



Mathematics Enhancement Programme

# Primary Demonstration Project

## 6A Handling Data

### Help Booklet



Support for Primary Teachers  
in Mathematics

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*Mathematics Enhancement Programme*

## Help Module 6

# HANDLING DATA

## Part A

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Preface

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# PREFACE

This is one of a series of *Help Modules* designed to help you gain confidence in mathematics. It has been developed particularly for primary teachers (or student teachers) but it might also be helpful for non-specialists who teach mathematics in the lower secondary years. It is based on material which is already being used in the *Mathematics Enhancement Programme: Secondary Demonstration Project*.

The complete module list comprises:

- |              |                       |
|--------------|-----------------------|
| 1. ALGEBRA   | 6. HANDLING DATA      |
| 2. DECIMALS  | 7. MENSURATION        |
| 3. EQUATIONS | 8. NUMBERS IN CONTEXT |
| 4. FRACTIONS | 9. PERCENTAGES        |
| 5. GEOMETRY  | 10. PROBABILITY       |

Notes for overall guidance:

- Each of the 10 modules listed above is divided into 2 parts. This is simply to help in the downloading and handling of the material.
- Though referred to as 'modules' it may not be necessary to study (or print out) each one in its entirety. As with any self-study material you must be aware of your own needs and assess each section to see whether it is relevant to those needs.
- The difficulty of the material in **Part A** varies quite widely: if you have problems with a particular section do try the one following, and then the next, as the content is not necessarily arranged in order of difficulty. Learning is not a simple linear process, and later studies can often illuminate and make clear something which seemed impenetrable at an earlier attempt.
- In **Part B**, **Activities** are offered as backup, reinforcement and extension to the work covered in Part A. **Tests** are also provided, and you are strongly urged to take these (at the end of your studies) as a check on your understanding of the topic.
- The marking scheme for the revision test includes B, M and A marks.

Note that:

- |                |   |
|----------------|---|
| <b>M</b> marks | are for method;   |
| <b>A</b> marks | are for accuracy (awarded only following a correct M mark); |
| <b>B</b> marks | are independent, stand-alone marks.                         |

We hope that you find this module helpful. Comments should be sent to:

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The full range of Help Modules can be found at

[www.ex.ac.uk/cimt/help/menu.htm](http://www.ex.ac.uk/cimt/help/menu.htm)

## 6 *Handling Data*

## Introductory Notes

### Historical Background

The British statesman, *Benjamin Disraeli*, once stated, "There are lies, damned lies and statistics." This might have been a bright remark to make in a debate, but the status of statistics was given a quite underrated currency which persists, even today! Disraeli's statement is in fact the reverse of the truth, and a more apt expression would be "There are lies, damned lies and numerical statements made by people ignorant of statistics!"

Statistical analysis makes possible the testing of numerical data for relevance, reliability and validity. Statisticians must present data in such a form that others can utilise the relevant information to enable them to make judgements.

The study of Statistics is reported to have started with the Englishman, *John Graunt* (1620 – 1674), who collected and studied the death records in various cities of Britain. He was fascinated by the patterns he found in the whole population.

Much of current day statistical analysis is of quite recent development, the availability of cheap computing power acting as a catalyst for the development of appropriate ways of presenting and analysing data. In fact, the more advanced statistical analyses and tests are based on probability theory, developed over the past few centuries, but put into a more modern context by mathematical statisticians such as

*Karl Pearson* (1857 – 1936)

*Sir Ronald Fisher* (1890 – 1962)

*Jerzy Neyman* (1894 – 1981).

There are interesting bibliographies of these people on the internet.

### Key Issues

#### *Introduction*

You can divide statistical work into stages such as

- understanding the problem: postulating hypotheses
- collecting the relevant data
- presenting the data
- analysing the data
- making conclusions related to the original hypotheses.

In this module you will deal with all of these stages.

You should already be aware of the use made of data, which is to be found in abundance in newspapers, television and other media. For example, nearly all daily newspapers include statistical data on

*For further background information, see*

"How to lie with statistics"  
by *Darrell Huff, Pelican*,  
ISBN 0 14 021300 7

"Use and abuse of statistics"  
by *W. Reichmann, Pelican*  
ISBN 0 14 020707 4

"Figuring and society"  
by *Ronald Meek, Fontana*  
ISBN 0 00 632560

*See web site*

<http://www-groups.dcs.st-and.ac.uk:80/~history/>

- weather
- financial markets
- sports (football, cricket, tennis, etc.).

Much of our analysis here deals with what is termed 'measures of central tendency':

*mean, median and mode.*

You should understand the differences between the measures but should also appreciate that these measures can *all* be classified as an 'average' for a set of data. The important point to make is that the most appropriate measure of average should be used and this depends on the context of the data being studied.

*Language / Notation*

You should try to be consistent with the language and notation used. For example, tally charts should use the notation  $\text{||||}$   $\text{|||}$ , etc. both for accuracy and for ease of adding up.

A possible source of confusion arises when using grouped data, and deciding where the group boundaries actually lie. For example, a height (or weight) is usually given to a specified degree of accuracy, e.g. nearest cm; 172 cm actually means that the value lies in the interval

$$171.5 \text{ cm} \leq \text{height} < 172.5 \text{ cm} \quad (\text{we conventionally round up this end point})$$

So heights may be grouped into 'every 5 cm' but the 170 – 174 cm grouping is in reality,

$$169.5 \text{ cm} - 174.5 \text{ cm}$$

and this is what should be used in the display.

To really confuse matters, the same is not true for ages where, for example, 'age 10' means the age grouping

$$10 - 11 \text{ years}$$

(even someone who is age 10 years, 364 days is still recorded as age 10!). So great care must be taken, and common sense used.

A number of key terms are used and you should be familiar with their definition and use:

- mean
  - median
  - mode
  - range
- } all measures of average

*Key Points*

In general, the presentation of data should be

- clear
- straightforward
- unambiguous
- not biased.

- The median is the  $\left(\frac{n+1}{2}\right)$ th data point.

*Misconceptions*

- When finding the mean value of grouped data, you must divide by the total frequency, *not* the total sum of  $x$ 's.
- In finding the median, using the

$$\left(\frac{n}{2}\right)\text{th term}$$

is correct unless  $n$  is very large. What is actually needed is the

$$\left(\frac{n+1}{2}\right)\text{th term.}$$

## WORKED EXAMPLES and EXERCISES

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# 6 Data Handling

## 6.1 Tables and Timetables

This section is concerned with the use of tables and timetables.



### Worked Example 1

This timetable is part of a rail timetable for trains from the south west of England.

Saturdays									
Penzance	0715	—	—	—	—	0750	—	0846	
Plymouth	0912	—	0935	—	—	1000	—	1035	
Ivybridge	—	—	—	—	—	1014	—	—	
Totnes	—	—	1002	—	—	1030	—	—	
Paignton	—	0940	—	1001	1017	—	1030	—	
Torquay	—	0946	—	1008	1022	—	1040	—	
Torre	—	—	—	—	1025	—	—	—	
Newton Abbot	0948	0958	1015	1020	1035	1044	1052	1112	
Teignmouth	—	1005	1021	1026	1042	—	1101	—	
Dawlish	—	1010	1026	1033	1047	—	1108	—	
Dawlish Warren	—	—	—	1040	1051	—	1114	—	
Starcross	—	—	—	—	1054	—	—	—	
Exeter St. Thomas	—	—	—	—	1103	—	—	—	
Exeter St. David's	1007	1023	1038	1052	1107	—	1127	1131	
Tiverton Parkway	—	—	1055	—	—	—	—	1148	
Taunton	1037	1054	1108	1118	—	—	1207	1201	
Bristol Temple Meads	1115	1154	—	1158	—	—	1255	—	
London Paddington	—	—	1325	—	—	—	—	1410	

- (a) Stewart wants to travel from Penzance to London Paddington. Describe the different options shown on this timetable.
- (b) How can John get from Plymouth to Bristol Temple Meads?
- (c) How long does the 0715 train take to go from Penzance to Newton Abbot?  
Does the 0846 train take the same time?



### Solution

- (a) There are several possible options, including:
  - (i) leave Penzance at 0715 and arrive at Newton Abbot at 0948. Then leave Newton Abbot at 1015 and arrive at London Paddington at 1325;
  - (ii) leave Penzance at 0846 and travel direct to London Paddington, arriving at 1410.
- (b) Possible options include:
  - (i) leave Plymouth at 0912 and travel direct to Bristol Temple Meads, arriving at 1115;
  - (ii) leave Plymouth at 0935 and travel to Newton Abbot arriving at 1015. Then leave Newton Abbot at 1020 and travel to Bristol Temple Meads, arriving at 1158;



6.1

- (iii) leave Plymouth at 1000 and travel to Newton Abbot, arriving at 1044. Then leave Newton Abbot at 1052 and travel to Bristol Temple Meads, arriving at 1255.
- (c) The 0715 train takes 2 hours 33 minutes; the 0846 train is quicker, taking 2 hours 26 minutes.



**Worked Example 2**

The chart can be used to find the mileage between some Scottish towns and cities.

	Aberdeen						
		Edinburgh					
			Fort William				
				Glasgow			
					Inverness		
						Perth	
							Stranraer
115							
152	130						
142	44	103					
104	156	66	169				
81	42	103	61	115			
226	123	183	84	249	145		

Find the distances between:

- (a) Fort William and Perth,
- (b) Edinburgh and Stranraer
- (c) Which two places are furthest apart?



**Solution**

- (a) To find the distance between Fort William and Perth look in the square where the two highlighted lines meet. So the distance is 103 miles.
- (b)

	Aberdeen						
		Edinburgh					
			Fort William				
				Glasgow			
					Inverness		
						Perth	
							Stranraer
115							
152	130						
142	44	103					
104	156	66	169				
81	42	103	61	115			
226	123	183	84	249	145		

6.1

Using the same approach for Edinburgh and Stranraer gives 123 miles.

	Aberdeen					
115		Edinburgh				
152	130		Fort William			
142	44	103		Glasgow		
104	156	66	169		Inverness	
81	42	103	61	115		Perth
226	123	183	84	249	145	Stranraer

(c)

The largest number in the table is 249.

Using the highlight, this is the distance between Inverness and Stranraer. So these two places are furthest apart.

	Aberdeen					
115		Edinburgh				
152	130		Fort William			
142	44	103		Glasgow		
104	156	66	169		Inverness	
81	42	103	61	115		Perth
226	123	183	84	249	145	Stranraer



Exercises

1. Mike lives in Paignton and works in Exeter, close to the Central Station. Use the following timetable to answer these questions.
  - (a) He starts work at 9.00 am. Which train should he catch?
  - (b) One day he misses his normal train. What is the earliest time he can get to Exeter Central?
  - (c) Mike can walk from Exeter St. David's to his office in 15 minutes. What should he do if he misses his usual train?
  - (d) Sometimes Mike has to go to Bristol or London. How can he get to these places as early as possible and at what time does he arrive?

6.1

Mondays to Fridays									
Penzance	—	—	—	—	—	—	0641	—	—
Plymouth	—	0725	0735	—	—	—	0818	0832	—
Ivybridge	—	—	—	—	—	—	0832	—	—
Totnes	—	0752	0802	—	—	—	0848	—	—
Paignton	0723	—	—	0800	0826	—	—	—	0856
Torquay	0728	—	—	0805	0832	—	—	—	0901
Torre	0731	—	—	0808	—	—	—	—	0904
Newton Abbot	0740	0805	0815	0818	0844	0900	0908	0914	—
Teignmouth	0747	—	—	0825	—	—	—	—	0921
Dawlish	0752	—	—	0830	—	—	—	—	0926
Dawlish Warren	0756	—	—	—	—	—	—	—	0929
Starcross	0759	—	—	0835	—	—	—	—	0933
Exeter St. Thomas	0808	—	—	0844	—	—	—	—	0942
Exeter St. David's	0812	0824	0834	0846	0903	—	0930	0945	—
Exeter Central	0818	0836	0853	0853	0923	—	0940	0953	—
Exmouth	0848	—	0918	0918	0948	—	—	—	1018
Barnstaple	—	—	—	0955	—	—	—	—	—
Tiverton Parkway	—	0841	0851	—	0920	—	—	—	—
Taunton	—	0854	0904	—	0933	—	0959	—	—
Bristol Temple Meads	—	0932	—	—	1009	—	1049	—	—
London Paddington	—	—	1110	—	—	—	—	—	—

2. Use the timetable below to answer these questions about James' journey.
- James catches the 1927 at Reading. What time does this arrive at Cardiff Central?
  - How long does his journey take?
  - He wanted to arrive at Cardiff before 11.00 pm. Could he have caught a later train?
  - What is the latest train he could have caught from Reading to arrive at Cardiff before 11.00 pm?

