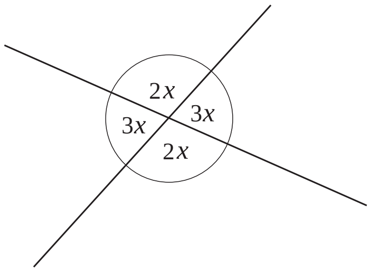
5.4

3. By considering each diagram, write down an equation and find the value of *x*.

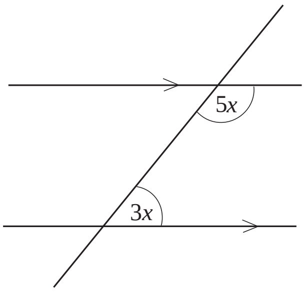
(a)



(b)



(c)



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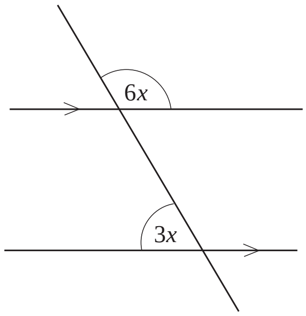

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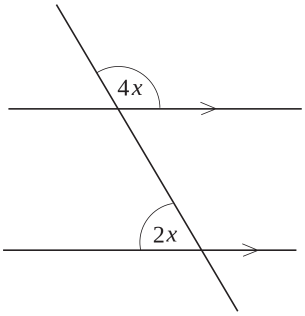
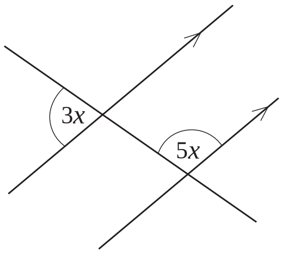
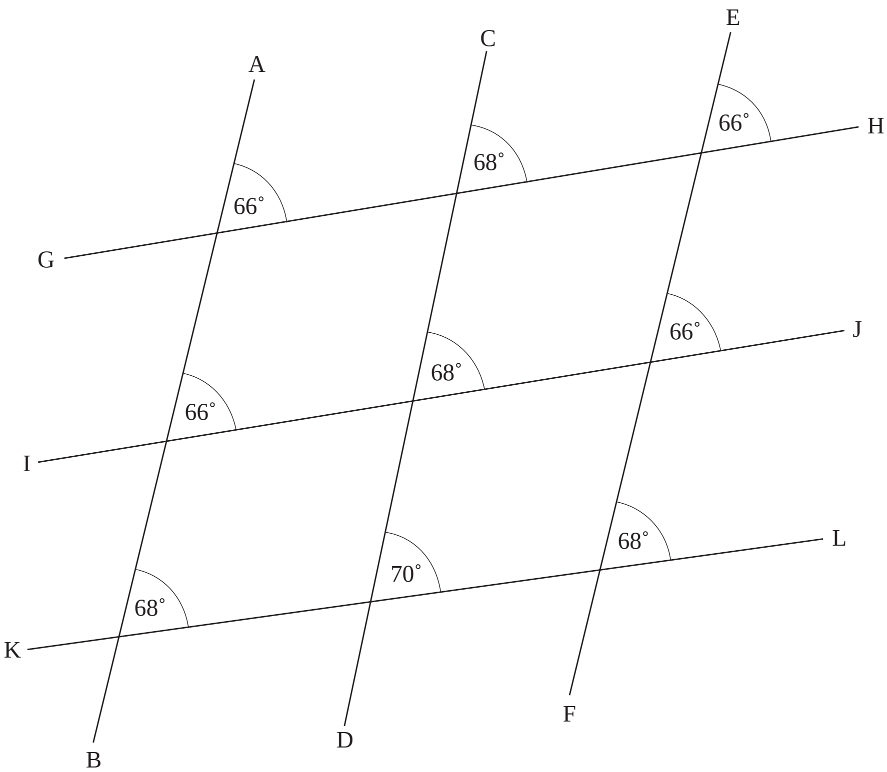


(f)

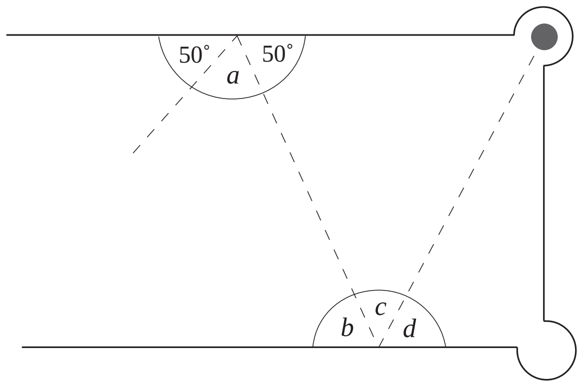


4. Which of the lines shown below are parallel?



5.4

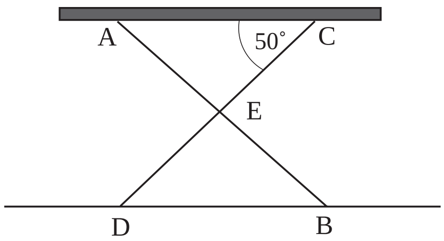
5. The diagram shows the path of a pool ball as it bounces off cushions on opposite sides of a pool table.



(a) Find the angles *a* and *b*.

(b) If, after the second bounce, the path is parallel to the path before the first bounce, find *c* and *d*.

6. A workbench is standing on a horizontal floor. The side of the workbench is shown.



The legs AB and CD are equal in length and joined at E. AE = EC

(a) Which two lines are parallel?

Angle ACD is $50°$.

(b) Work out the size of angle BAC giving a reason for your answer.

(SEG)

7. Here are the names of some quadrilaterals.

Square
Rectangle
Rhombus
Parallelogram
Trapezium
Kite

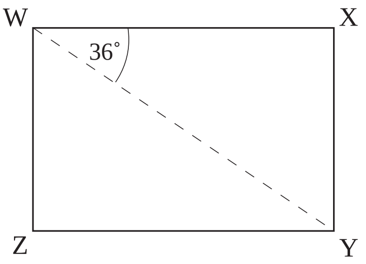
(a) Write down the names of the quadrilaterals which have two pairs of parallel sides.

(b) Write down the names of the quadrilaterals which must have two pairs of equal sides.

(LON)

5.4

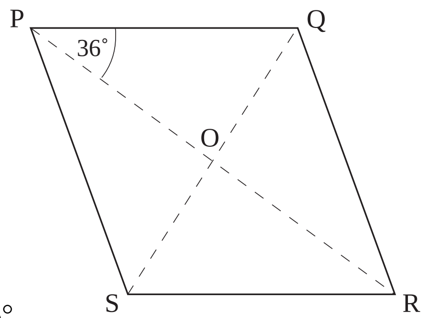
8. WXYZ is a rectangle.



Not to scale

(a) Angle XWY = 36°.
Work out the size of angle WYZ, giving a reason for your answer.

PQRS is a rhombus.



Not to scale

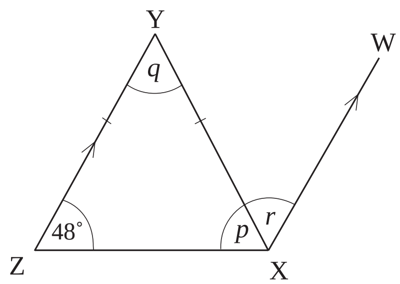
(b) Angle QPR = 36°.

The diagonals PR and QS cross at O.

Work out the size of angle PQS, giving a reason for your answer.

(SEG)

9. In the diagram, XY = ZY and ZY is parallel to XW.



Not to scale

(a) Write down the size of angle p.

(b) Calculate the size of angle q. Give a reason for your answer.

(c) Give a reason why angle q = angle r.

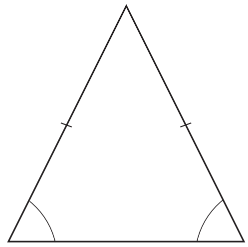
(SEG)

5.5 Squares and Triangles

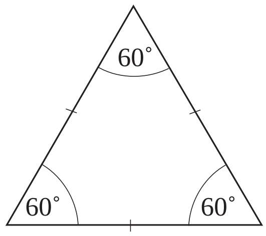
A *triangle* is a geometric shape with three sides and three angles. Some of the different types of triangles are described in this Unit.

A *square* is a four-sided geometric shape with all sides of equal length. All four angles are the same size.

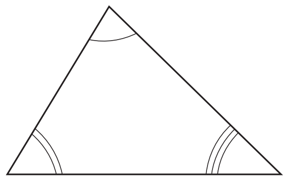
An *isosceles* triangle has two sides that are the same length, and the two base angles are equal.



All the sides of an *equilateral* triangle are the same length, and the angles are all $60°$.

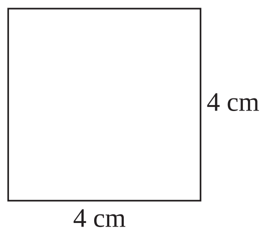


A *scalene* triangle has sides that all have different lengths, and has 3 different angles.



Worked Example 1

Find the area of the square shown in the diagram.



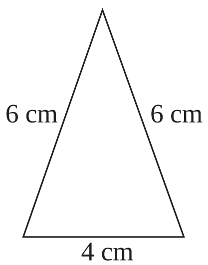
Solution

$$\text{Area} = 4 \times 4$$
$$= 16 \text{ cm}^2$$

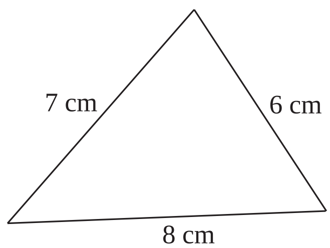
Exercises

1. For each triangle below, state whether it is *scalene*, *isosceles* or *equilateral*.

(a)

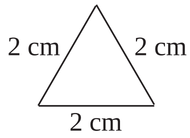


(b)

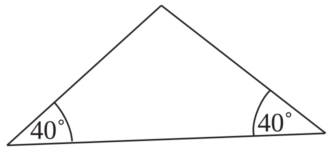


5.5

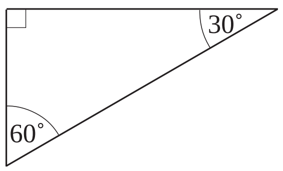
(c)



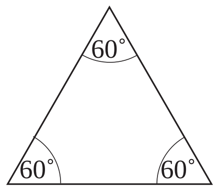
(d)



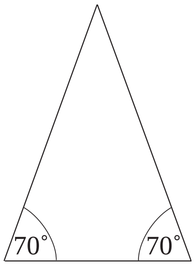
(e)



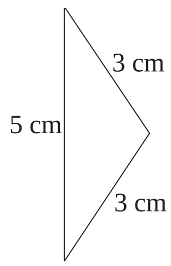
(f)



(g)



(h)

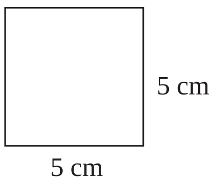


2. (a) When a square is cut in half diagonally, two triangles are obtained. Are these triangles scalene, isosceles or equilateral?

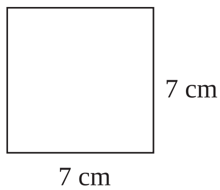
   (b) What type of triangle do you get if you cut a rectangle in half diagonally?

3. Find the area of each square below.

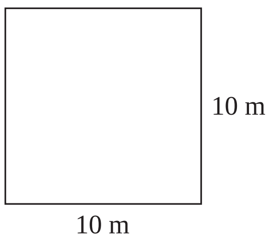
   (a)



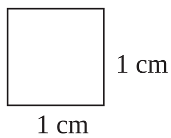
   (b)



   (c)



   (d)



4. Find the areas of the squares with sides of length:

   (a) 2 m (b) 100 m (c) 15 cm (d) 17 cm

5. Find the lengths of the sides of a square that has an area of:

   (a) 9 cm^2 (b) 25 m^2 (c) 100 m^2

   (d) 64 cm^2 (e) 1 cm^2 (f) 400 m^2

5.5

6. Two squares of side 4 cm are joined together to form a rectangle. What is the area of the rectangle?

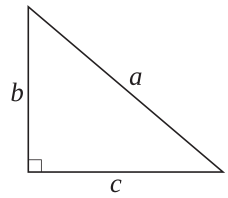
7. A square of side 12 cm is cut in half to form two triangles. What is the area of each triangle?