Mondays to Fridays continuation									
London Paddington	1815	1830	1845	1900	1915	2000	2015	2100	2115
Reading	1840	1857	1910	1927	1942	2027	2043	2127	2145
Didcot Parkway	—	1913	1925	—	1957	—	2058	—	2203
Swindon	1910	1934	1946	1959	2018	2059	2119	2200	2224
Chippenham	1923	—	1959	—	2032	—	2133	—	2238
Bath Spa	1938	—	2012	—	2044	—	2146	—	2251
Bristol Parkway	—	2002	—	2027	—	2127	—	2228	—
Bristol Temple Meads	1956	—	2025	—	2100	—	2201	—	2306
Weston-super-Mare	2023	—	—	—	2140	—	2235	—	—
Newport	—	2024	—	2049	—	2149	—	2250	—
Cardiff Central	—	2041	—	2106	—	2206	—	2312	—
Bridgend	—	2101	—	2126	—	2226	—	2332	—
Port Talbot Parkway	—	2112	—	2137	—	2237	—	2343	—
Neath	—	2120	—	2145	—	2245	—	2351	—
Swansea	—	2135	—	2200	—	2300	—	0005	—

6.1

3. Farharaz arrives by taxi at London Paddington at 0800. Use the timetable below to answer the following questions.

<b>London - Reading - Bristol → Taunton</b>									
<b>Exeter - Torbay - Plymouth - Penzance</b>									
<b>Mondays to Fridays</b>									
London Paddington	0530	0740	0815	0845	0935	0945	1035	1135	1235
Reading	0610	0805	0845	0913	1003	1015	1103	1203	1303
Newbury	—	—	—	—	1018	—	—	—	—
Pewsey	—	—	—	—	1037	—	—	—	—
Westbury	—	—	—	—	1056	—	—	1248	1348
Castle Cary	—	—	—	—	—	—	—	1305	—
Bristol Temple Meads	0750	0915	1008	1028	—	1127	—	—	—
Taunton	0843	0952	1051	1108	1133	—	—	1328	1423
Tiverton Parkway	0857	—	—	1120	1145	—	—	1340	—
Exeter St Davids	0913	1016	1117	1135	1200	1235	1239	1355	1449
Dawlish	0928	—	—	1150	—	—	—	—	—
Teignmouth	0933	—	—	1155	—	—	—	—	—
Newton Abbot	0941	1040	1141	1203	1224	—	—	1419	1513
Totnes	0952	1052	—	—	1235	—	—	1433	—
Plymouth	1023	1120	1222	1240	1310	—	1339	1505	1551
Liskeard	1051	1149	1311	—	—	—	1404	1547	1619
Bodmin Parkway	1102	1201	1323	—	—	—	1416	1559	1631
Par	1114	1211	1334	—	—	—	1427	1611	1644
St Austell	1121	1219	1341	—	—	—	1435	1618	1652
Truro	1138	1237	1359	—	—	—	1453	1636	1710
Redruth	1150	1249	1411	—	—	—	1505	1649	1722
Camborne	1156	1256	1417	—	—	—	1512	1656	1729
St Erth	1207	1307	1428	—	—	—	1523	—	1741
Penzance	1217	1320	1439	—	—	—	1535	1716	1755

- (a) Can he get to Totnes by train to meet his friend Lucy at noon?  
 (b) What is the earliest time he can arrive in Totnes?  
 (c) Lucy can meet Farharaz at Newton Abbot. What is the earliest time he can be there?

4. The table gives the distances, in miles, between some towns and cities.

- (a) Find the distances between:  
 (i) Birmingham and Leeds  
 (ii) Manchester and Bradford  
 (iii) Sheffield and Coventry.
- (b) Nargis travels from Birmingham to Leeds and then to Manchester.  
 (i) How far does he travel?  
 (ii) How would the distance he travels be reduced if he went to Manchester and then to Leeds?

							Birmingham
							Bradford
							Coventry
							Derby
							Doncaster
							Leeds
							Manchester
							Sheffield
108							
18	114						
40	74	40					
93	34	92	54				
109	9	110	70	28			
80	34	94	59	51	40		
77	37	78	37	18	33	38	

- (c) Denise is going to Birmingham from Leeds. She must stop in either Bradford or Sheffield on the way. Which is the shorter route?

6.1

5. The table gives the distances in km between 3 ports, Calais, St. Malo and Boulogne, and some holiday destinations in Europe.

	Calais	St. Malo	Boulogne					
	1422	1560	1430	Florence				
	811	1169	819	612	Interlaken			
	1370	1561	1388	307	563	Venice		
	729	244	707	1708	1172	1710	Quimper	
	704	344	670	1494	995	1355	400	Ile de Re
	1110	729	1152	1358	1233	1504	787	409
								Biarritz

- (a) The Eccles family decide to go to Biarritz for their holiday. Which of the three ports (Calais, St. Malo or Boulogne) is closest to Biarritz?
- (b) They decide to start their holiday at Calais, and also to visit Venice before they return to Calais. How far do they have to travel in total?
- (c) Their friends, the Morse family, decide to travel from St. Malo to Biarritz, then Interlaken and return to Boulogne. How far do they have to travel?
- (d) Which holiday destination is closest to St. Malo and which is closest to Calais?
6. In a school 30 students took GCSE exams in both Maths and Physics. Their results are given in this table.

		<i>Maths Grade</i>				
		<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>
<i>Physics Grade</i>	<b>A</b>	2	3			
	<b>B</b>	1	1	4		
	<b>C</b>		2	3	2	
	<b>D</b>			4	2	2
	<b>E</b>		1	0	2	1

6.1

- (a) How many students got the same grade in both subjects?
- (b) How many students got a higher grade in Physics than in Maths?
- (c) Which was the most common grade in Physics?

7. The table shows the sports options selected by a group of students in each of their years in secondary school. In each year, each student chose just one sport.

	<i>Hockey</i>	<i>Football</i>	<i>Tennis</i>	<i>Swimming</i>
<i>Year 7</i>	6	18	5	14
<i>Year 8</i>	5	16	7	15
<i>Year 9</i>	7	14	10	12
<i>Year 10</i>	2	12	10	19
<i>Year 11</i>	8	13	12	10

- (a) How many chose tennis in Year 8?
- (b) How many more chose football in Year 7 than in Year 10?
- (c) In which years was football the most popular sport?
- (d) (i) In which year was swimming the most popular sport?  
(ii) How many students were there in this group?

8. The table below shows the cost of a week at a large holiday centre. The cost varies according to the number of people in the party and the type of accommodation booked. There is a £20 reduction for each child.

		<i>Accommodation Type</i>		
		<i>Saver</i>	<i>Comfortable</i>	<i>Luxury</i>
<i>Number of people in party</i>	4	£180	£260	£368
	5	£220	£320	£454
	6	£265	£385	£541
	7	£305	£449	£630
	8	£340	£507	£704

6.1

- (a) How much would it cost for Mr and Mrs Jones and their 4 children to stay in 'Comfortable' accommodation for one week?
- (b) How much more would it cost if they booked 'Luxury' accommodation?
- (c) How much would they save in 'Saver' accommodation compared with 'Luxury' accommodation?
- (d) (i) How much would it cost if two grandparents came with the family and they all stayed in 'Saver' accommodation?  
(ii) By how much does this differ from the total in part (a)?

9. In a privatised railway company, there are 84 male conductors and 56 female conductors. Conductors can either be senior or standard. There is a total of 28 senior conductors and there are 48 female standard conductors.

- (a) Copy and complete the two-way table to show the number of male and female conductors who are senior or standard.

	<i>Male</i>	<i>Female</i>
<i>Standard</i>		
<i>Senior</i>		

- (b) Comment on the results.

10. Each student in a class chose *one* sport. The numbers of choices were put in a table.

		Outdoor Sports		Indoor Sports	
		Hockey	Tennis	Badminton	Squash
Year 11	Girls	12	10	15	5
	Boys	10	15	7	19
Year 10	Girls	14	9	17	3
	Boys	15	12	11	13

- (a) How many students chose hockey?
- (b) How many more girls chose tennis than squash?
- (c) One girl says that boys usually prefer outdoor sports. Do the figures in the table support this view? Explain your answer.

(SEG)

11. The two-way table shows the number of students achieving grades **A** to **E** in examinations in English and French.

		English grade				
		E	D	C	B	A
French grade	A			1	1	2
	B		1		5	2
	C		1	7	2	
	D		2	3	1	
	E	3	1			

6.1

- (a) How many of the students who achieved grade **B** in English achieved a different grade in French?
- (b) How many students achieved the same grade in both subjects.
- (c) What does the table suggest about the grades achieved in English and French?

(SEG)

12. St. Margaret's School entered Y11 and Y10 pupils for the NEAB mathematics examination at levels P, Q and R as shown in the table.

- (a) How many Y10 pupils have entered for the examination?
- (b) (i) What was the total number of pupils entered for level Q?
- (ii) What percentage of the pupils entered at level Q were Y10 pupils?

		Level P	Level Q	Level R
Y11	Boys	25	18	7
	Girls	10	45	11
Y10	Boys	0	13	3
	Girls	0	14	1

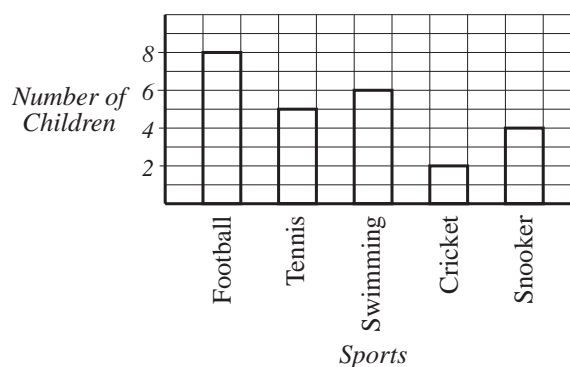
(NEAB)

## 6.2 Pictograms and Bar Charts

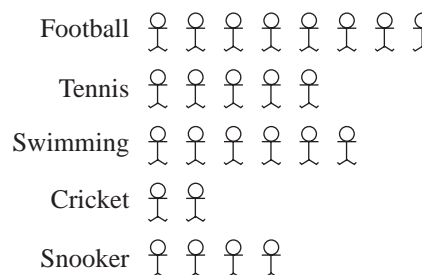
Bar charts and pictograms can be used for displaying data when the data are in discrete categories.

For example, the bar chart and pictogram below show the favourite sports of 25 children.

Bar Chart



Pictogram



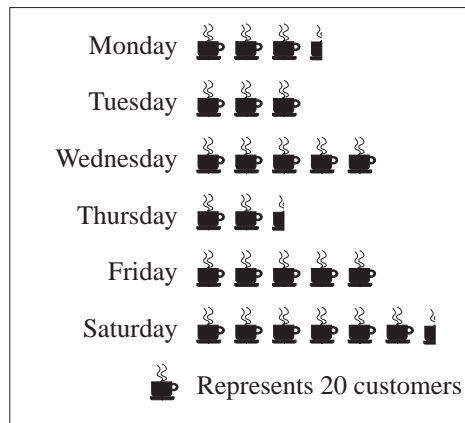


6.2



**Worked Example 1**

The pictogram shows the number of customers using a coffee shop during one week.



- (a) How many customers used the shop on Wednesday?
- (b) How many customers used the shop on Monday?
- (c) How many customers visited the coffee shop during the week?



**Solution**

- (a) For Wednesday there are 5 symbols, so the number of customers was

$$5 \times 20 = 100$$

- (b) For Monday there are  $3\frac{1}{2}$  symbols, so the number of customers was

$$3\frac{1}{2} \times 20 = 70$$

- (c) The total for the week is given by

$$70 + 60 + 100 + 50 + 100 + 130 = 510 \text{ customers.}$$



**Worked Example 2**

John asked each person in the class what their shoe size was. He obtained these results.

7	5	6	8	4	$5\frac{1}{2}$
$6\frac{1}{2}$	7	8	$7\frac{1}{2}$	$5\frac{1}{2}$	6
$6\frac{1}{2}$	$5\frac{1}{2}$	7	6	$6\frac{1}{2}$	8
7	5	$6\frac{1}{2}$	6	$7\frac{1}{2}$	7
$5\frac{1}{2}$	6	5	$5\frac{1}{2}$	6	$7\frac{1}{2}$

Draw a bar chart to show this data.



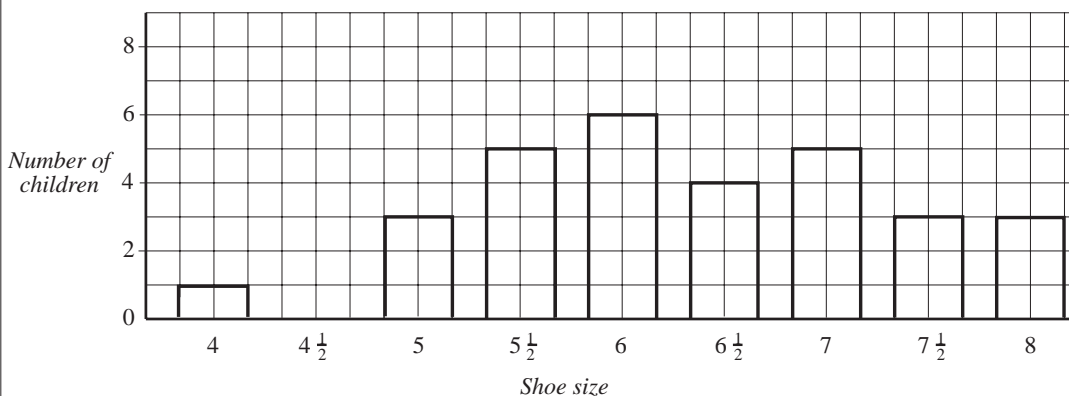
**Solution**

First the data can be entered into a tally chart.

Shoe Size	Tally	Total
4		1
$4\frac{1}{2}$		0
5		3
$5\frac{1}{2}$		5
6		6
$6\frac{1}{2}$		4
7		5
$7\frac{1}{2}$		3
8		3
		30

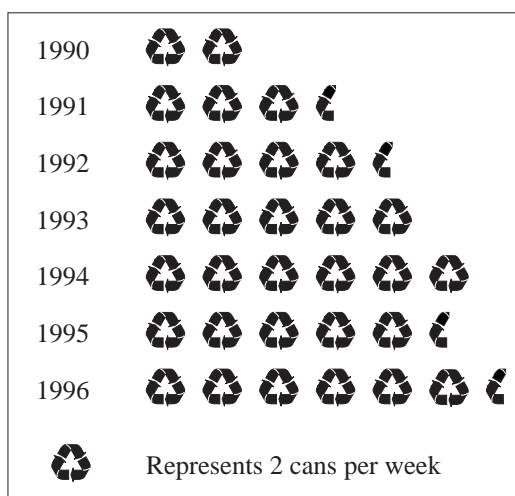
6.2

The bar chart can be drawn as shown below.

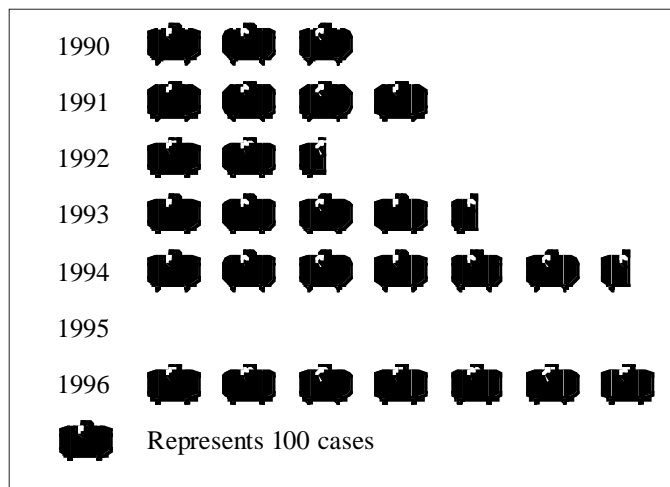


Exercises

- Jenny kept a record of the average number of cans she recycled each week over a number of years. The pictogram shows her results.
  - In which year did she recycle most cans?
  - How many cans did she recycle each week in;
    - 1993,
    - 1991,
    - 1994?
  - In which year did she recycle an average of 11 cans per week?



- The pictogram shows how many suitcases were sold by a shop from 1990 to 1996, with one row missing.
  - How many cases were sold in 1991?
  - What is the smallest number of cases sold in a year?



- How many cases were sold in 1991?
- What is the smallest number of cases sold in a year?

## 6.2

- (c) What is the greatest number of cases sold in a year?
- (d) In 1995 a total of 550 cases were sold. How many cases should appear in the missing row?
- (e) How many suitcases have been sold altogether?

3. A class conducted a survey to find their favourite ice creams. The results were:

<i>Favourite Ice Cream</i>	<i>Number of Children</i>
Solero	9
Magnum	12
Mars	7
Feast	4

- (a) Draw a pictogram to show these results.
- (b) Represent this information in a bar chart.

What are the advantages of each type of representation of the data?

4. A group of students recorded the number of vehicles passing their school in one hour. The results are recorded below.

<i>Vehicle Type</i>	<i>Number of Vehicles</i>
Cars	20
Vans	8
Lorries	3
Motorbikes	5
Buses	2

Represent this information with a bar chart.

5. Draw a bar chart to show the data given in the table about the hours of sunshine per day at a number of resorts.

<i>Resort</i>	<i>Hours of Sunshine per Day</i>
Algarve	6
Benidorm	6
Eilat	7
Majorca	5
Mombasa	9
Tenerife	6
Torremolinos	6

6.2

6. The children on a school bus were asked which year group they were in. Their replies were:

10	7	7	10	11	9	8	7	8	9
7	9	11	11	8	8	9	7	10	10
11	8	9	7	10	11	11	11	11	7
7	7	8	7	8	9	10	10	9	8

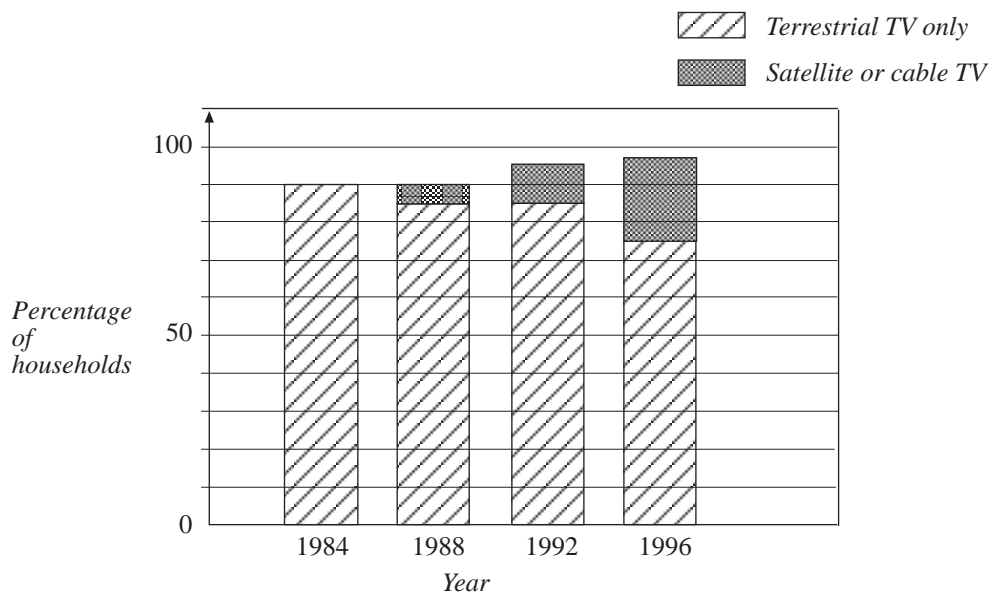
Draw a bar chart to show this data.

7. A headteacher asked a class of Year 7 students how many younger brothers and sisters each student had. The results were:

0	1	2	1	0	0	1	2	1	1
2	0	0	1	1	2	3	4	1	1
2	1	2	0	0	3	2	1	5	1

Draw a bar chart to illustrate this data.

8. A group of households were asked to give information about access to satellite, cable and terrestrial TV. The results are summarised below.

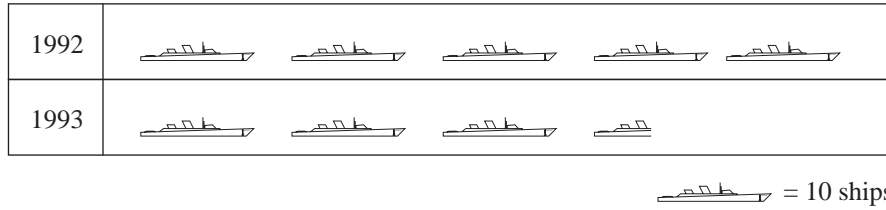


Use this presentation of the results to answer the following questions.

- (a) What percentage of households had some form of TV in
  - (i) 1984
  - (ii) 1996?
- (b) What percentage of households had satellite or cable TV in
  - (i) 1984
  - (ii) 1996?
- (c) Comment on the trends in the results from the survey.

6.2

9.

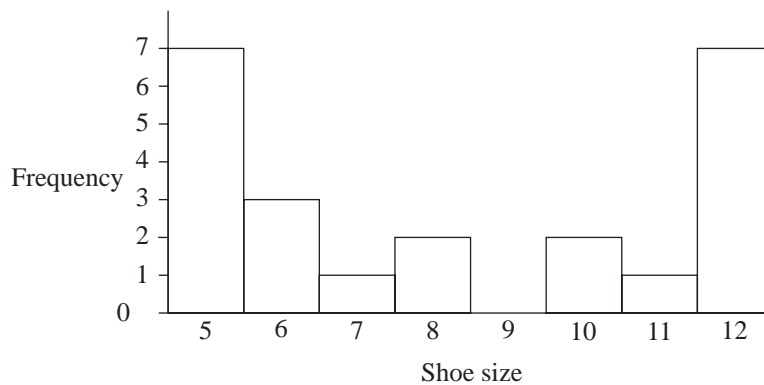


The diagram shows how many ships were in Mathsland's navy in 1992 and 1993. Use the diagram to answer these questions.

- (a) How many ships were there in Mathsland's navy in 1992?
- (b) How many ships were there in Mathsland's navy in 1993?

(LON)

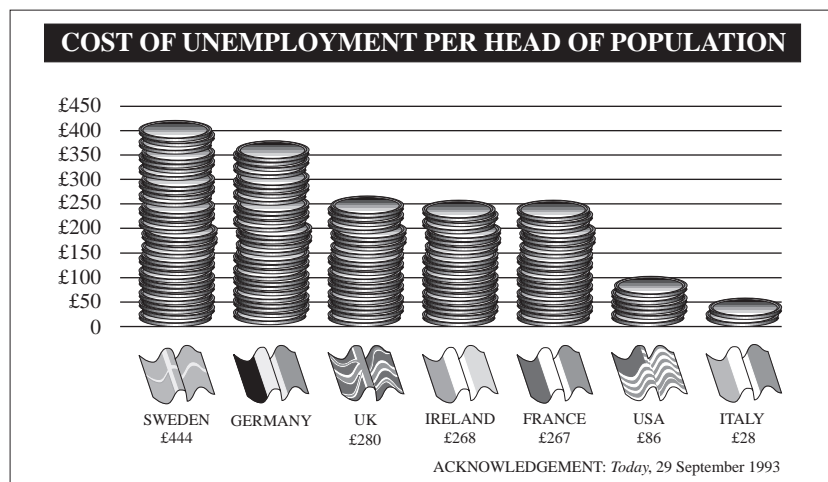
10. The bar chart below shows the shoe sizes of a group of 16 year old boys.



- (a) How many boys are there in the group?
- (b) Comment on the shape of the bar chart, saying whether or not this is the shape you would expect.

(MEG)

11.

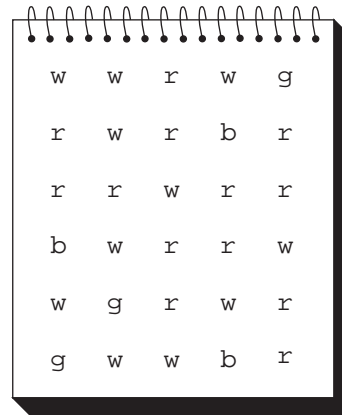


The pictogram represents, approximately, the cost of unemployment per head of population in seven developed countries. The exact figures for six of them are shown under their names.

6.2

- (a) There are 19 coins in the pile for Ireland? To the nearest whole number, how many pounds does one coin represent?
- (b) Estimate the cost of unemployment per head of population in Germany.
- (c) The cost of unemployment per head of population in Japan is £218. How many coins would there be in the pile for Japan?
- (d) Explain why the pictogram is only approximate.

12. Sally did a survey of car colours.  
The notebook shows all her results.

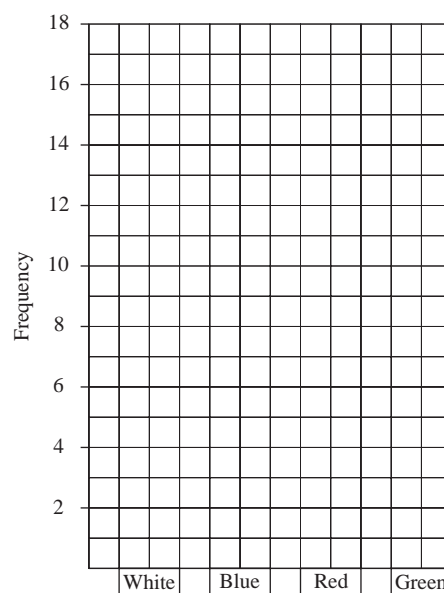


Key:  
w white  
b blue  
r red  
g green

(a) Copy and complete the frequency table.