8. A square of side 6 cm is cut into quarters to form 4 smaller squares. What is the area of each of these squares?

5.6 Pythagoras' Theorem

Pythagoras' Theorem gives a relationship between the lengths of the sides of a right angled triangle. For the triangle shown,

$$a^2 = b^2 + c^2$$

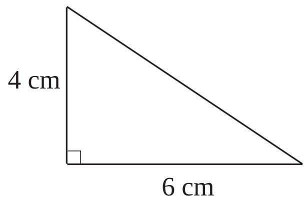


Note

The longest side of a right angled triangle is called the *hypotenuse*.

Worked Example 1

Find the length of the hypotenuse of the triangle shown in the diagram. Give your answer correct to 2 decimal places.



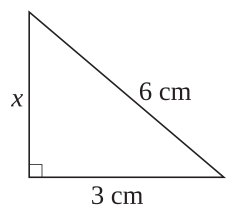
Solution

As this is a right angled triangle, Pythagoras' Theorem can be used. If the length of the hypotenuse is a, then $b = 4$ and $c = 6$.

So
$$a^2 = b^2 + c^2$$
$$a^2 = 4^2 + 6^2$$
$$a^2 = 16 + 36$$
$$a^2 = 52$$
$$a = \sqrt{52}$$
$$= 7.2 \text{ cm}$$ (to one decimal place)

Worked Example 2

Find the length of the side of the triangle marked x in the diagram.



Solution

As this is a right angled triangle, Pythagoras' Theorem can be used. Here the length of the hypotenuse is 6 cm, so $a = 6$ cm and $c = 3$ cm with $b = x$.

5.6

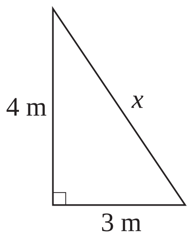
So

$$
\begin{aligned}
a^2 &= b^2 + c^2 \\
6^2 &= x^2 + 3^2 \\
36 &= x^2 + 9 \\
36 - 9 &= x^2 \\
36 - 9 &= x^2 \\
27 &= x^2 \\
\sqrt{27} &= x \\
x &= 5.2 \text{ cm} \qquad \text{(to one decimal place)}
\end{aligned}
$$

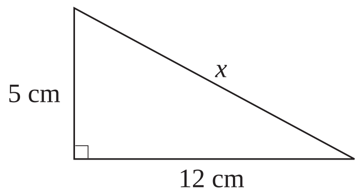
Exercises

1. Find the length of the side marked x in each triangle.

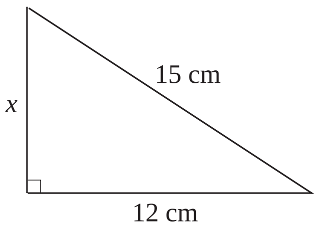
(a)



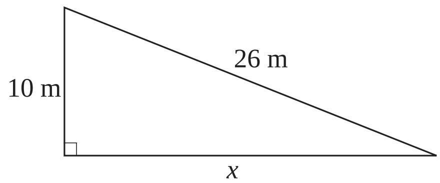
(b)



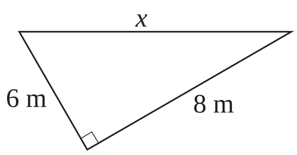
(c)



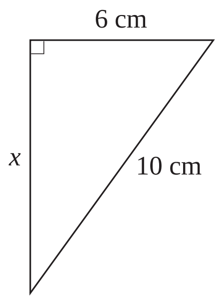
(d)



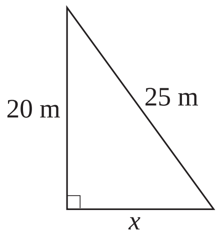
(e)



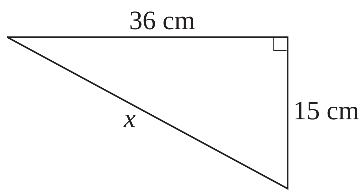
(f)



(g)



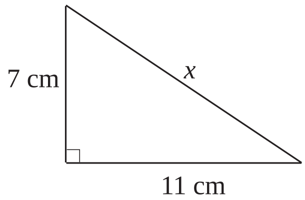
(h)



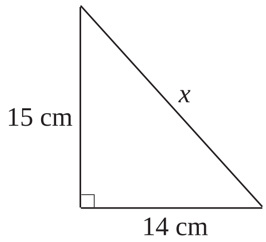
5.6

2. Find the length of the side marked x in each triangle. Give your answers correct to 2 decimal places.

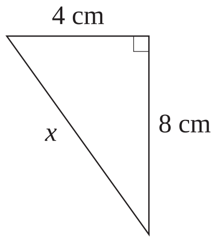
(a)



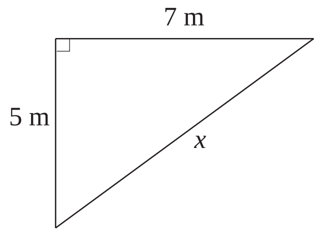
(b)



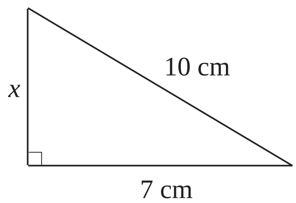
(c)



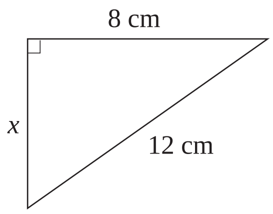
(d)



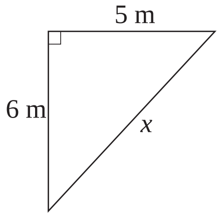
(e)



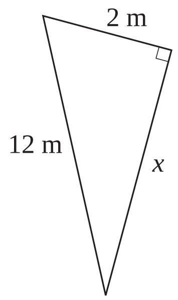
(f)



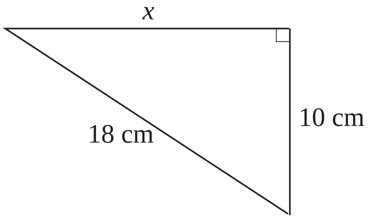
(g)



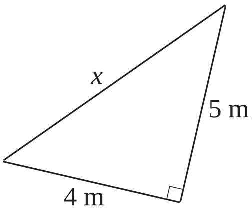
(h)



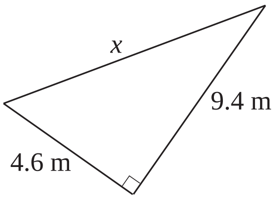
(i)



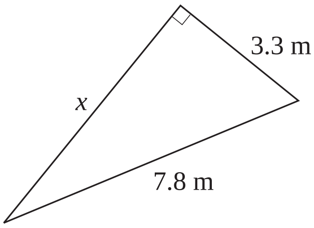
(j)



(k)

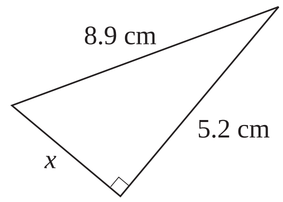


(l)

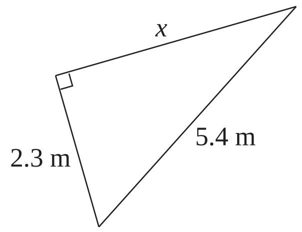


5.6

(m)



(n)

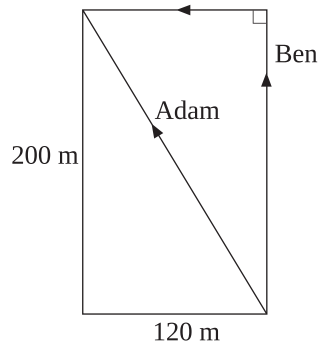


3. Adam runs diagonally across a school field, while Ben runs around the edge.

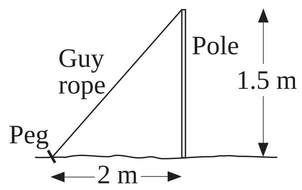
   (a) How far does Ben run?

   (b) How far does Adam run?

   (c) How much further does Ben run than Adam?



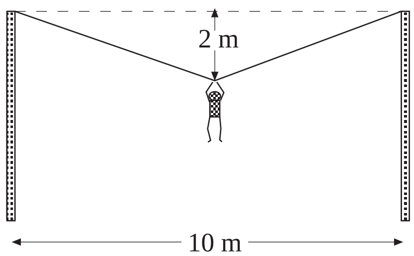
4. A guy rope is attached to the top of a tent pole, at a height of 1.5 metres above the ground, and to a tent peg 2 metres from the base of the pole. How long is the guy rope?



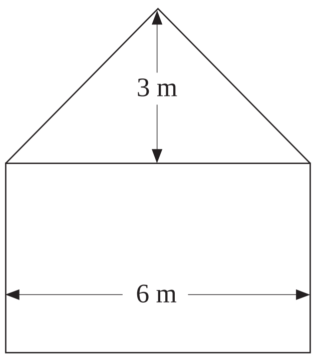
5. Farida is 1.4 metres tall. At a certain time her shadow is 2 metres long. What is the distance from the top of her head to the top of her shadow?

6. A rope of length 10 metres is stretched from the top of a pole 3 metres high until it reaches ground level. How far is the end of the line from the base of the pole?

7. A rope is fixed between two trees that are 10 metres apart. When a child hangs on to the centre of the rope, it sags so that the centre is 2 metres below the level of the ends. Find the length of the rope.



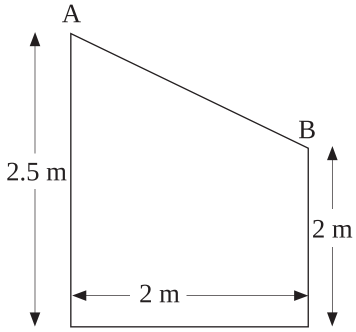
8. The roof on a house that is 6 metres wide peaks at a height of 3 metres above the top of the walls. Find the length of the sloping side of the roof.



5.6

9. The picture shows a garden shed.

   Find the length, AB, of the roof.



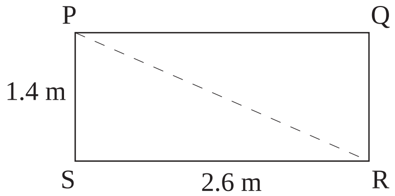
10. Miles walks 3 km east and then 10 km north.

    (a) How far is he from his starting point?

    (b) He then walks east until he is 20 km from his starting point. How much further east has he walked?

11. Ali is building a shed. It should be rectangular with sides of length 3 metres and 6 metres. He measures the diagonal of the base of the shed before he starts to put up the walls. How long should the diagonal be?

12. Pauline is building a greenhouse. The base PQRS of the greenhouse should be a rectangle measuring 2.6 metres by 1.4 metres.



    To check that the base is rectangular, Pauline has to measure the diagonal, PR.

    (a) Calculate the length of PR when the base is rectangular.

    (b) When building the greenhouse Pauline finds angle $PSR > 90°$. She measures PR. Which of the following statements is *true*?

    X: PR is greater than it should be. Y: PR is less than it should be.