COLOUR	TALLY	FREQUENCY
White		
Blue		
Red		
Green		

(b) Show this information as a bar chart of the form shown here.



(SEG)

## 6.3 Pie Charts

Pie charts, which represent quantities as sectors of a circle, can be used to illustrate data. They are particularly effective if there is only a small number of items to illustrate. In total a complete circle, i.e.  $360^\circ$ , must always be used.



### Worked Example 1

Tracey uses her pocket money of £18 per month in the following way.

Magazines	£4
Sweets	£3
Swimming	£6
Bus fares	£4
Money box	£1

Draw a pie chart to show how Tracey uses her pocket money.



### Solution

Tracey has a total of £18.

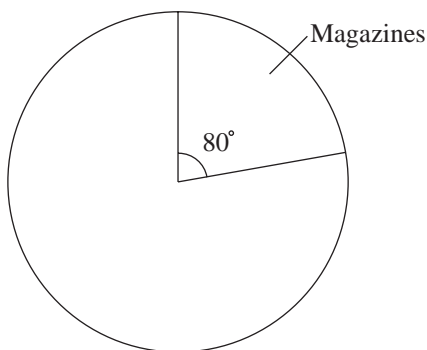
$$\frac{360^\circ}{18} = 20^\circ$$

So  $20^\circ$  should be used for each £1. The angles needed are given in this list.

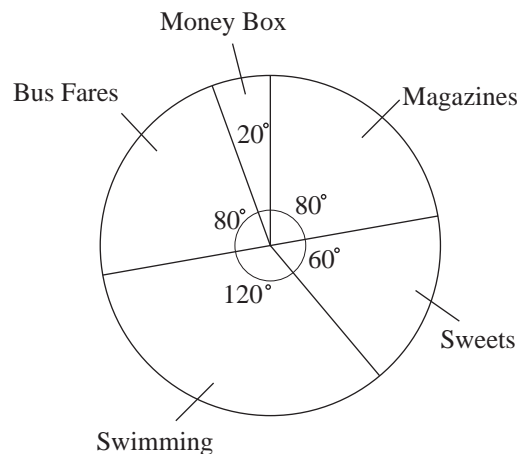
Magazines	$4 \times 20^\circ = 80^\circ$
Sweets	$3 \times 20^\circ = 60^\circ$
Swimming	$6 \times 20^\circ = 120^\circ$
Bus fares	$4 \times 20^\circ = 80^\circ$
Money box	$1 \times 20^\circ = 20^\circ$

The pie chart can now be drawn.

The diagram below shows the first section for magazines.



This is the completed pie chart.



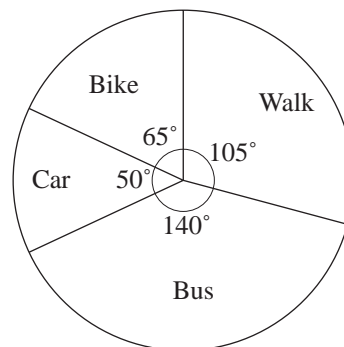
6.3



### Worked Example 2

The pie chart was constructed by asking 72 children how they travel to school.

- (a) How many children travel to school by:
  - (i) car, (ii) bike, (iii) bus?
- (b) What percentage walk to school?



### Solution

- (a) There are 72 children so

$$\frac{360^\circ}{72} = 5^\circ \text{ per child.}$$

- (i) The angle for travelling by car is  $50^\circ$  so  $\frac{50^\circ}{5^\circ} = 10$  children travel by car.
- (ii) The angle for travelling by bike is  $65^\circ$  so  $\frac{65^\circ}{5^\circ} = 13$  children travel by bike.
- (iii) The angle for travelling by bus is  $140^\circ$  so  $\frac{140^\circ}{5^\circ} = 28$  children travel by bus.

- (b) The number who walk to school is given by

$$\frac{105^\circ}{5^\circ} = 21$$

so the percentage who walk is

$$\frac{21}{72} \times 100 \approx 29.1\%$$



### Note

Alternatively, you could just use the angles in the pie chart to give

$$\frac{105}{360} \times 100 \approx 29.1\%$$



### Exercises

1. In an opinion poll 360 people were asked who they would vote for in the next election. Their responses are:

Labour	150
Liberal Democrat	60
Conservative	100
Other	50

Draw a pie chart to show this.



6.3

2. Sarah recorded how she spent the last 24 hours. Her results are below.

Sleeping	9 hours
School	7 hours
Homework	2 hours
Watching TV	3 hours
Eating	1 hour
Travelling	2 hours

Draw a pie chart to show this information.

3. Emma has 720 stamps in her stamp collection. She has sorted them into three groups.

*UK 400 stamps, Europe 200 stamps, Other countries 120 stamps*

Draw a pie chart to show this information.

4. The 30 pupils in a class state their favourite sport. Their results are listed below.

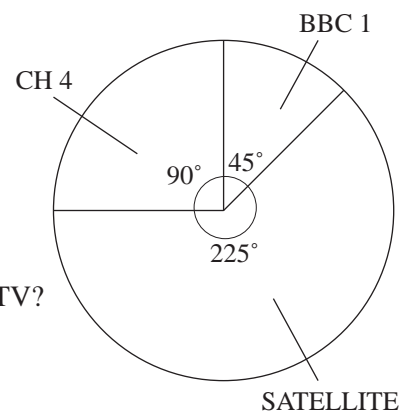
Snooker	3
Football	9
Netball	6
Squash	2
Tennis	10

Draw a pie chart to show this information.

5. The pie chart shows how the time Ron spends watching television is split between different channels, for one day.

Ron spends 1 hour watching BBC1.

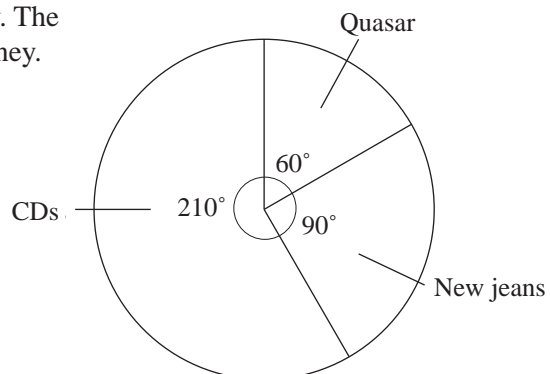
- (a) How long does he spend watching CH4?  
 (b) How long does he spend watching satellite TV?



6. Ahmed was given £60 on his birthday. The pie chart shows how he spent this money.

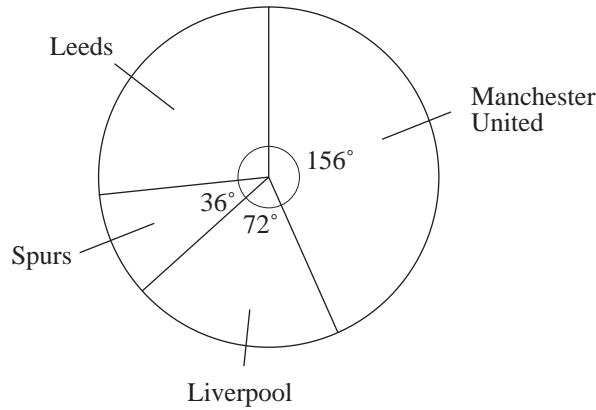
How much did he spend on:

- (a) Quasar,  
 (b) his new jeans,  
 (c) CDs?



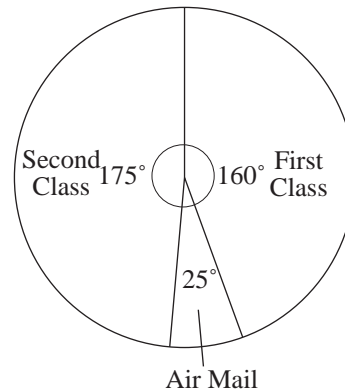
6.3

7. The pie chart shows the football teams supported by a class of children. There are 3 children who support Spurs.



- (a) What is the angle representing Leeds?
- (b) How many children support Liverpool?
- (c) How many children are there in the class?
- (d) How many children support Manchester United?

8. A postman collects 720 letters from the letter boxes in a small town. They are sorted into First Class, Second Class and Air Mail. The pie chart shows the different numbers of each type. How many letters of each type did he collect?



9. Sita spent £90. The table shows what she spent it on.

Items	Amount spent
Bus fares	£12
Going out	£25
Clothes	£30
Records	£15
Others	£8
<b>Total Spending</b>	<b>£90</b>

Sita is asked to construct a pie chart to show her spending.

- (a) Work out the angle of each sector in the pie chart.

6.3

Items	Angle of sector
Bus fares	
Going out	
Clothes	
Records	
Others	
Total of angles	360°

- (b) Construct the pie chart to illustrate the data.
- (c) What fraction of Sita's spending was on clothes?

(LON)

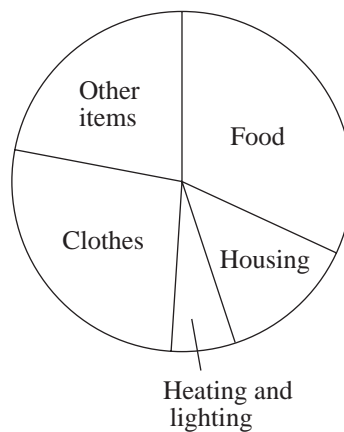
10. Arthur spends £180 per week.

Item	Spending £
Food	30
Heating and lighting	12
Clothes	20
Other items	20
Housing	98
Total spending	180

The way in which he spends his money is shown in the table.

- (a) Draw a pie chart to show how Arthur spends his money.

This pie chart shows how the average person spends money.



- (b) Describe **one** way in which Arthur's spending differs from the average person's spending.

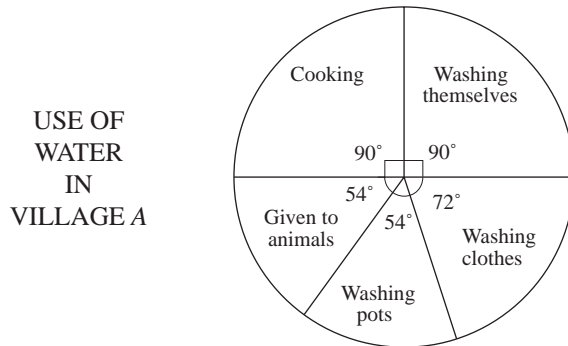
(SEG)

6.3

11. This question is about the way water is used in two Mozambique villages.

(a) In village A, 324 litres of water are used each day.

The pie chart shows how the water is used.



(i) How much water (in litres) is used each day for cooking?

(ii) What fraction of the water used is given to animals?

(b) In village B, the water is used as follows:

Cooking	20%
Washing themselves	50%
Washing clothes	20%
Washing pots	10%

Represent this information in a pie chart.

(MEG)

12. (a) (i) 500 000 cars were stolen in England and Wales in 1990.

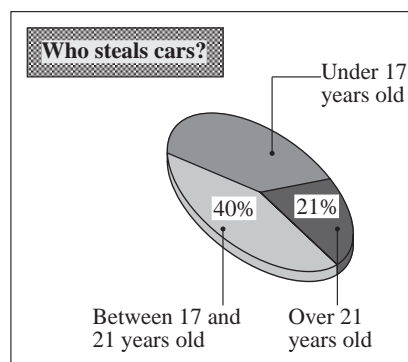
Write the number 500 000 in words.

(ii) 40% of the cars stolen in 1990 were never found.

Calculate the number of cars that were never found.

(b) The chart shows information about the people who stole cars in 1990.

Use the information in the diagram to answer these questions about car theft in 1990.



6.3

- (i) What percentage of the cars stolen were taken by people aged under 17?
- (ii) Which age group was the least likely to be involved in stealing cars?
- (iii) What is the probability that a stolen car was taken by a person who was aged 17 or over?

Give your answer as a decimal or a fraction.