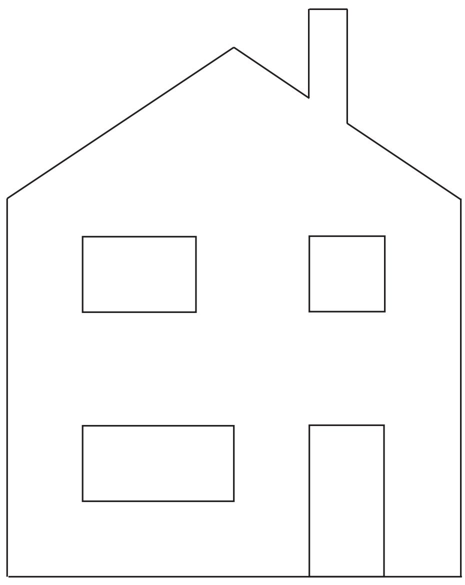
    Z: PR is the right length.

(SEG)

5.7 Scale Drawings

Scale drawings are often used to produce plans for houses or new kitchens.

For example, a scale of 1: 40 means that 1 cm on the plan /is equivalent to 40 cm in reality.



Worked Example 1

The diagram shows a scale drawing of the end wall of a house. The scale used is 1:100.

Find:

(a) the height of the top of the chimney,

(b) the height of the door,

(c) the width of the house.

5.7

Solution

As the scale is 1:100, every 1 cm on the drawing represents 100 cm or 1 m in reality.

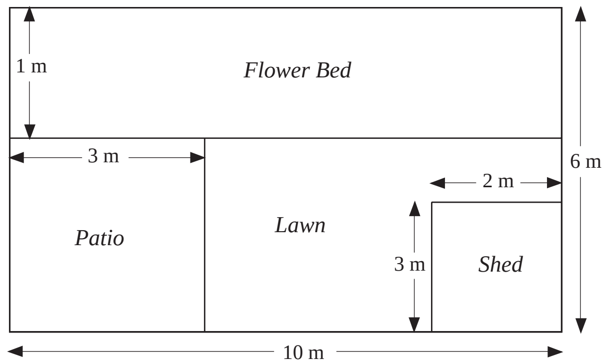
(a) The height of the top of the chimney is 7.5 cm on the drawing. This corresponds to $7.5 \times 100 = 750$ cm or 7.5 m in reality.

(b) The height of the door is 2 cm on the drawing and so is 2 m in reality.

(c) The width of the house is 6 cm on the drawing and so is 6 m in reality.

Worked Example 2

The diagram shows a rough sketch of the layout of a garden.



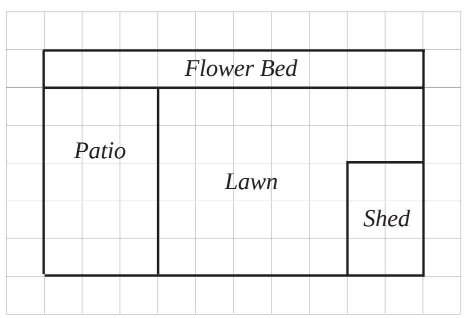
Produce a scale drawing with a scale of 1: 200.

Solution

A scale of 1: 200 means that 1 cm on the plan represents 200 cm or 2 m in reality.
The table below lists the sizes of each part of the garden and the size on the scale drawing.

Area	*Real Size*	*Size on Drawing*
Garden	10 m $\times$ 6 m	5 cm $\times$ 3 cm
Patio	5 m $\times$ 3 m	2.5 cm $\times$ 1.5 cm
Lawn	7 m $\times$ 5 m	3.5 cm $\times$ 2.5 cm
Flower Bed	10 m $\times$ 1 m	5 cm $\times$ 0.5 cm
Shed	3 m $\times$ 2 m	1.5 cm $\times$ 1 cm

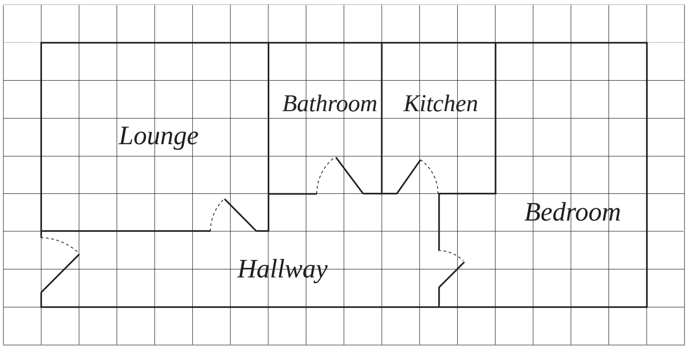
A scale drawing can now be produced and is shown opposite drawn on squared paper.



5.7

Exercises

1. The scale drawing below of a flat has been drawn on a scale of 1: 200.

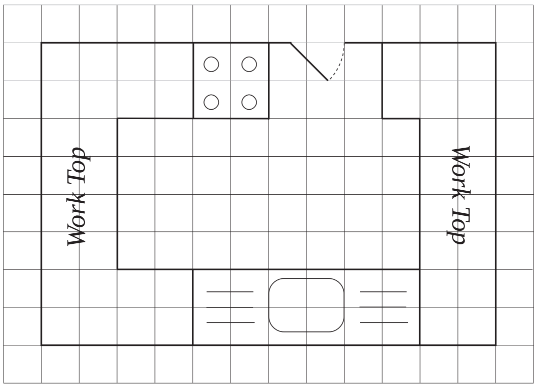


(a) Find the actual sizes of the lounge and bathroom.

(b) Find the area of the bedroom floor.

(c) Find the length of the hallway.

2.



The scale drawing shows a plan of a kitchen on a scale of 1: 60.

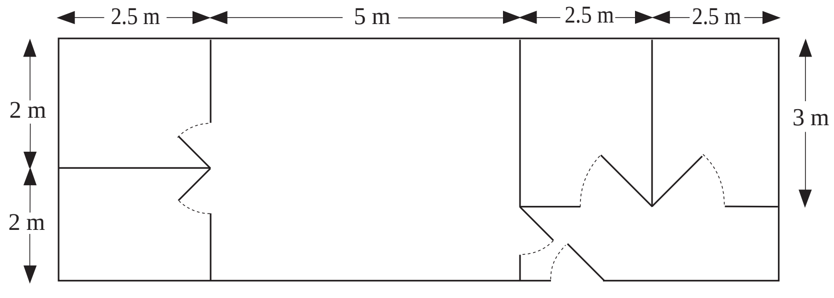
(a) What are the length and width of the kitchen?

(b) What is the size of the cooker?

(c) What is the size of the sink?

(d) Find the area of the worktops in the kitchen.

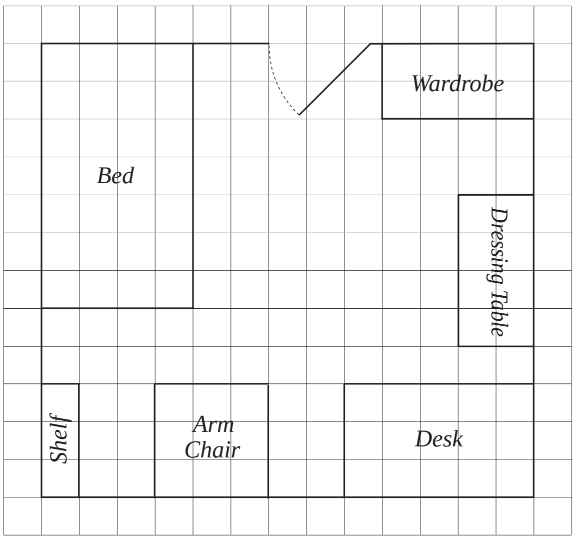
3. A rough sketch is made of a set of offices. It is shown below.



Use the information given to produce a scale drawing with a scale of 1: 200.

5.7

4. Hannah produces a scale drawing of her ideal bedroom on a scale of 1: 50. The plan is shown below.



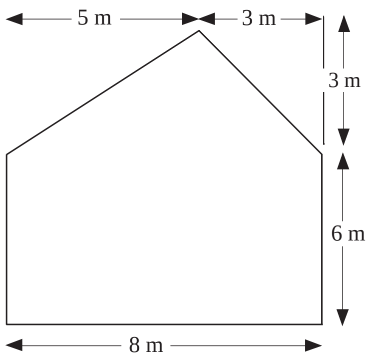
(a) What is the size of the room?

(b) How long is her bed?

(c) What is the area of the top of her desk?

(d) What is the floor area of the room?

5. The diagram shows a rough sketch of the end wall of a house.

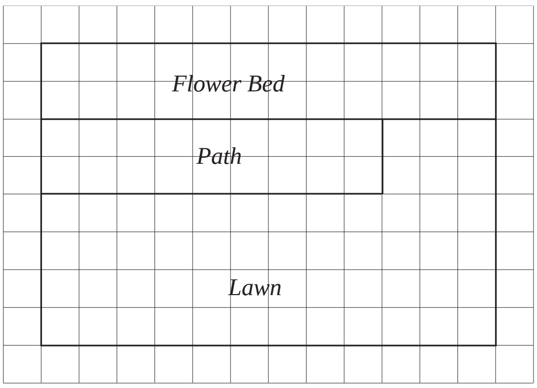


(a) Produce a scale drawing using a scale of 1:100.

(b) Use the drawing to find the sloping lengths of both sides of the roof.

6. The diagram shows a scale drawing of a garden.

It is drawn with a scale of 1: 80.



(a) How wide is the garden?

(b) How long is the path?

(c) A shed with a base of size 2 m by 1.5 m is to be added to the plan.

Find the size of the rectangle that should be drawn on the plan.

(d) The garden also contains a pond of radius 0.6 m. What would be the radius of the circle which should be added to the plan?

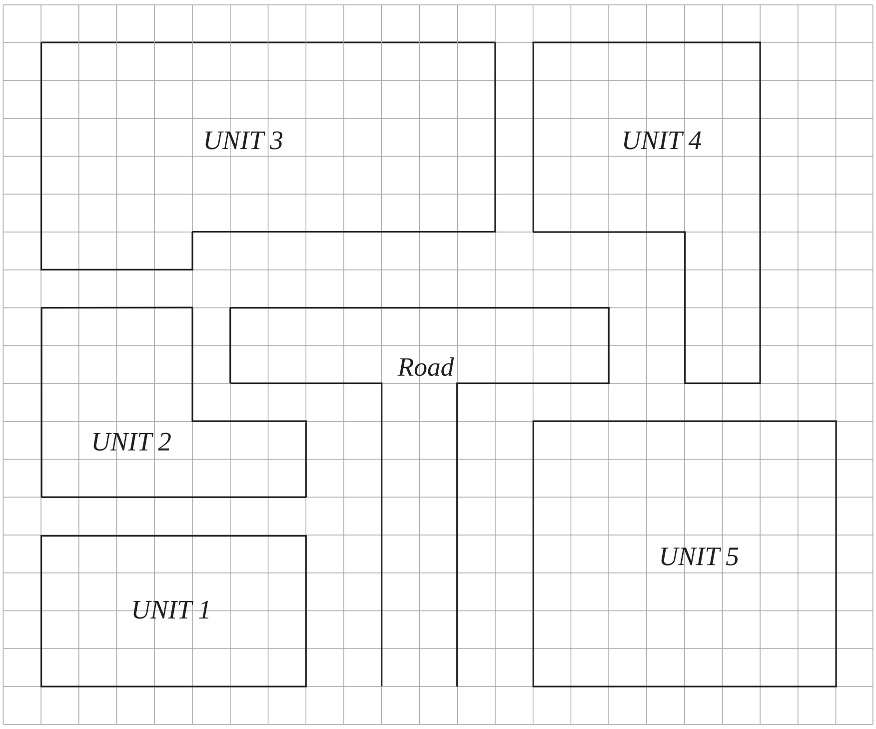
5.7

7. A classroom is rectangular with width 4 m and length 5 m.

What would be the size of the rectangle used to represent the classroom on plans with a scale of:

(a) 1: 50 (b) 1: 25 (c) 1: 100?

8. The diagram shows a scale plan of a small industrial estate drawn on a scale of 1: 500.



(a) Find the floor area of each unit.

(b) Re-draw the plan with a scale of 1:1000.

9. The plan of a house has been drawn using a scale of 1: 20.

(a) (i) On the plan, the length of the lounge is 25 cm. What is the actual length of the lounge in metres?

(ii) The actual lounge is 3.2 m wide. How wide is the lounge on the plan?

(b) The actual kitchen is 2.6 m wide. Estimate, in feet, the width of the actual kitchen.

(SEG)

10. A classroom is drawn on a plan using a scale of 1: 50.

(a) On the plan, how many centimetres represent one metre?

(b) The width of the classroom is 6.7 m. How many centimetres represent this width on the plan?

(SEG)

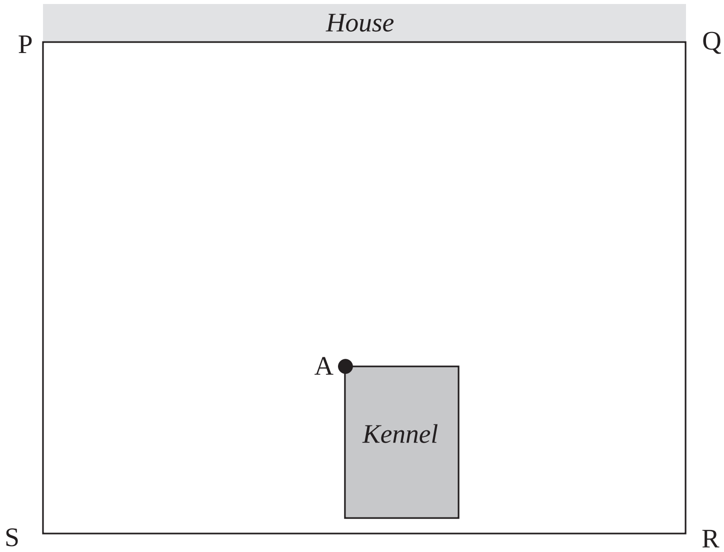
5.7

11. Byron and Shelley are two dogs.

(a) Byron's lead is 1 m long. One end of the lead can slide along a railing, which is fixed to the wall of the house.

Shelley's lead is 1.5 m long. One end of this lead is attached to a post, A, at the corner of his kennel.

The scale diagram below represents the fenced garden, PQRS, where the dogs live.

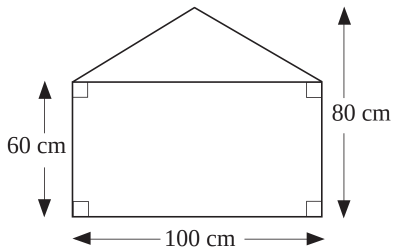


Scale

2 cm represents 1 m

Copy the diagram and show on your drawing all the possible positions of each dog if the leads remain tight.

(b)



Not to scale

The diagram represents the cross-section of Shelley's kennel. Calculate the area of this cross-section, giving your answer in cm^2.

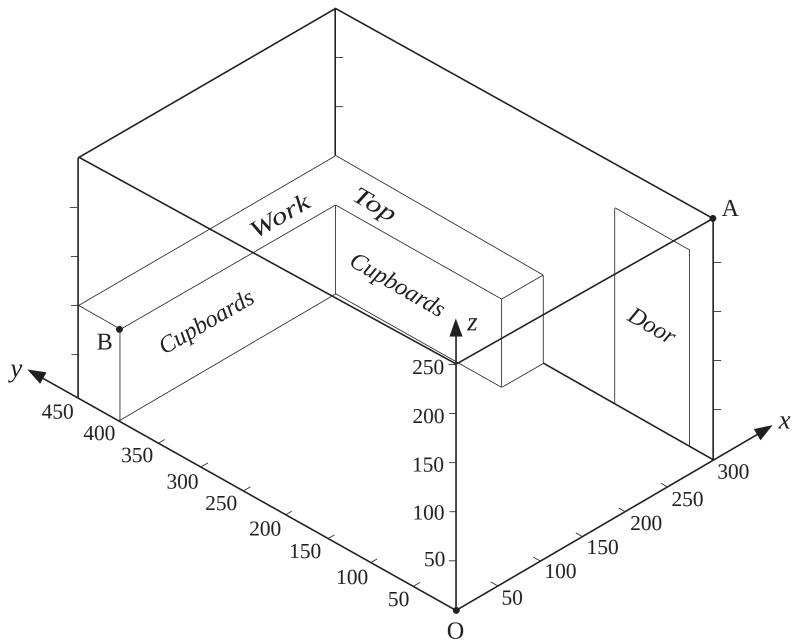
(MEG)

Investigation

Look at an atlas and find out the scales used in maps of the UK, Europe and the world. Are the same scales used for all the different maps? If not, why not?

5.7

12. The diagram is an isometric drawing of a kitchen with cupboards at floor level.
The kitchen is a cuboid 300 cm wide, 450 cm long and 250 cm high.
The work top above the cupboards is 100 cm above the floor and 50 cm wide.



(a) Another cupboard is to be fixed above the work top on the shorter wall. It is a cuboid 300 cm long, 50 cm high and 25 cm from back to front. Its top is to be 50 cm below the ceiling.

Draw this cupboard on a copy of the diagram.

(b) Axes, O*x*, O*y*, O*z* are taken as shown in the diagram. Write down the coordinates of

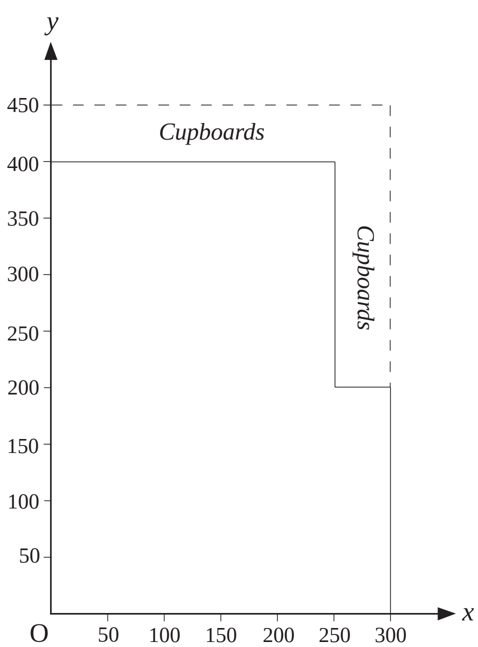
(i) the point A (ii) the point B.

(c)

This is a scale drawing of the floor of the kitchen.

The part of the floor within 50 cm of the cupboards must be kept clear.

On a copy of the diagram, show clearly and accurately this part of the floor.



(MEG)

5.8 Constructing Triangles and Other Shapes

A protractor and a compass can be used to produce accurate drawings of triangles and other shapes.

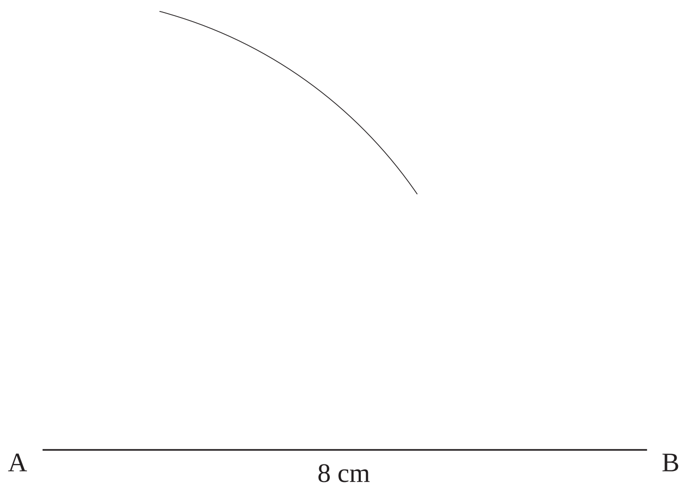
Worked Example 1

Draw a triangle with sides of length 8 cm, 6 cm and 6 cm.

Solution

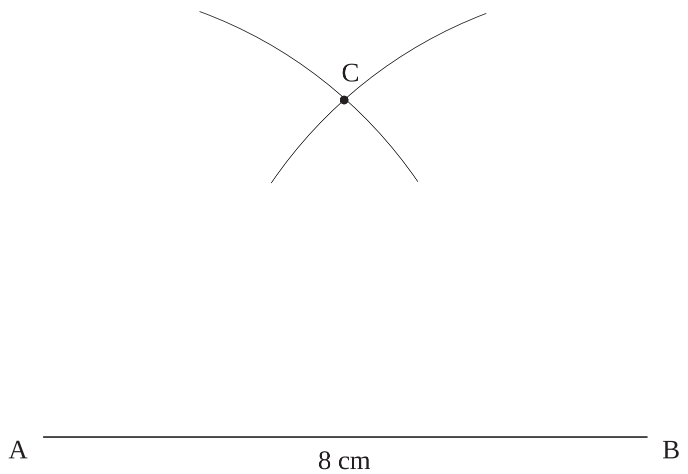
First draw a line of length 8 cm.

Then set the distance between the point and pencil of your compass to 6 cm and draw an arc with centre A as shown below.



The arc is a distance of 6 cm from A.

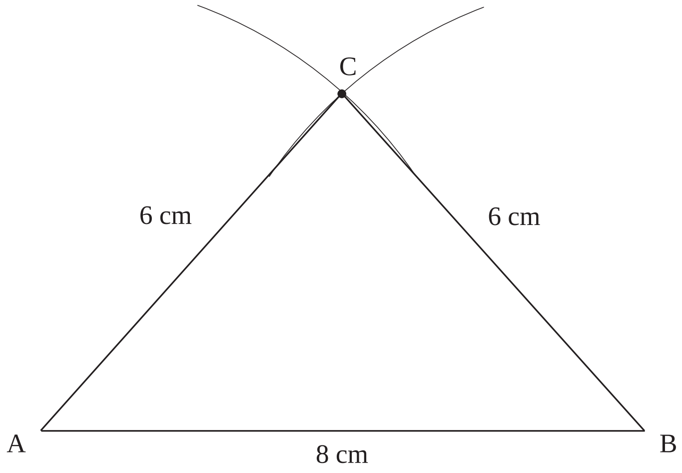
With your compass set so that the distance between the point and the pencil is still 6 cm, draw an arc centred at B, as shown below.



The point, C, where the two arcs intersect is the third corner of the triangle.

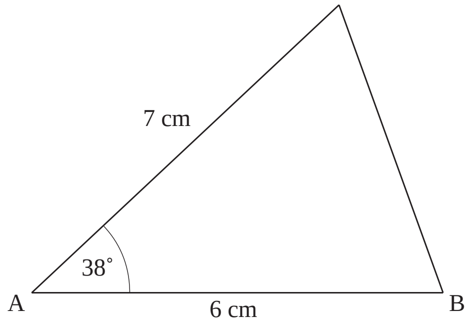
5.8

The triangle can now be completed.



Worked Example 2

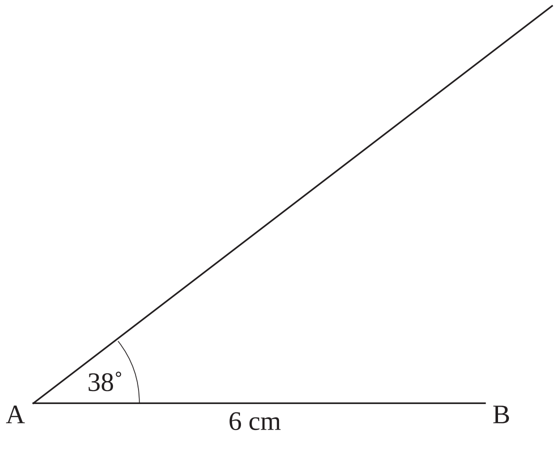
The diagram shows a rough sketch of a triangle.



Make an accurate drawing of the triangle and find the length of the third side.