(NEAB)

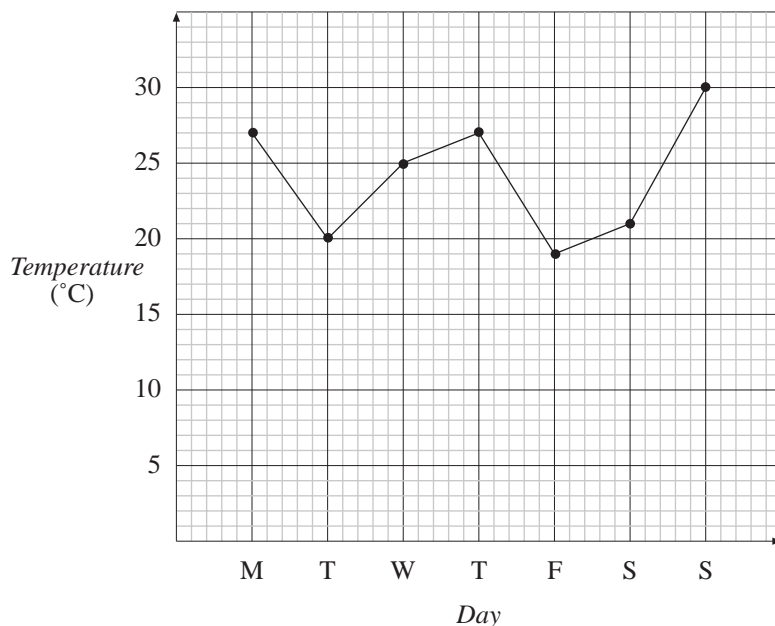
## 6.4 Line Graphs

A *line graph* is drawn by plotting data points and joining them with *straight* lines. It is really only the actual data points that count, but by drawing the lines you get a better impression of the trend in the data points. This method of representation is particularly useful when illustrating trends over time.

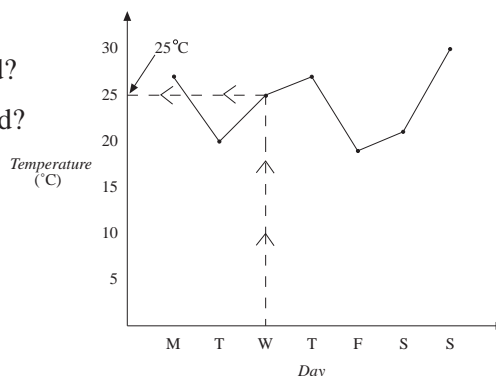


### Worked Example 1

Stuart recorded the temperature in his greenhouse at 6 pm each day for a week. His records are shown on this line graph.



- (a) What was the temperature on Wednesday?
- (b) What was the lowest temperature recorded?
- (c) What was the highest temperature recorded?

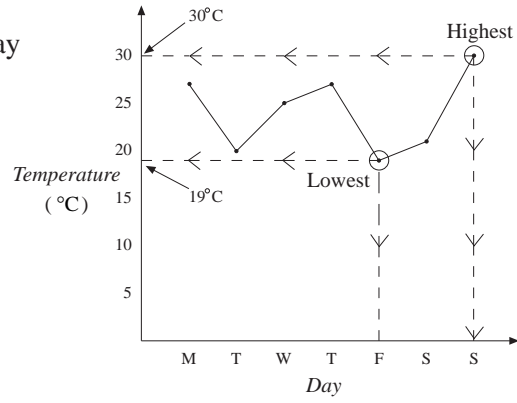


### Solution

- (a) For Wednesday the temperature can be read as 25°C.

6.4

- (b) The lowest temperature occurred on Friday and was 19°C.
- (c) The highest temperature occurred on Sunday and was 30°C.



**Worked Example 2**

As part of a science project Evan records the height of a plant every week. His results are in this table.

<i>Week</i>	0	1	2	3	4	5	6
<i>Height (cm)</i>	0	1	3	4	6	8	9

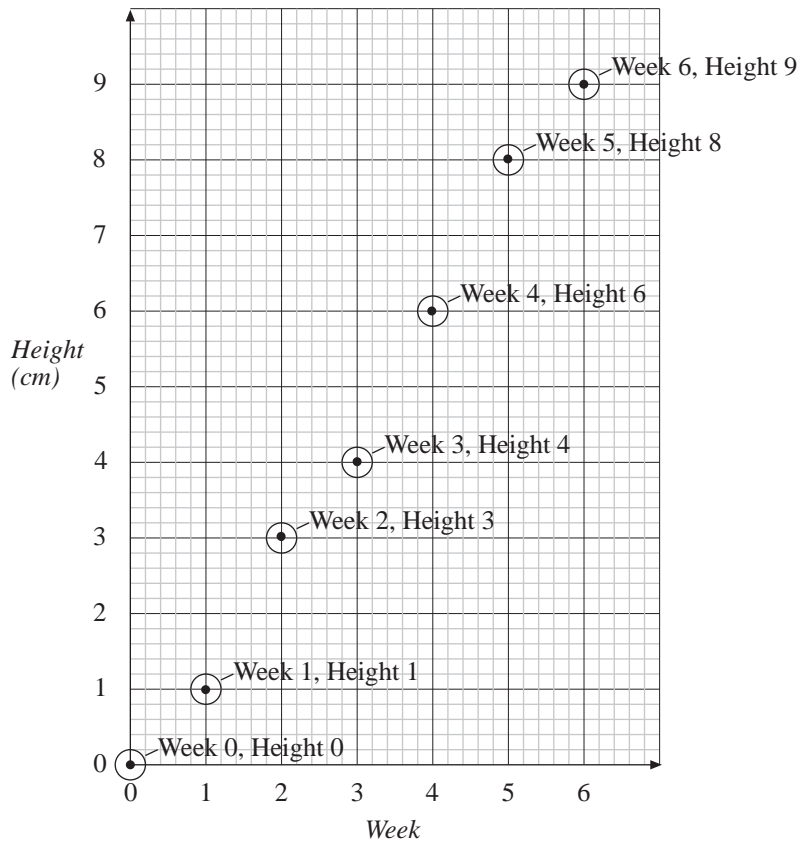
Draw a line graph.



**Solution**

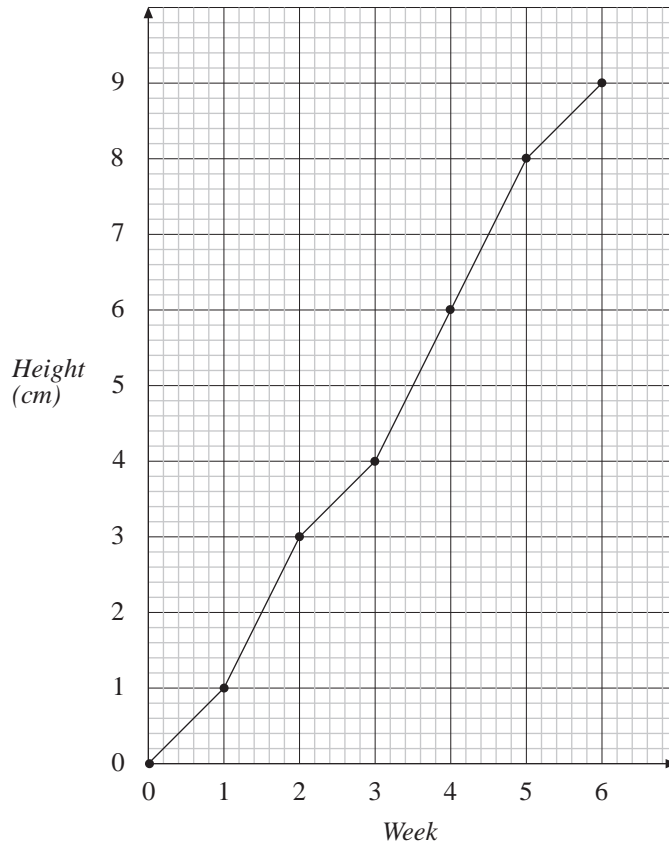
First draw a suitable set of axes.

Then plot a point for each measurement as shown below.



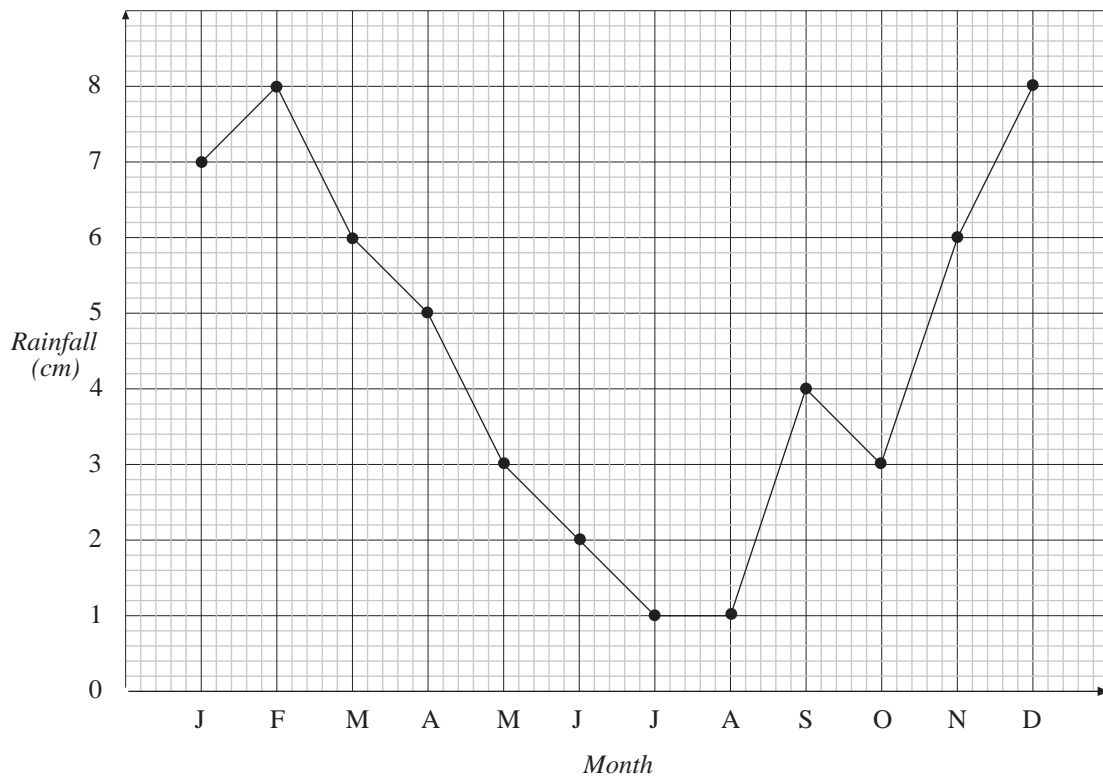
The points can then be joined with straight lines as shown in the next graph.

6.4



Exercises

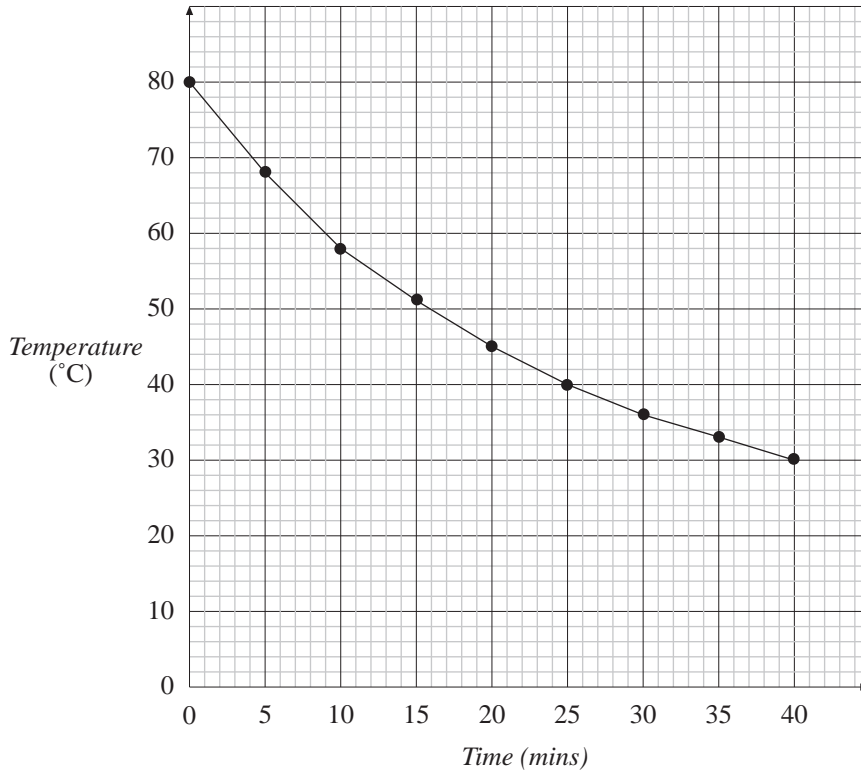
- The line graph shows the monthly rainfall for a town.



6.4

- (a) How much rain was there in September?
- (b) In which month was the rainfall 5 cm?
- (c) Which months were the wettest?
- (d) Which months were the driest?