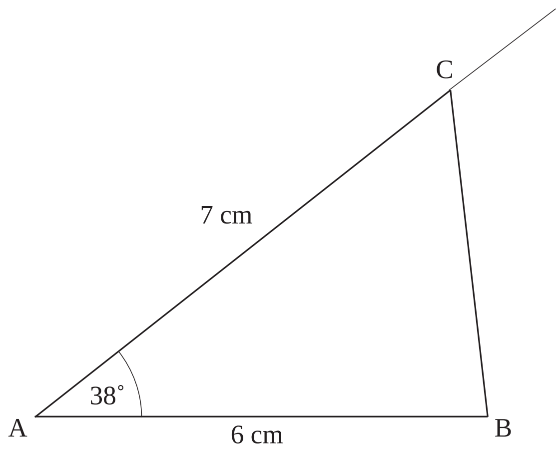
Solution

First draw a line of length 6 cm and measure an angle of $38°$.



Then measure 7 cm along the line and the triangle can be completed.

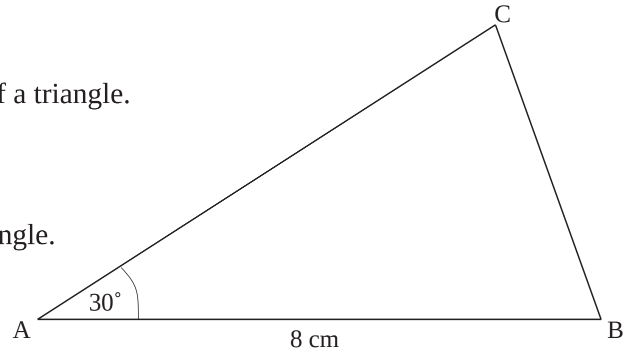
5.8



The third side of the triangle can then be measured as 4.3 cm.

Worked Example 3

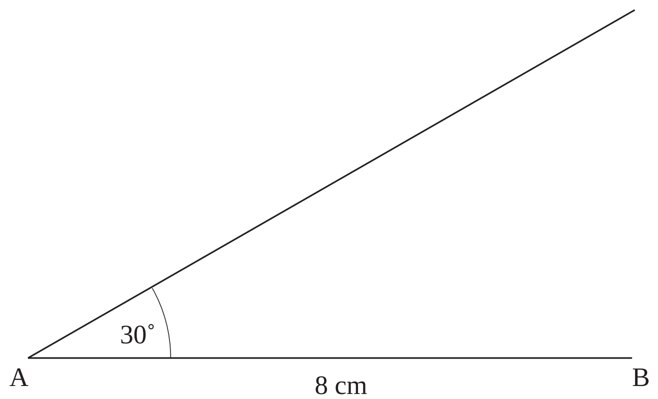
The diagram shows a rough sketch of a triangle.



Make an accurate drawing of the triangle.

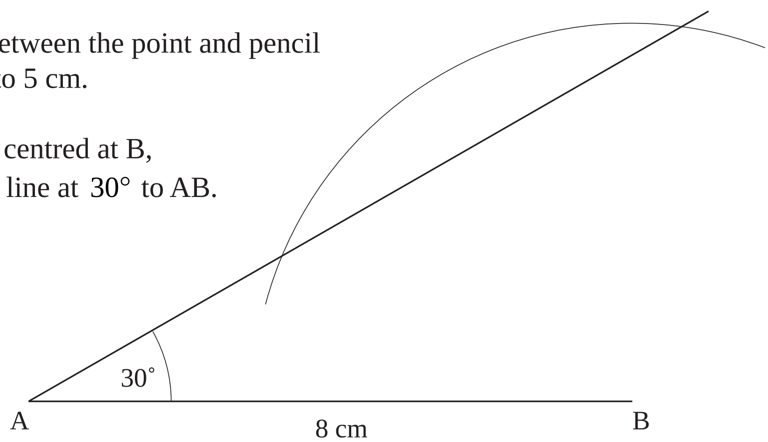
Solution

First draw the side of length 8 cm and measure the angle of $30°$, as shown below.



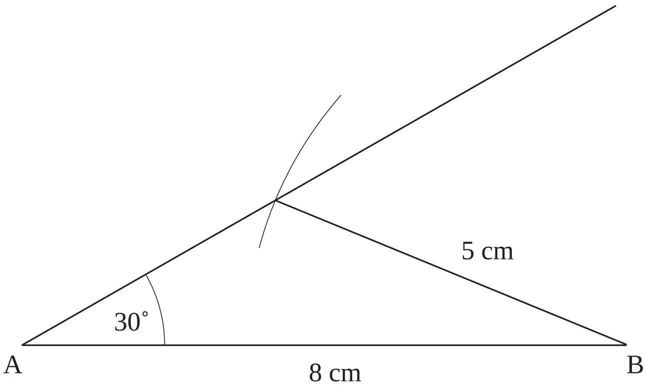
Set the distance between the point and pencil of your compass to 5 cm.

Then draw an arc centred at B, which crosses the line at $30°$ to AB.

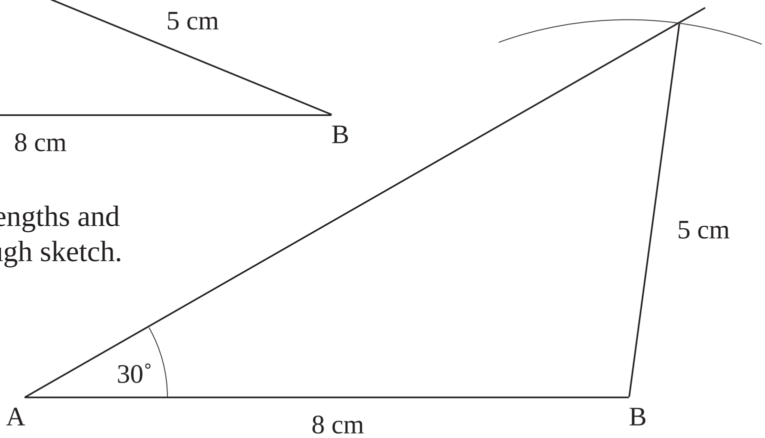


5.8

As the arc crosses the line in two places, there are two possible triangles that can be constructed as shown below.



Both triangles have the lengths and angle specified in the rough sketch.



Note

An arc must be taken when constructing triangles to ensure that all possibilities are considered.

Exercises

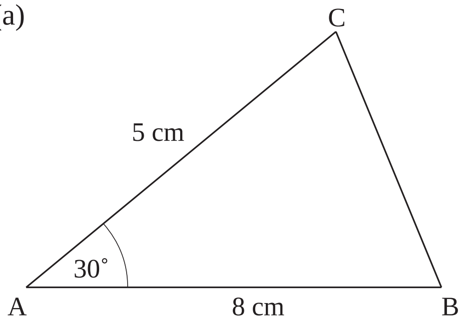
1. Draw triangles with sides of the following lengths.

   (a) 10 cm, 6 cm, 7 cm (b) 5 cm, 3 cm, 6 cm

   (c) 4 cm, 7 cm, 6 cm

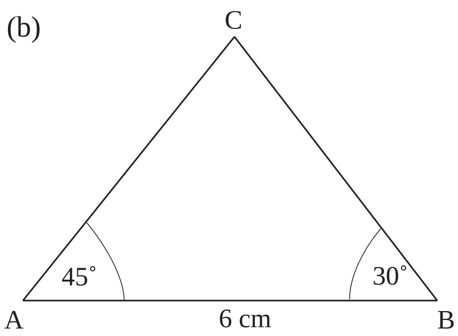
2. Draw accurately the triangles shown in the rough sketches below and answer the question given below each sketch.

   (a)



How long is the side BC?

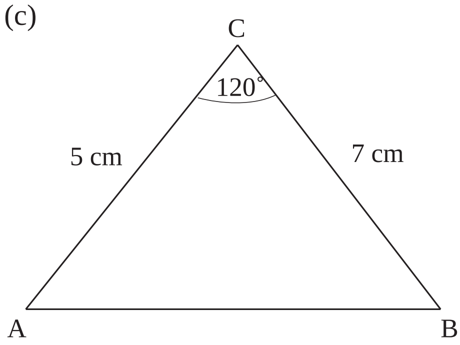
   (b)



How long are the sides AC and BC?

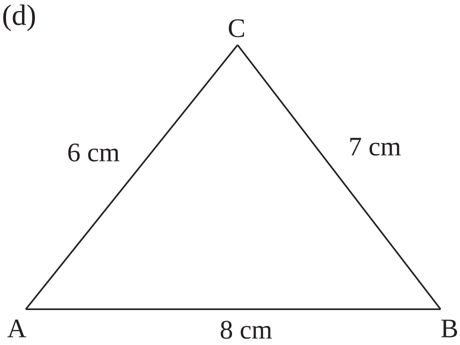
5.8

(c)



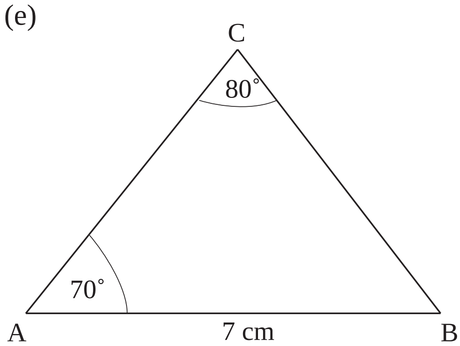
How long is the side AB?

(d)



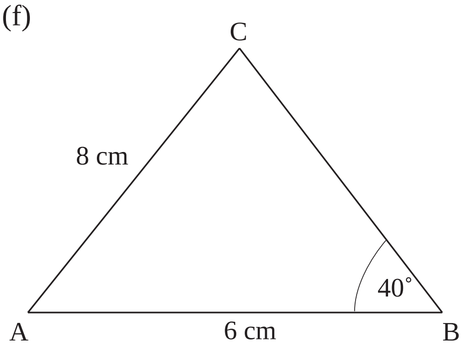
What is the size of the angle ABC?

(e)



How long is the side AC?

(f)



How long is the side BC?

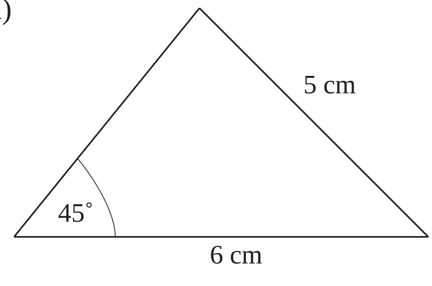
3. An isosceles triangle has a base of length 6 cm and base angles of $50°$. Find the lengths of the other sides of the triangle.

4. An isosceles triangle has 2 sides of length 8 cm and one side of length 4 cm. Find the sizes of all the angles in the triangle.

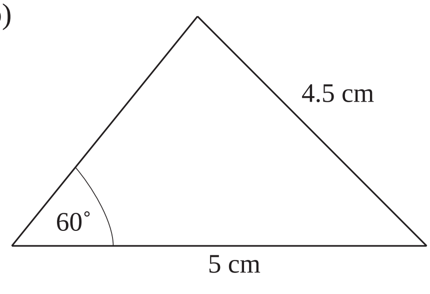
5. Draw an equilateral triangle with sides of length 5 cm.

6. For each rough sketch shown below, draw two possible triangles.

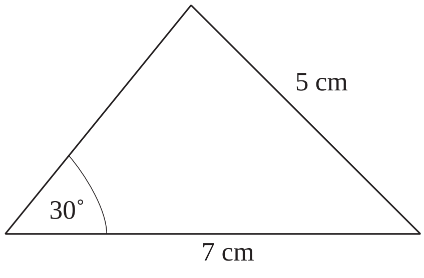
(a)



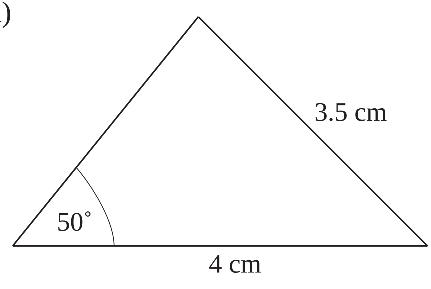
(b)



(c)

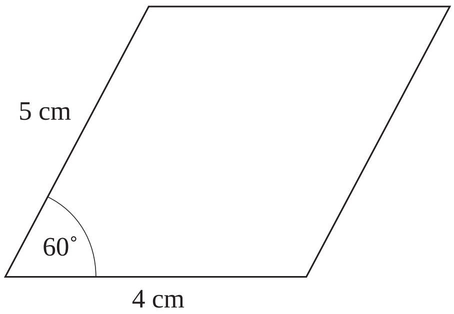


(d)



5.8

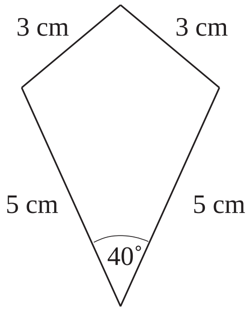
7. (a) Draw accurately the parallelogram shown below.