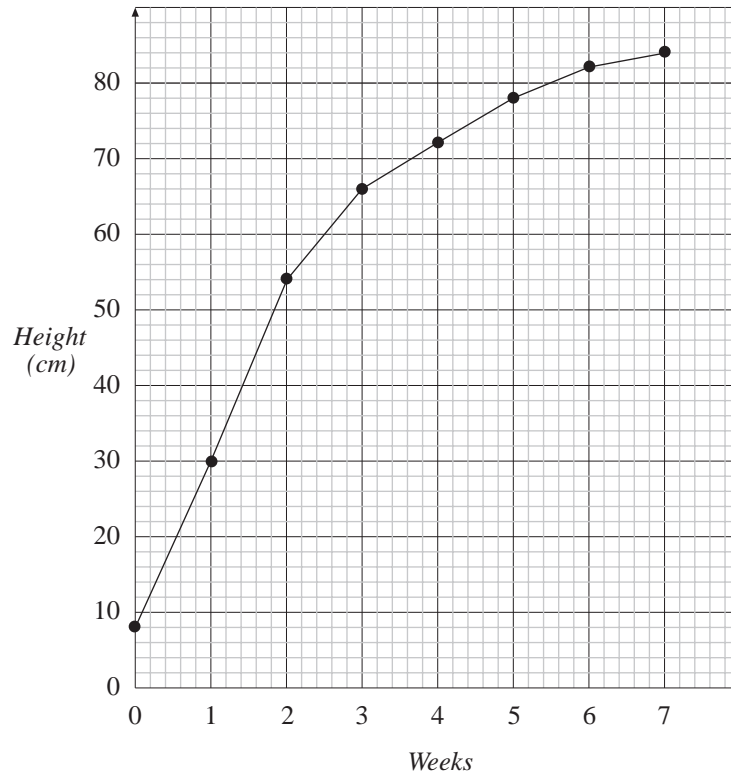
2. A mug was filled with hot water and the temperature was recorded every 5 minutes. The graph below shows the results.



- (a) What was the temperature after 25 minutes?
  - (b) What was the temperature at the start of the experiment?
  - (c) When was the temperature 45°C?
  - (d) How long did it take for the temperature to drop from 68°C to 36°C?
3. The following graph shows how the height of a sunflower plant changed since it was planted in a garden.
- (a) What was the height of the plant when it was planted in the garden?
  - (b) How much did the plant grow in the first week?
  - (c) What is the greatest height that the graph shows?
  - (d) How long did it take for the height to increase from 54 cm to 78 cm?



6.4



4. Paul recorded the temperature outside his house at 8.00 am every day. His results are in the table.

<i>Day</i>	M	T	W	T	F	S	S
<i>Temperature (°C)</i>	8	5	4	6	7	5	3

Draw a line graph for this data.

5. Karen counted the number of cars that drove past her while she was waiting at the bus stop each morning on her way to work.

<i>Day</i>	M	T	W	T	F	S
<i>Number of cars</i>	18	12	22	36	4	10

Draw a line graph for this data.

6. Anna recorded the time it took her to walk to school every day for a week.

<i>Day</i>	M	T	W	T	F
<i>Time taken (mins)</i>	8	9	15	12	7

Draw a line graph for this data.

## 6.4

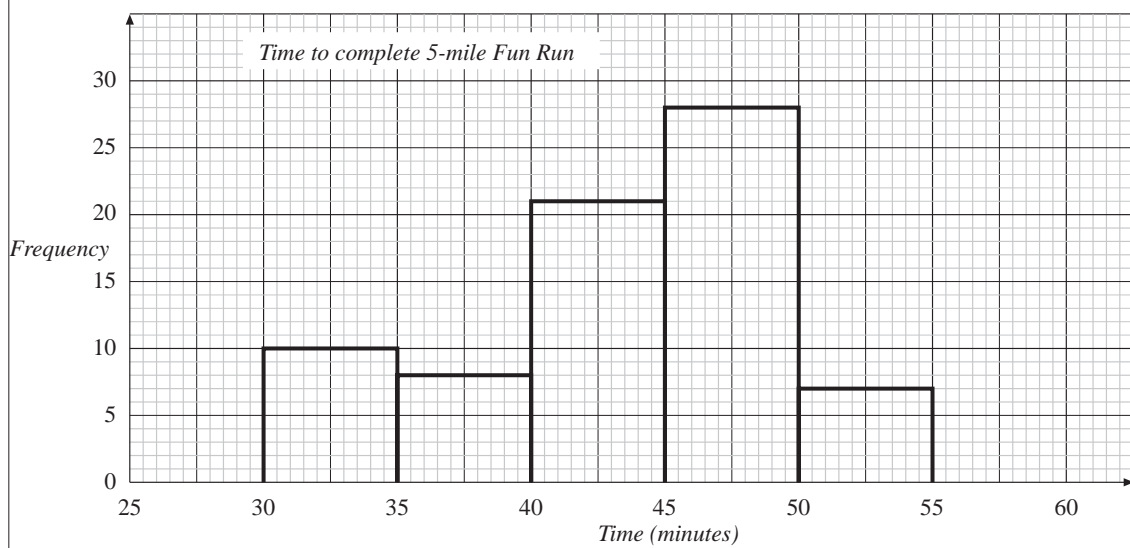
7. Stuart is training to run a marathon. Each week he recorded the time it took him to run 5 miles.

<i>Week</i>	0	1	2	3	4	5	6
<i>Time (mins)</i>	52	50	46	44	40	36	34

Draw a line graph for this data.

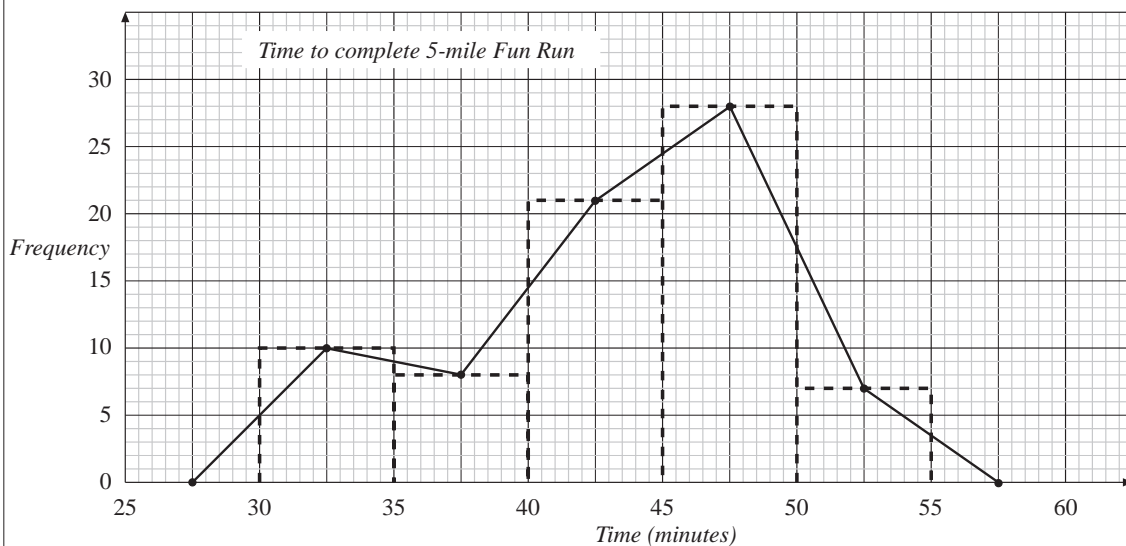
## 6.5 Frequency Graphs

For *continuous data*, when any value over a range of values is possible, a *frequency graph* like the one below should be used, rather than a bar chart which is used for discrete data.



A graph like this is often called a *histogram*, and is characterised by having a continuous scale along the horizontal axis. Note that in this case the widths of the bars are all the same, but this is not always the case, as you will see in the next section. Care though must be taken about the end points. For example, the first *class interval* (in minutes) would normally be  $30 \leq \text{time} < 35$ , so that a time of 35 minutes would be in the second class interval.

A *frequency polygon* could also be used to show the same data, as on the following graph. Note how it is related to the histogram.



6.5



### Worked Example 1

Use the data shown on the graphs above to answer these questions.

- (a) How many people completed the Fun Run in between 40 and 45 minutes?
- (b) How many people completed the Fun Run in less than 40 minutes?
- (c) How many people completed the Fun Run in less than 1 hour?



### Solution

- (a) The 40-45 minute interval contains 21 people.
- (b) The 30-35 and 35-40 minute intervals must be considered.  
There are 10 people in the 30-35 minute interval.  
There are 8 people in the 35-40 minute interval.  
So there are  $10 + 8 = 18$  people who complete the run in less than 40 minutes.
- (c) The number in each interval is needed.  
So the number of people is:

$$10 + 8 + 21 + 28 + 7 = 74$$



### Worked Example 2

A group of students measured the reaction times of 50 other students. The times are given below correct to nearest hundredth of a second.

0.44	0.32	0.31	0.47	0.27	0.31	0.40	0.28	0.16	0.26
0.33	0.46	0.41	0.33	0.31	0.28	0.38	0.29	0.17	0.26
0.29	0.40	0.29	0.24	0.41	0.22	0.25	0.47	0.31	0.36
0.49	0.21	0.42	0.43	0.28	0.36	0.24	0.37	0.34	0.27
0.49	0.16	0.29	0.30	0.41	0.27	0.29	0.28	0.40	0.42

Draw a histogram for this data.



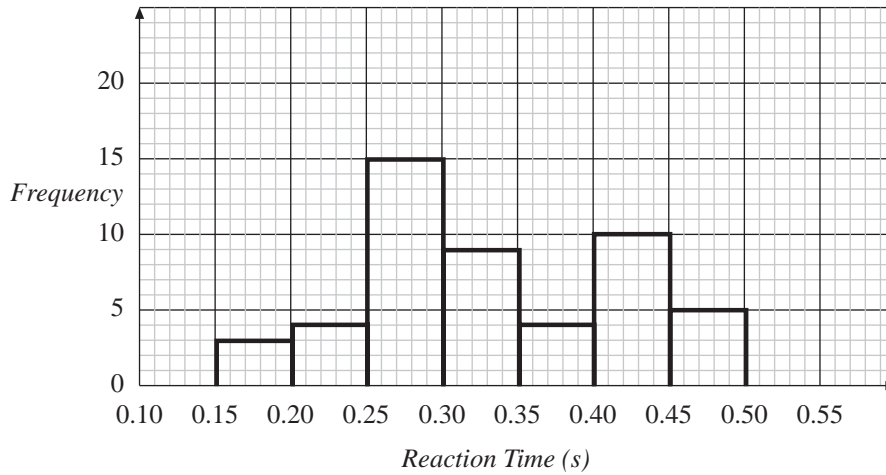
### Solution

First the data must be collected into groups, using a tally chart.

<i>Reaction Time(s)</i>	<i>Tally</i>	<i>Frequency</i>
$0.15 \leq t < 0.20$		3
$0.20 \leq t < 0.25$		4
$0.25 \leq t < 0.30$		15
$0.30 \leq t < 0.35$		9
$0.35 \leq t < 0.40$		4
$0.40 \leq t < 0.45$		10
$0.45 \leq t < 0.50$		5

Now that the data has been collected in this way, the following histogram can be drawn:

6.5



**Worked Example 3**

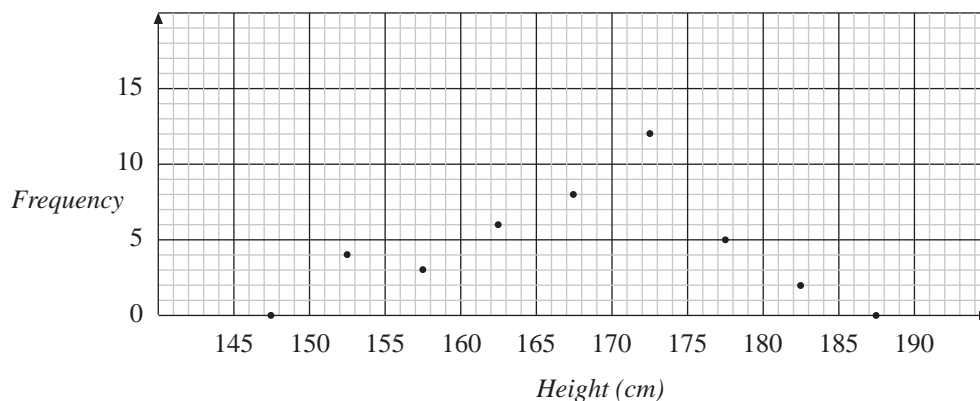
Draw a frequency polygon for the data on the height of children given in the table below.