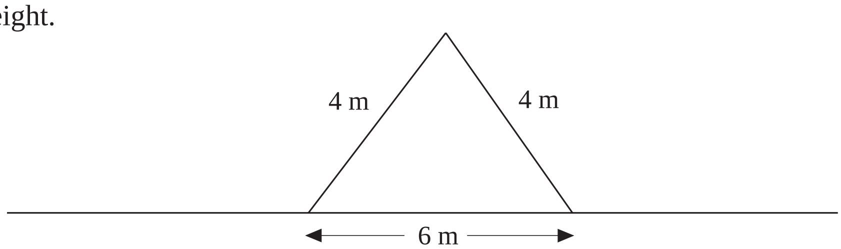
(b) Measure the two diagonals of the parallelogram.

8. (a) Draw the kite shown in the rough sketch opposite.

(b) Check that the diagonals of the kite are at right angles.

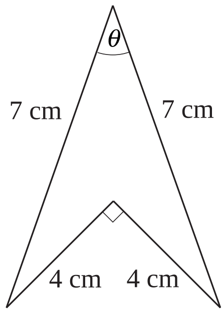


9. A pile of sand has the shape shown below. Using an accurate diagram, find its height.



10. Draw accurately the shape shown opposite.

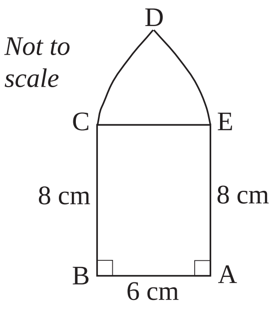
Find the size of the angle marked θ.



11. The sketch shows the design for a church window.

CB and EA are perpendicular to BA.
CD is part of a circle, centre E.
DE is part of a circle, centre C.

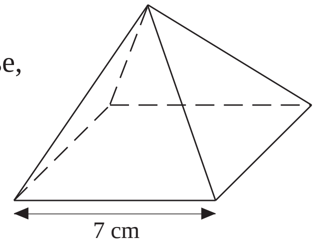
Using a ruler and compasses draw the design accurately.

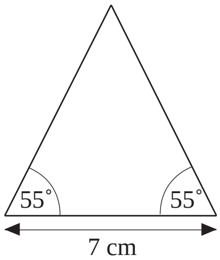


(SEG)

5.8

12. John is required to construct a pyramid with a square base, as shown opposite.



(a) 

Each sloping face is a triangle with base angles of $55°$.

Construct one of these triangles accurately and to full size.

(b) Construct the square base of the pyramid accurately and to full size.

(MEG)

13. A rectangle has sides of 8 cm and 5 cm.

(a) Calculate the perimeter of the rectangle.

(b) Construct the rectangle accurately.

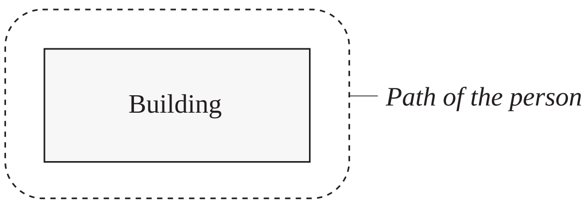
(MEG)

14. Construct a rhombus ABCD with the line AB as base and with $B\hat{A}D = 50°$.

(MEG)

5.9 Construction of Loci

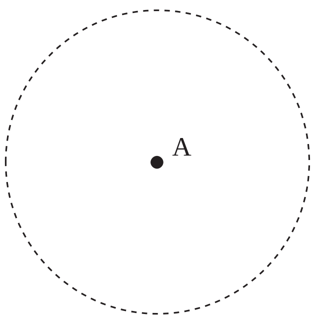
When a person moves so that they always satisfy a certain condition, their possible path is called a *locus*. For example, consider the path of a person who walks around a building, always keeping the same distance away from the building.



The dotted line in the diagram shows the path taken – i.e. the *locus*.

Worked Example 1

Draw the locus of a point which is always a constant distance from another point.



Solution

The fixed point is marked A.
The locus is a circle around the fixed point.

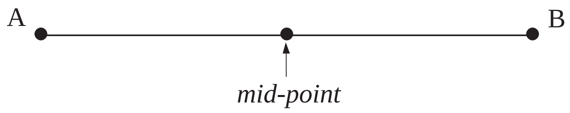
5.9

Worked Example 2

Draw the locus of a point which is always the same distance from A as it is from B.

Solution

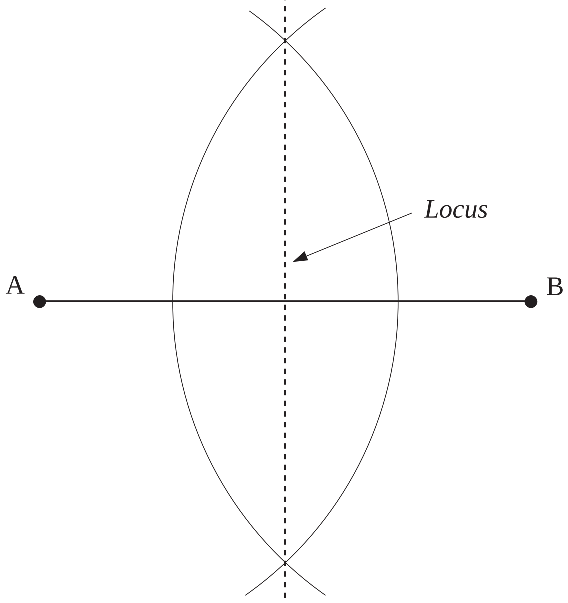
The locus of the line AB will be the same distance from both points.



However, any point on a line perpendicular to AB and passing through the mid-point of AB will also be the same distance from A and B.

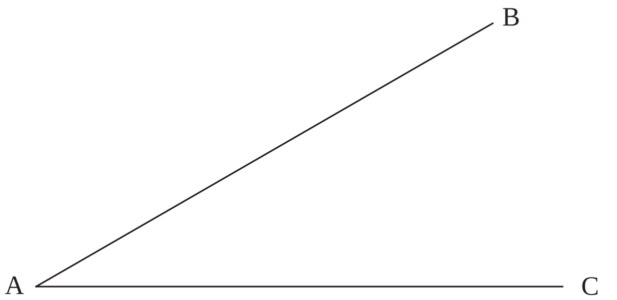
The diagram shows how to construct this line.

This line is called the *perpendicular bisector* of AB.



Worked Example 2

Draw the locus of a point that is the same distance from the lines AB and AC shown in the diagram below.

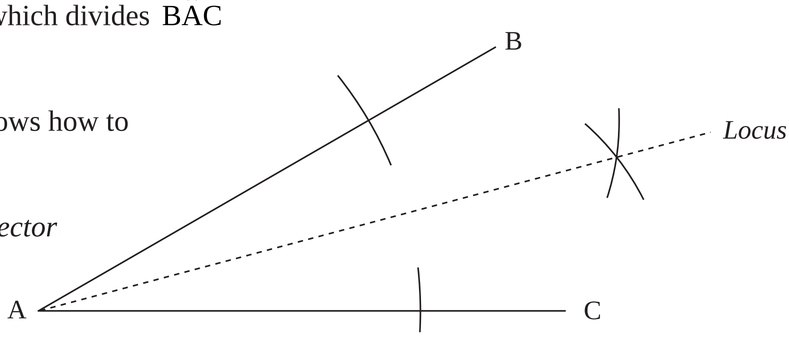


Solution

The locus will be a line which divides $B\hat{A}C$ into two equal angles.

The diagram opposite shows how to construct this locus.

This line is called the *bisector* of $B\hat{A}C$.



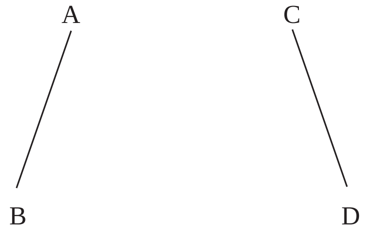
5.9

Exercises

1. Draw the locus of a point which is always a distance of 4 cm from a fixed point A.

2. The line AB is 4 cm long. Draw the locus of a point which is always 2 cm from the line AB.

3. Copy the diagram and draw in the locus of a point which is the same distance from the line AB as it is from CD.

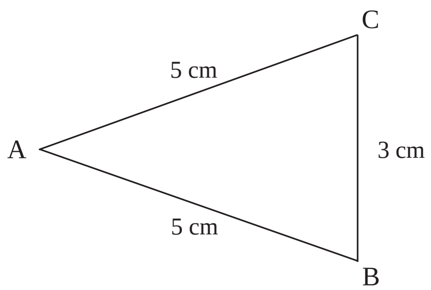


4. (a) Draw an equilateral triangle with sides of length 5 cm.

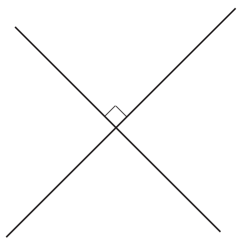
   (b) Draw the locus of a point that is always 1 cm from the sides of the triangle.

5. Copy the triangle opposite.

   Draw the locus of a point which is the same distance from AB as it is from AC.



6. Draw the locus of a point that is the same distance from both lines shown in the diagram below.



7. (a) Draw 2 parallel lines.

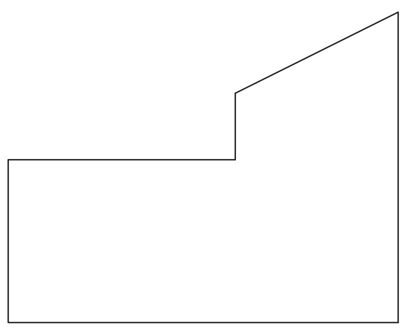
   (b) Draw the locus of a point which is the same distance from both lines.

8. The diagram below shows the boundary fence of a high security army base.

   (a) Make a copy of this diagram.

   (b) A security patrol walks round the *outside* of the base, keeping a constant distance from the fence. Draw the locus of the patrol.

   (c) A second patrol walks *inside* the fence, keeping a constant distance from the fence. Draw the locus of this patrol.



5.9

9. The points A and B are 3 cm apart. Draw the locus of a point that is twice as far from A as it is from B.

10. A ladder leans against a wall, so that it is almost vertical. It slides until it is flat on the ground. Draw the locus of the mid-point of the ladder.

11. The points A and B are 4 cm apart.

    (a) Draw the possible positions of the point, P, if

    (i) $AP = 4$ cm and $BP = 1$ cm

    (ii) $AP = 3$ cm and $BP = 2$ cm

    (iii) $AP = 2$ cm and $BP = 3$ cm

    (iv) $AP = 1$ cm and $BP = 4$ cm

    (b) Draw the locus of the point P, if $AP + BP = 5$ cm.

    (c) Draw the locus of the point P, if $AP + BP = 6$ cm.