Height (cm)	Frequency
$150 \leq h < 155$	4
$155 \leq h < 160$	3
$160 \leq h < 165$	6
$165 \leq h < 170$	8
$170 \leq h < 175$	12
$175 \leq h < 180$	5
$180 \leq h < 185$	2



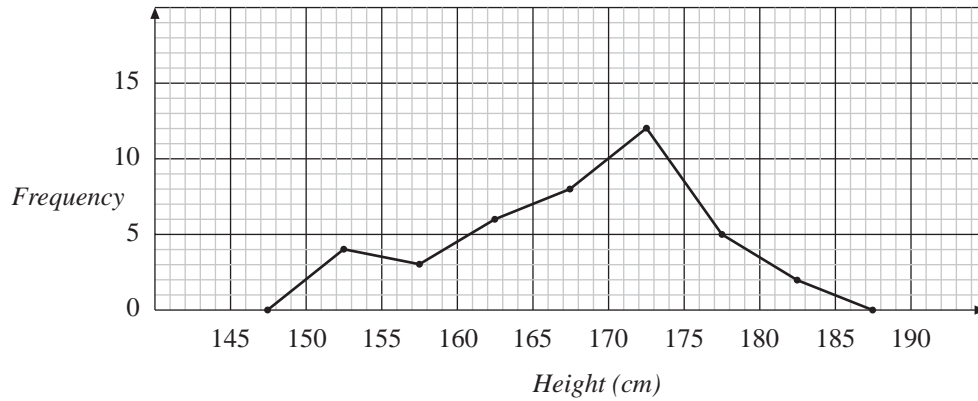
**Solution**

Points should be placed above the centre of each interval. The height is given by the frequency. The graph below shows these points.



Note that points have been placed on the horizontal axis in the intervals that have frequencies of 0. The points can then be joined to give the following frequency polygon:

6.5



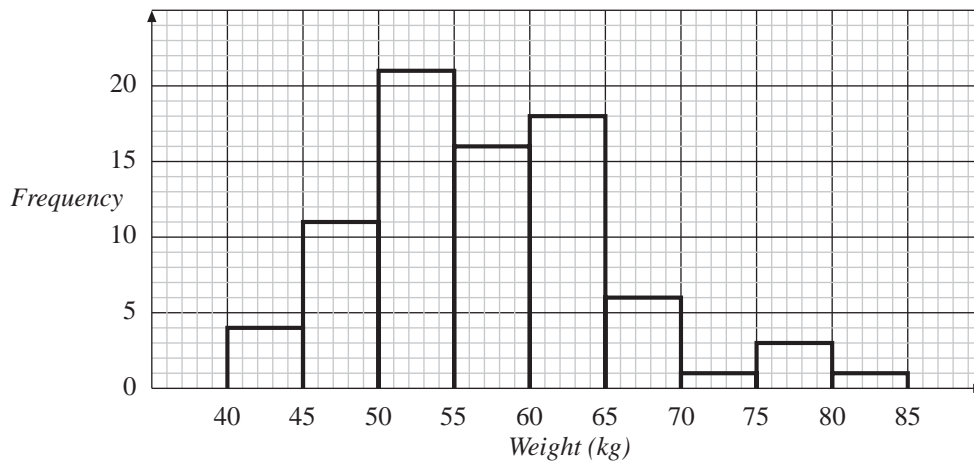
Investigation

Choose an article of at least 500 words from a newspaper. Record the word lengths (in terms of number of letters in a word) of 500 successive words in the article. Tabulate the results in a frequency distribution. Find the mean, mode and median for the word lengths used.



Exercises

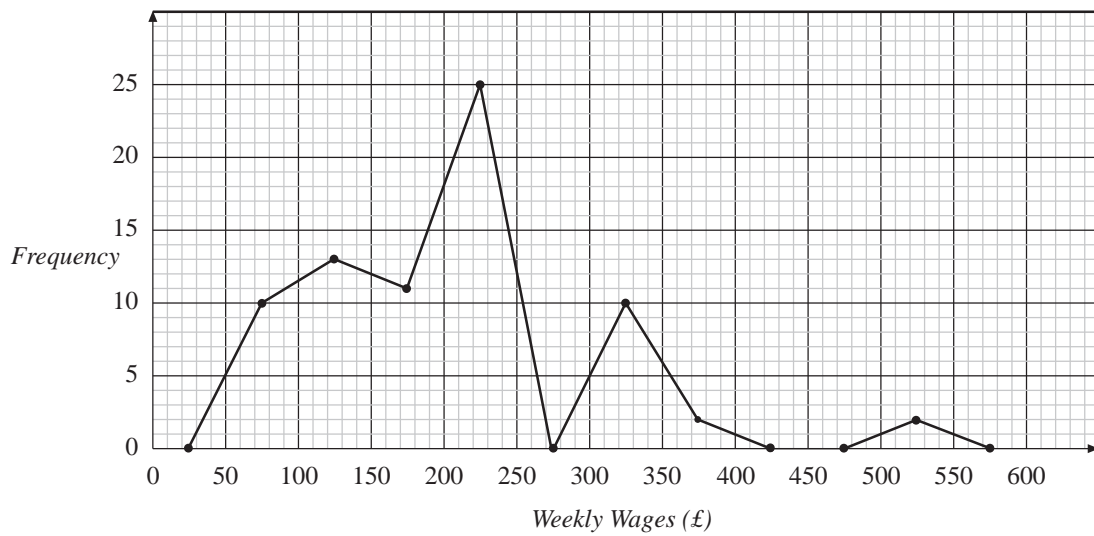
- The histogram below shows how the weights of children in one year group were distributed.



- How many children had a weight greater than 70 kg?
- How many children had a weight between 50 and 65 kg?
- How many children had a weight less than 50 kg?
- How many children were there in the year group?

6.5

2. The frequency polygon shows the weekly wages of a large firm.



- (a) How many people earn between £300 and £350 per week?
- (b) How many people earn between £100 and £300 per week?
- (c) How many people are employed by the firm?
- (d) What are the largest and smallest possible weekly wages that the graph shows could be paid?

3. An orchard contains 100 apple trees. The weight of apples produced by each tree in one year was recorded. The results are given in the table .

<i>Mass of apples (kg)</i>	<i>Frequency</i>
$50 < m \leq 60$	5
$60 < m \leq 70$	7
$70 < m \leq 80$	13
$80 < m \leq 90$	10
$90 < m \leq 100$	20
$100 < m \leq 110$	22
$110 < m \leq 120$	18
$120 < m \leq 130$	5

Draw a histogram for the data.

4. A psychologist uses a test in which people have to solve a puzzle. He records the time it took people to solve the puzzle.

<i>Time taken (mins)</i>	$0 \leq t < 1$	$1 \leq t < 2$	$2 \leq t < 3$	$3 \leq t < 4$	$4 \leq t < 5$
<i>Frequency</i>	5	32	18	7	12

Draw a histogram for the data.

6.5

5. The finishing times for a cross country race were recorded to the nearest minute. Draw a suitable histogram for the data.

23	38	43	47
27	39	43	48
31	39	43	48
32	40	43	48
32	40	44	50
32	40	44	50
33	41	46	51
34	41	46	51
35	42	46	52
37	42	47	53

6. At the end of a football season a newspaper calculated the average number of goals scored per match for 100 top footballers.

2.7	1.4	2.9	1.3	2.1	2.1	2.9	2.1	1.7	3.1
1.2	2.0	0.6	2.2	2.2	2.2	2.7	1.6	1.3	1.9
1.3	1.4	1.9	2.1	1.8	0.9	2.1	2.3	1.9	1.3
1.3	0.3	1.2	2.0	1.4	1.7	2.4	1.9	1.7	1.5
2.7	2.2	0.7	1.5	1.6	1.4	2.7	2.0	1.9	2.7
2.1	1.6	0.7	1.5	2.0	0.9	1.6	1.6	1.5	2.6
1.1	2.2	1.8	1.5	1.8	2.6	0.2	1.2	1.4	1.9
1.7	1.4	2.1	2.2	1.5	2.1	2.4	0.5	0.9	1.4
1.8	0.6	1.9	2.6	1.6	2.1	2.0	1.8	1.3	2.0
1.2	2.2	1.4	2.1	1.2	0.4	2.6	1.9	0.9	2.1

Use the data given to draw a suitable histogram and then draw a frequency polygon on top of the histogram.

7. Two students recorded the time it took drivers of cars to find a space and park in a car park. They also noted if the drivers were male or female.

<i>Time Taken (mins)</i>	$0 < t \leq 2$	$2 < t \leq 4$	$4 < t \leq 6$	$6 < t \leq 8$	$8 < t \leq 10$	$10 < t \leq 12$
<i>Male Drivers</i>	4	11	24	11	3	2
<i>Female Drivers</i>	0	16	21	5	5	8

- (a) On the same set of axes draw frequency polygons for male and female drivers.
- (b) What evidence does the graph provide to support the claim that females take longer to find a space and park than males?

6.5

8. The age distribution in a town is given in the table.  
Draw a histogram to show the data.

<i>Age</i>	<i>Frequency</i>
$0 \leq a < 10$	1800
$10 \leq a < 20$	1500
$20 \leq a < 30$	1450
$30 \leq a < 40$	1600
$40 \leq a < 50$	1250
$50 \leq a < 60$	1150
$60 \leq a < 70$	800
$70 \leq a < 80$	500
$80 \leq a < 90$	150

9. A hire company owns three types of car which are classified as small, medium and large. The distance travelled by each car is always recorded.

<i>Distance (miles)</i>	$0 < t \leq 100$	$100 < t \leq 200$	$200 < t \leq 300$	$300 < t \leq 400$	$400 < t \leq 500$
<i>Small Cars</i>	80	50	30	2	1
<i>Medium Cars</i>	30	45	67	70	10
<i>Large Cars</i>	5	12	16	24	12

- (a) On the same set of axes draw frequency polygons for each type of car.  
 (b) Comment on the graphs you have drawn.
10. A large number of children entered a mathematics competition. Their scores are listed below.

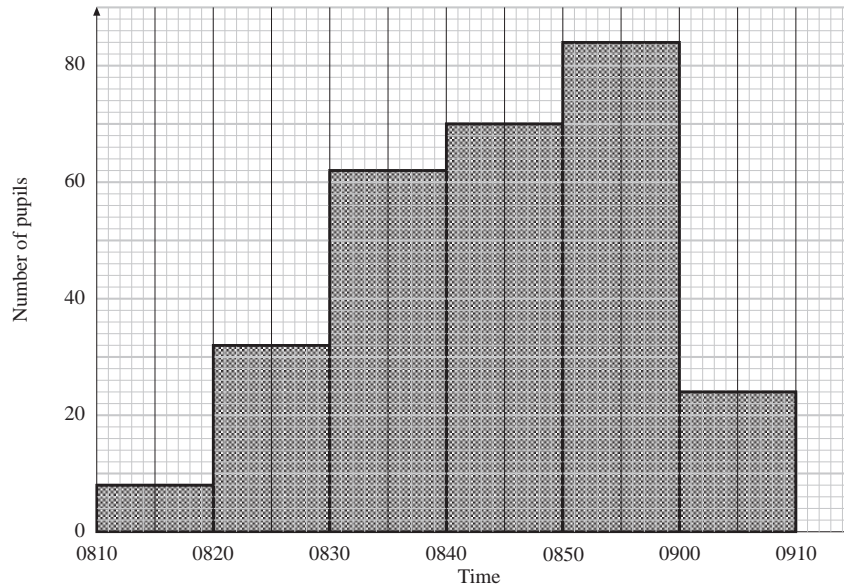
<i>Score</i>	$0 \leq s < 20$	$20 \leq s < 40$	$40 \leq s < 60$	$60 \leq s < 80$	$80 \leq s < 100$
<i>Frequency</i>	82	166	342	220	54

Draw a histogram for the data.



6.5

11. The graph shows the result of a survey of the times at which pupils arrive at school one day.

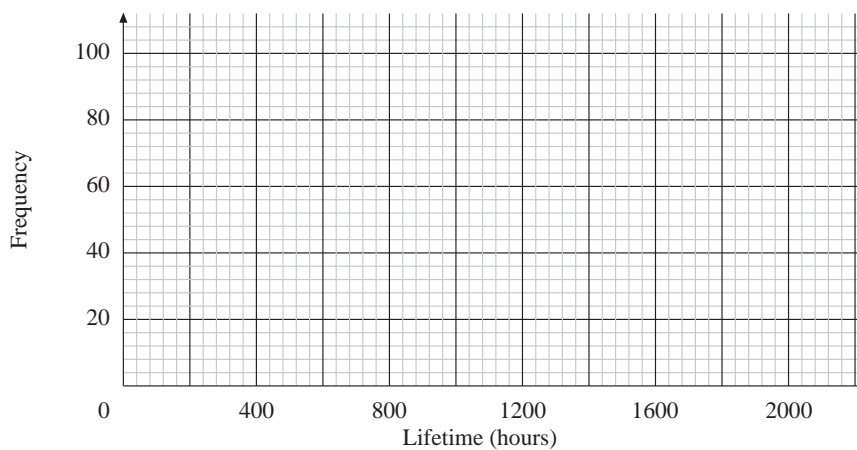


How many pupils arrived for school between 0830 and 0850? (SEG)

12. The table below gives information about the expected lifetimes, in hours, of 200 light bulbs.

Lifetime ( $t$ )	$0 < t \leq 400$	$400 < t \leq 800$	$800 < t \leq 1200$	$1200 < t \leq 1600$	$1600 < t \leq 2000$
Frequency	32	56	90	16	6

- (a) Mr Jones buys one of the light bulbs.
- (i) What is the probability that it will not last more than 400 hours?
  - (ii) What is the probability that it will last at least 800 hours but not more than 1600 hours?
- (b) Using axes similar to those below, draw a frequency polygon to illustrate the information in the table.