Answers to Exercises

5.1 Measuring Angles

Note that measured angles are approximate answers

1. (a) 78° (b) 120° (c) 60° (d) 130° (e) 125° (f) 60°

3. (a) 315° (b) 195° (c) 240° (d) 325° (e) 264°
   (f) 350°

5. (a) $a = 62^\circ$, $b = 118^\circ$ (b) $a = 58^\circ$, $b = 76^\circ$, $c = 46^\circ$
   (c) $a = 104^\circ$, $b = 76^\circ$ (d) $a = 42^\circ$, $b = 74^\circ$, $c = 64^\circ$
   The angles add up to 180°

6. (a) 50°, 60°, 70° (b) 31°, 59°, 90° (c) 15°, 19°, 147°
   (d) 33°, 40°, 107° The three angles add up to 180°

7. (a) $a = 150^\circ$, $b = 90^\circ$, $c = 120^\circ$ (b) $a = 152^\circ$, $b = 116^\circ$, $c = 63^\circ$, $d = 29^\circ$
   (c) $a = 48^\circ$, $b = 154^\circ$, $c = 35^\circ$, $d = 123^\circ$ (d) $a = 45^\circ$, $b = 45^\circ$, $c = 270^\circ$
   In each case the angles add up to 360°

8. (c) 7.7 cm and 6.4 cm, 90° 9. (b) 11.5 cm, 34°, 66°

10. (a) 34°, 34°, 51°, 241° (b) 25°, 29°, 98°, 208°
    In both cases the angles add up to 360°

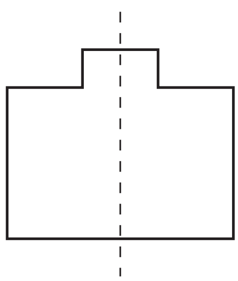
11. The interior angles will always add up to 540°

5.2 Line and Rotational Symmetry

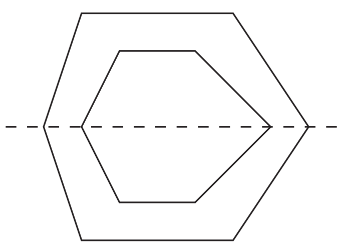
1. (a) B - 2 lines, D - 2 lines, E - 1 line, F - 1 line, G - 4 lines, I - 1 line
   (b) A - order 4, B - order 2, D - order 2, G - order 4, H - order 3

2. A - has symmetry, no lines, order 3
   B - has symmetry, 1 line, no order
   C - has symmetry, 1 line, no order
   D - has symmetry, 1 line, no order
   E - has symmetry, 1 line, no order
   F - has symmetry, 4 lines, order 8
   G - has symmetry, 1 line, no order
   H - has symmetry, no lines, order 4
   I - no symmetry, no lines, no order

3. (a)



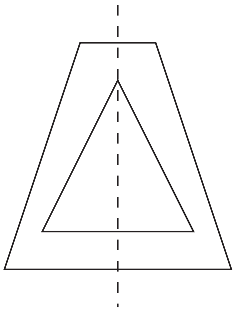
(b)



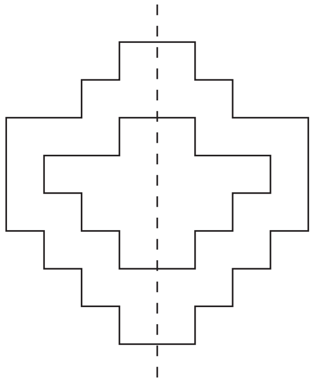
Answers

5.2

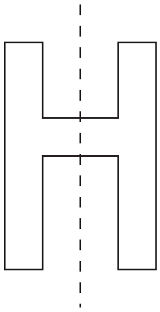
(c)



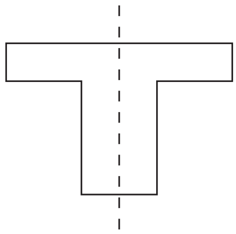
(d)



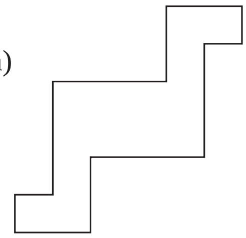
(e)



(f)

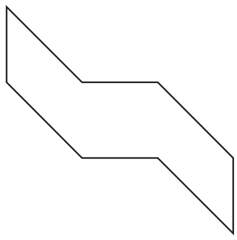


4. (a)



order 2

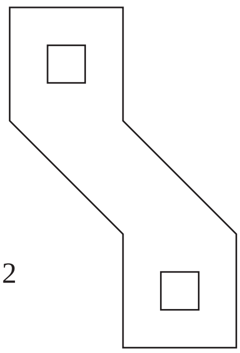
(b)



order 2

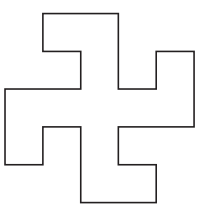
(c) Not possible

(d)



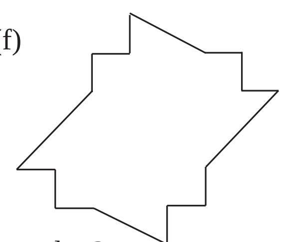
order 2

(e)



order 4

(f)

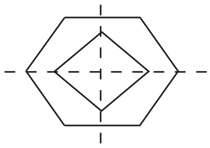


order 2

Answers

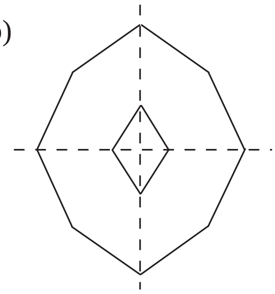
5.2

5. (a)



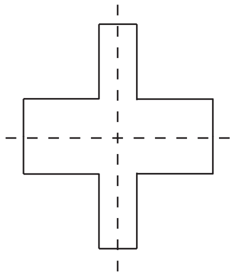
Order 2

(b)



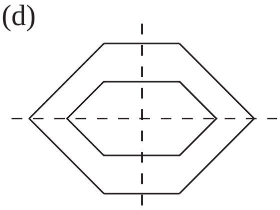
Order 2

(c)



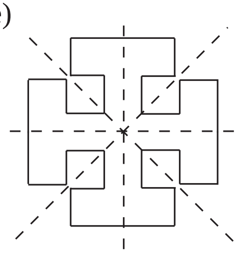
Order 2

(d)



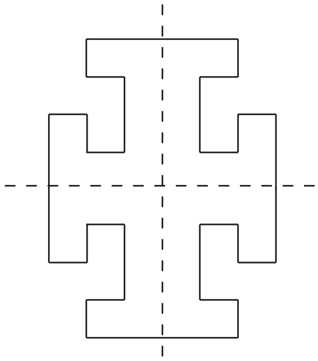
Order 2

(e)



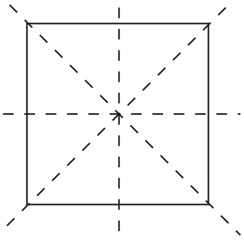
Order 4

(f)

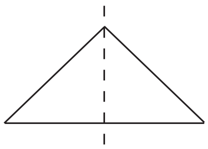


Order 2

6.

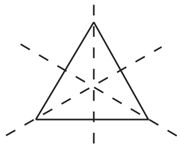


7 (a) (i)



(ie. any isosceles triangle)

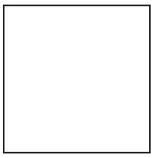
(ii)



(ie. any equilateral triangle)

(b) No.

8.

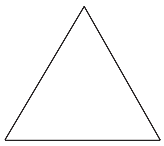


(eg. a square)

9. (a)

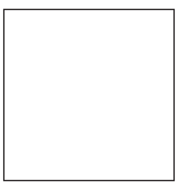
Rotational symmetry order 2

(b)



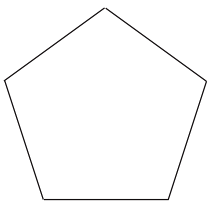
Rotational symmetry order 3

(c)



Rotational symmetry order 4

(d)

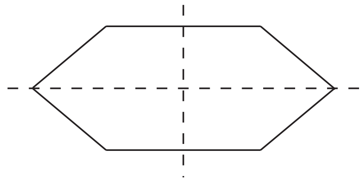


Rotational symmetry order 5

Answers

5.2

10. (a) No (b)

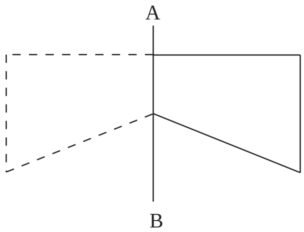


(c) No

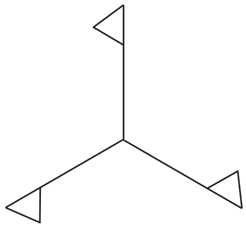
11. Letter I has rotational symmetry.

12. Designs (a), (b) and (d) have line symmetry.

13. (a)



(b)



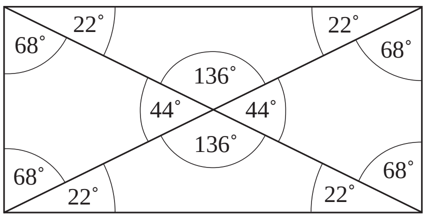
(c) Rotational symmetry of order 2.

5.3 Angle Geometry

1. (a) $a = 50°$ (b) $x = 130°$ (c) $b = 92°$ (d) $a = 80°$
   (e) $a = 111°$ (f) $x = 82°$ (g) $x = 110°$ (h) $a = 45°$
   (i) $x = 55°$ (j) $a = b = 70°$ (k) $a = b = c = 60°$
   (l) $a = 50°$, $b = 80°$ (m) $a = 109°$ (n) $x = 114°$ (o) $x = 87°$

2. (a) (i) $a = 70°$, $b = 110°$ (ii) $a = 53°$, $b = 127°$ (iii) $a = 48°$, $b = 132°$
   (b) b is equal to the sum of the two opposite angles in the triangle.
   (c) (i) $b = 105°$ (ii) $b = 106°$ (iii) $b = 135°$

3.



4. (a) $a = 75°$, $b = 75°$, $c = 30°$, $d = 75°$
   (b) $a = 60°$, $b = 60°$, $c = 30°$, $d = 60°$, $e = 60°$, $f = 60°$, $g = 30°$
   (c) $a = 80°$, $b = 45°$, $c = 45°$, $d = 55°$, $e = 80°$
   (d) $a = 30°$, $b = 20°$, $c = 10°$, $d = 80°$, $e = 80°$, $f = 60°$

5. (a) $a = 65°$, $b = 80°$ (b) $a = 40°$,
   (c) $a = 60°$, $b = 60°$, $c = 60°$, $d = 120°$, $e = 30°$
   (d) $a = 65°$, $b = 65°$, $c = 58°$, $d = 90°$, $e = 35°$
   (e) $a = 90°$, $b = 97°$, $c = 41.5°$, $d = 41.5°$, $e = 69°$, $f = 69°$, $g = 104°$, $h = 38°$
   (f) $a = 60°$, $b = 60°$, $c = 60°$, $d = 80°$, $e = 100°$, $f = 40°$, $g = 40°$, $h = 120°$, $i = 38°$

Answers

5.3

6. $a = 44°,\ b = 68°,\ c = 68°,\ d = 112°,\ e = 112°,\ f = 68°$

7. $a = 50°,\ b = 40°,\ c = 70°,\ d = 20°,\ e = 65°,\ f = 50°$

8. $a = 25°,\ b = 110°,\ c = 45°,\ d = 65°,\ e = 70°,\ f = 25°,\ g = 25°$

9. (a) $9x = 180°,\ x = 20°$ (b) $3x - 30 = 180°,\ x = 70°$
   (c) $3x + 30 = 180°,\ x = 50°$ (d) $5x = 360°,\ x = 72°$
   (e) $4x + 20 = 180°,\ x = 40°$ (f) $4x = 360°,\ x = 90°$
   (g) $17x + 20 = 360°,\ x = 20°$ (h) $2x = 30°,\ x = 15°$
   (i) $5x + 90 = 360°,\ x = 54°$ (j) $10x + 80 = 180°,\ x = 10°$
   (k) $6x = 150°,\ x = 25°$ (l) $13x + 22 = 360°,\ x = 26°$

10. (a) order = 6 (b) (i) AOB = 60° (ii) Equilateral triangle

11. BCD = 134° ABC = 77°

5.4 Angles with Parallel and Intersecting Lines

1. (a) $a = 38°$, Opposite angles
   (b) $a = 57°$, Opposite angles, $b = 123°$, Straight line
   (c) $a = 60°$, Straight line, $b = 120°$, Opposite angles, $c = 60°$, Opposite angles
   (d) $a = 100°$, Straight line, $b = 100°$, Opposite angles
   (e) $a = 145°$, Straight line, $b = 35°$, Opposite angles, $c = 145°$, Opposite angles
   (f) $a = 50°$, Corresponding angles
   (g) $a = 40°$, Corresponding angles, $b = 140°$, Straight line
   (h) $a = 60°$, Straight line, $b = 60°$, Corresponding angles, $c = 120°$, Straight line
   (i) $a = 42°$, Opposite angles, $b = 138°$, Supplementary angles,
   $c = 42°$, Corresponding angles
   (j) $a = 100°$, Straight line, $b = 80°$, Opposite angles, $c = 100°$, Opposite angles
   $d = 80°$, Corresponding angles
   (k) $a = 25°$, Opposite angles, $b = 155°$, Straight line,
   $c = 25°$, Corresponding angles
   (l) $a = 124°$, Alternate angles, $b = 56°$, Straight line
   (m) $a = 37°$, Corresponding angles, $b = 143°$, Straight line,
   $c = 37°$, Opposite angles
   (n) $a = 56°$, Corresponding then Opposite angles, $b = 124°$, Straight line,
   $c = 124°$, Corresponding then Opposite angles,
   (o) $a = 160°$, Straight line, $b = 160°$, Corresponding angles,
   $c = 20°$, Alternate angles

2. (a) $a = 70°,\ b = 140°$
   (b) $a = 60°,\ b = 110°,\ c = 70°,\ d = 120°$
   (c) $a = 52°,\ b = 128°,\ c = 52°,\ d = 128°$
   (d) $a = 75°,\ b = 105°,\ c = 75°,\ d = 105°$

Answers

5.4

(e) $a = 60°,\ b = 80°,\ c = 80°$
(f) $a = 70°,\ b = 50°,\ c = 60°,\ d = 70°,\ e = 70°$
(g) $a = 74°,\ b = 100°,\ c = 41°,\ d = 115°$
(h) $a = 48°,\ b = 48°,\ c = 132°,\ d = 138°,\ e = 42°,\ f = 48°$
(i) $a = 64°,\ b = 52°,\ c = 64°$
(j) $a = 38°,\ b = 52°,\ c = 52°$

3. (a) $4x = 180°,\ x = 45°$ (b) $10x = 360°,\ x = 36°$
   (c) $8x = 180°,\ x = 22.5°$ (d) $9x = 180°, x = 20°$
   (e) $6x = 180°,\ x = 30°$ (f) $8x = 180°,\ x = 22.5°$

4. AB is parallel to EF , GH is parallel to IJ

5. $a = 80°,\ b = 50°,\ c = 80°,\ d = 50°$

6. (a) AC and BD are parallel (b) BAC = 50° because AEC is isosceles

7. (a) Square, Rectangle, Rhombus and Parallelogram
   (b) Rectangle, Parallelogram, Kite, Rhombus and Square

8. (a) 36°; alternate angles (b) 54°; angle POQ is 90°

9. (a) $p = 48°$ (b) $q = 84°$ (c) Alternate angles

5.5 Squares and Triangles

1. (a) Isosceles (b) Scalene (c) Equilateral (d) Isosceles
   (e) Scalene (f) Equilateral (g) Isosceles (h) Isosceles

2. (a) Isosceles (b) Scalene

3. (a) 25 cm^2 (b) 49 cm^2 (c) 100 m^2 (d) 1 cm^2

4. (a) 4 m^2 (b) 10 000 m^2 (c) 225 cm^2 (d) 289 cm^2

5. (a) 3 cm (b) 5 m (c) 10 m (d) 8 cm (e) 1 cm (f) 20m

6. 32 cm^2 7. 72 cm^2 8. 9 cm^2

5.6 Pythagoras' Theorem

1. (a) 5 m (b) 13 m (c) 9 cm (d) 24 m (e) 10 m (f) 8 cm
   (g) 15 m (h) 39 cm

2. (a) 13.04 cm (b) 20.52 cm (c) 8.94 cm (d) 8.60 m (e) 7.14 cm
   (f) 8.94 cm (g) 7.81 m (h) 11.83 m (i) 14.97 cm (j) 6.40 m
   (k) 10.47 m (l) 7.07 m (m) 7.22 cm (n) 4.89 m

3. (a) 320 m (b) 233.2 m (c) 86.8 m

4. 2.5 m 5. 2.44 m 6. 9.54 m 7. 10.77 m 8. 4.24 m 9. 2.06 m

10. (a) 10.44 km (b) 14.32 km 11. 6.71 m

12. (a) 295 m (b) X

Answers

5.7 Scale Drawings

1. (a) 6 m by 5 m (b) 32.5 m^2 (c) 10.5 m

2. (a) 3.6 m, 2.4 m (b) 60 cm by 60 cm (c) 60 cm by 180 cm
   (d) 3.78 m^2

4. (a) 3 m by 3.25 m (b) 1.75 m (c) 0.9375 m^2 (d) 9.75 m^2

5. (b) 4.2 m, 5.8 m

6. (a) 4.8 m (b) 3.6 m (c) 2.5 cm by 1.875 cm (d) 0.75 cm

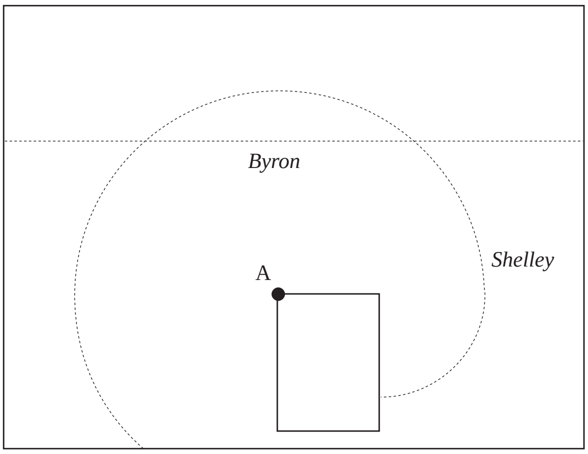
7. (a) 8 cm by 10 cm (b) 16 cm by 20 cm (c) 4 cm by 5 cm

8. (a) 1 : 175 m^2, 2 : 162.5 m^2, 3 : 400 m^2, 4 : 237.5 m^2, 5 : 350 m^2

9. (a) (i) 5 m (ii) 16 cm (b) $8\frac{1}{2}$ feet

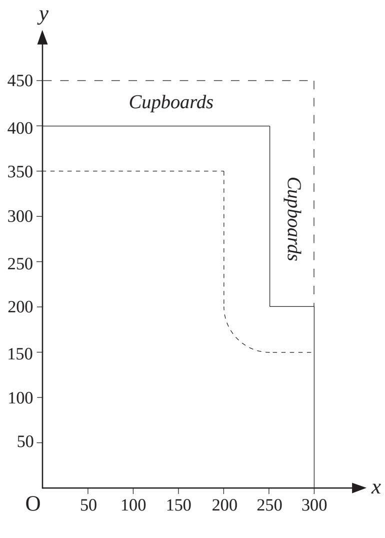
10. (a) 2 cm (b) 13.4 cm

11. (a)



(b) 7000 cm^2

12. (b) (i) (300, 0, 250) (ii) (0, 400, 100) (c)



Answers

5.8 Constructing Triangles and other Shapes

2. (a) 4.4 (b) AC ≈ 3.1 cm, BC ≈ 4.4 cm (c) AB ≈ 10.4 cm
   (d) 46.6° (e) AC ≈ 3.6 cm (f) BC ≈ 11.6 cm

3. 4.7 cm

4. 29.0°, 75.5°, 75.5°

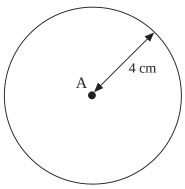
7. (b) 4.6 cm, 7.8 cm

9. 2.6 cm

10. 48°

13. (a) 26 cm

5.9 Construction of Loci

1.


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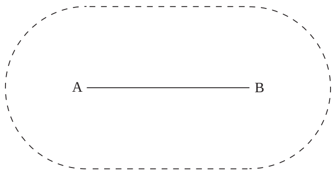

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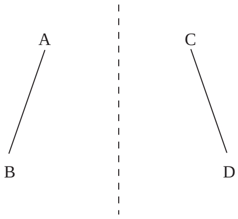

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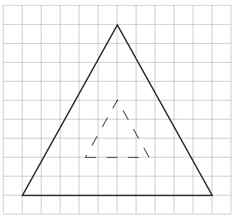

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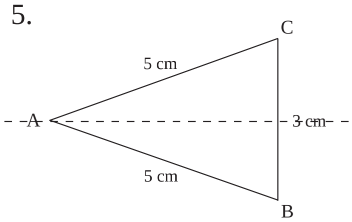

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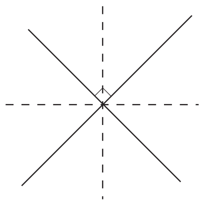

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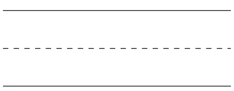

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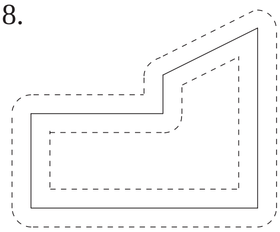

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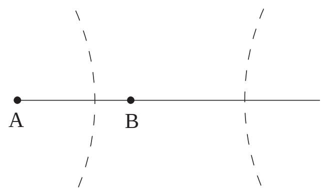

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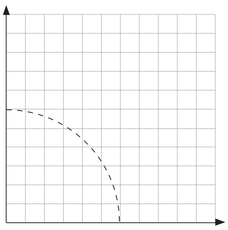
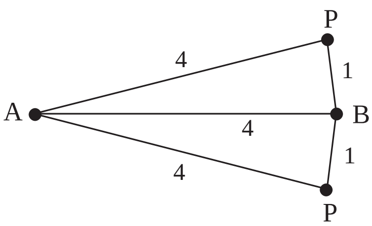


11. (a) (i)


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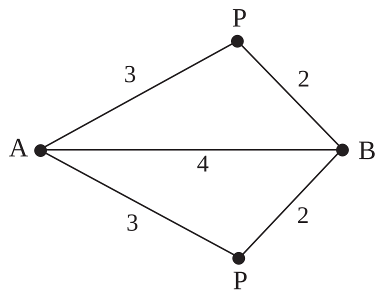
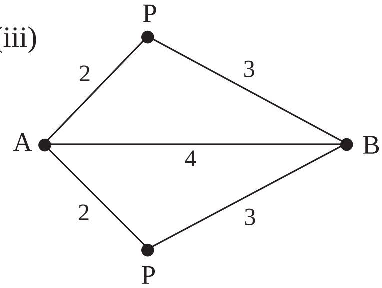


Answers

5.9

(iii)



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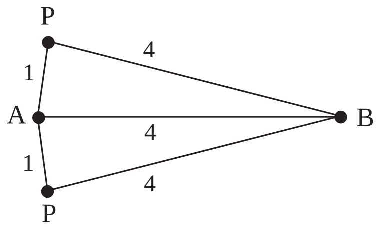
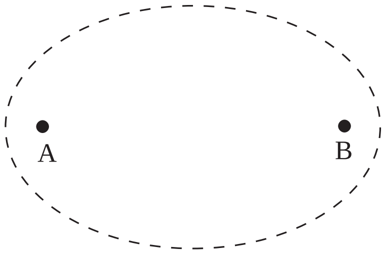


(b)



(c)