(MEG)

6.5

13. The height of some pupils is recorded.

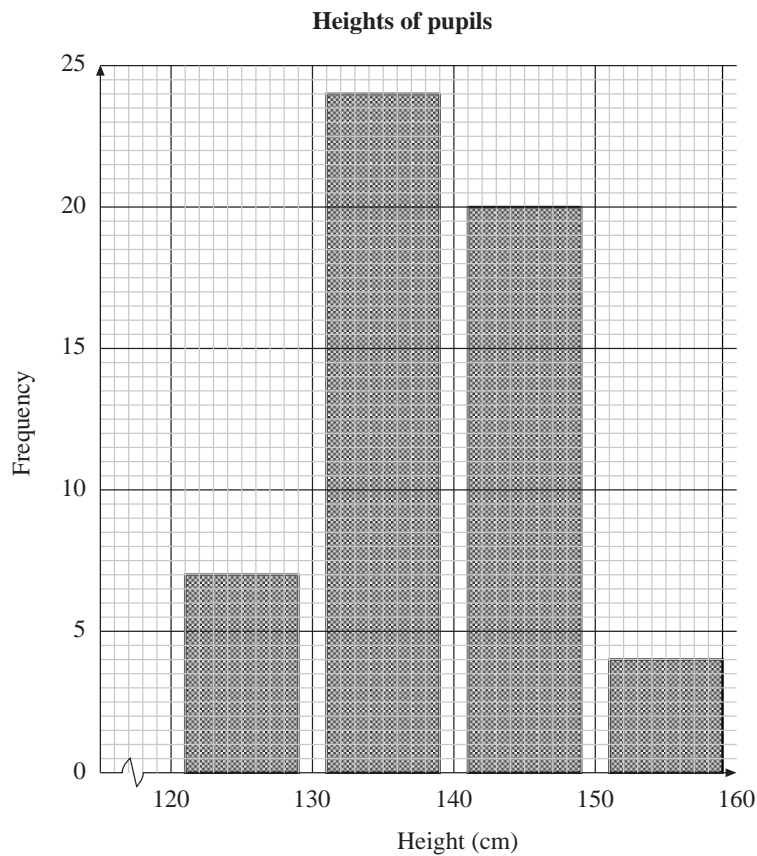
Height $h$ (cm)	Frequency
$120 \leq h < 125$	2
$125 \leq h < 130$	5
$130 \leq h < 135$	8
$135 \leq h < 140$	14
$140 \leq h < 145$	11
$145 \leq h < 150$	9
$150 \leq h < 155$	3
$155 \leq h < 160$	1

Ann records the data using class intervals of 10 cm.

(a) Copy and complete Ann's table.

Height $h$ (cm)	Frequency
$120 \leq h < 130$	
$130 \leq h < 140$	
$140 \leq h < 150$	
$150 \leq h < 160$	

Ann draws a frequency diagram of her data.



6.5

Ann has made two mistakes in drawing her diagram.

(b) What are the two mistakes?

Another pupil is included.

The pupil has a height of 150 cm.

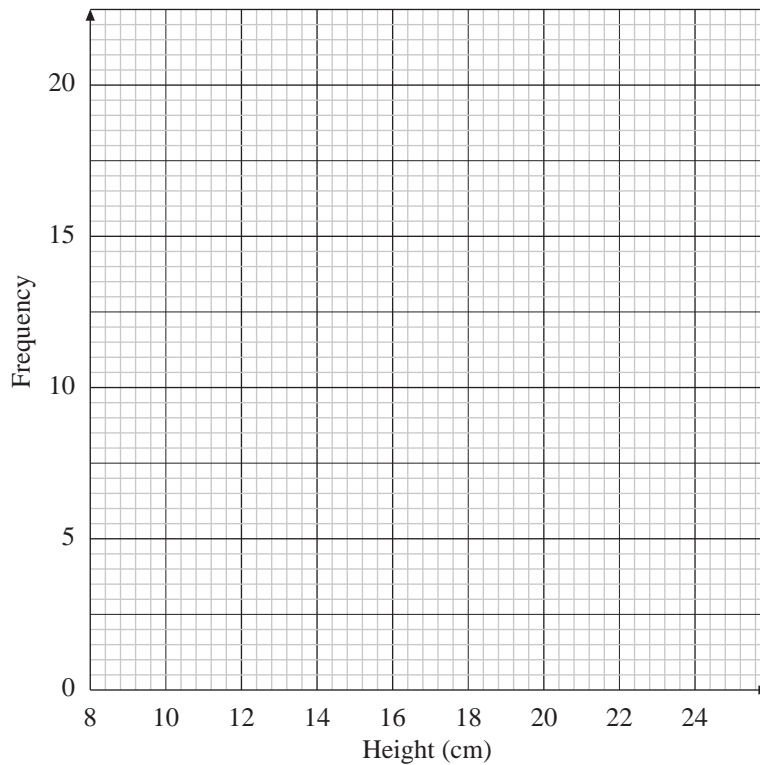
(c) Into which of Ann's class intervals should the pupil be placed?

(SEG)

14. The height of each of 60 plants of type A was measured and recorded.

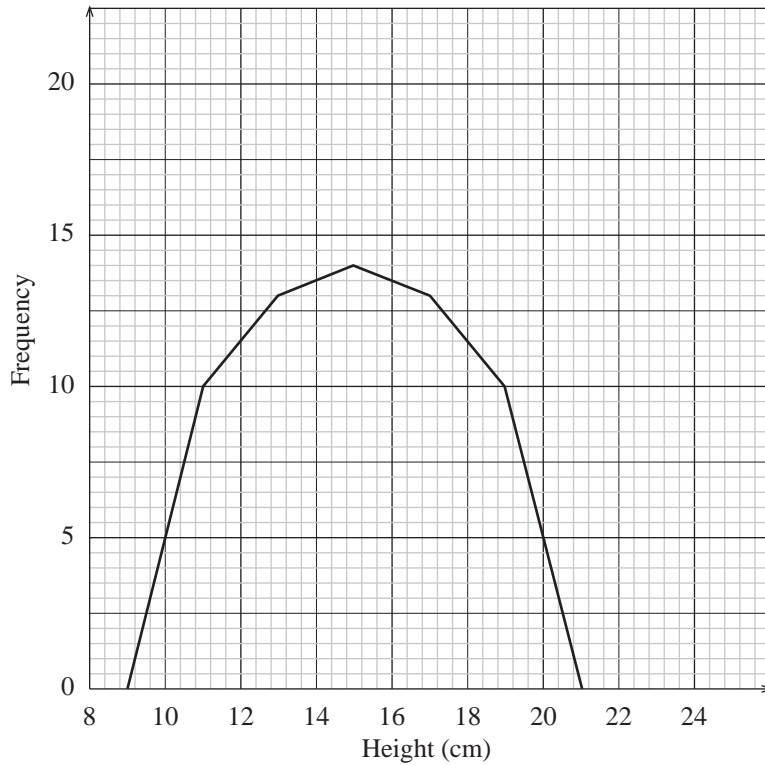
Height of plant (cm)	8—	10—	12—	14—	16—	18—	20-22
Number of plants	0	2	3	18	19	18	0

(a) Draw the frequency polygon of these results on a grid like the one below.



The following graph shows a frequency polygon of 60 plants of type B.

6.5



(b) Write down **two** differences between the two types of plant shown by the frequency polygons.

(SEG)

## 6.6 Mean, Median, Mode and Range

In the first part of this Unit, you have been looking at ways of collecting and representing data. Now you will go one step further and find out how to calculate statistical quantities which summarise the important characteristics of the data.

The *mean*, *median* and *mode* are three different ways of describing the average.

- To find the *mean*, add up all the numbers and divide by the number of numbers.
- To find the *median*, place all the numbers in order and select the middle number.
- The *mode* is the number which appears most often.
- The *range* gives an idea of how the data are spread out and is the difference between the smallest and largest values.



### Worked Example 1

Find

- (a) the mean      (b) the median      (c) the mode      (d) the range  
of this set of data.

5, 6, 2, 4, 7, 8, 3, 5, 6, 6

## 6.6

**Solution**

- (a) The mean is

$$\begin{aligned} & \frac{5+6+2+4+7+8+3+5+6+6}{10} \\ &= \frac{52}{10} \\ &= 5.2. \end{aligned}$$

- (b) To find the median, place all the numbers in order.

2, 3, 4, 5, 5, 6, 6, 6, 7, 8

As there are *two* middle numbers in this example, 5 and 6,

$$\begin{aligned} \text{median} &= \frac{5+6}{2} \\ &= \frac{11}{2} \\ &= 5.5. \end{aligned}$$

- (c) From the list above it is easy to see that 6 appears more than any other number, so

$$\text{mode} = 6.$$

- (d) The range is the difference between the smallest and largest numbers, in this case 2 and 8. So the range is
- $8 - 2 = 6$
- .

**Worked Example 2**

Five people play golf and at one hole their scores are

3, 4, 4, 5, 7.

For these scores, find

- (a) the mean      (b) the median      (c) the mode      (d) the range .

**Solution**

- (a) The mean is

$$\begin{aligned} & \frac{3+4+4+5+7}{5} \\ &= \frac{23}{5} \\ &= 4.6. \end{aligned}$$

## 6.6

- (b) The numbers are already in order and the middle number is 4. So

$$\text{median} = 4.$$

- (c) The score 4 occurs most often, so,

$$\text{mode} = 4.$$

- (d) The range is the difference between the smallest and largest numbers, in this case 3 and 7, so

$$\begin{aligned} \text{range} &= 7 - 3 \\ &= 4. \end{aligned}$$



## Exercises

1. Find the mean median, mode and range of each set of numbers below.

(a) 3, 4, 7, 3, 5, 2, 6, 10

(b) 8, 10, 12, 14, 7, 16, 5, 7, 9, 11

(c) 17, 18, 16, 17, 17, 14, 22, 15, 16, 17, 14, 12

(d) 108, 99, 112, 111, 108

(e) 64, 66, 65, 61, 67, 61, 57

(f) 21, 30, 22, 16, 24, 28, 16, 17

2. Twenty children were asked their shoe sizes. The results are given below.

$$\begin{array}{cccccccccccc} 8, & 6, & 7, & 6, & 5, & 4\frac{1}{2}, & 7\frac{1}{2}, & 6\frac{1}{2}, & 8\frac{1}{2}, & 10 \\ 7, & 5, & 5\frac{1}{2}, & 8, & 9, & 7, & 5, & 6, & 8\frac{1}{2}, & 6 \end{array}$$

For this data, find

- (a) the mean  
 (b) the median  
 (c) the mode  
 (d) the range.

## 6.6

3. Eight people work in a shop. They are paid hourly rates of  
£2, £15, £5, £4, £3, £4, £3, £3.

- (a) Find  
(i) the mean      (ii) the median      (iii) the mode.  
(b) Which average would you use if you wanted to claim that the staff were:  
(i) well paid      (ii) badly paid?  
(c) What is the range?

4. Two people work in a factory making parts for cars. The table shows how many complete parts they make in one week.

<i>Worker</i>	<i>Mon</i>	<i>Tue</i>	<i>Wed</i>	<i>Thu</i>	<i>Fri</i>
Fred	20	21	22	20	21
Harry	30	15	12	36	28

- (a) Find the mean and range for Fred and Harry.  
(b) Who is most consistent?  
(c) Who makes the most parts in a week?
5. A gardener buys 10 packets of seeds from two different companies. Each pack contains 20 seeds and he records the number of plants which grow from each pack.

<i>Company A</i>	20	5	20	20	20	6	20	20	20	8
<i>Company B</i>	17	18	15	16	18	18	17	15	17	18

- (a) Find the mean, median and mode for each company's seeds.  
(b) Which company does the mode suggest is best?  
(c) Which company does the mean suggest is best?  
(d) Find the range for each company's seeds.
6. Adrian takes four tests and scores the following marks.  
65, 72, 58, 77
- (a) What are his median and mean scores?  
(b) If he scores 70 in his next test, does his mean score increase or decrease? Find his new mean score.  
(c) Which has increased most, his mean score or his median score?
7. Richard keeps a record of the number of fish he catches over a number of fishing trips. His records are:  
1, 0, 2, 0, 0, 0, 12, 0, 2, 0, 0, 1, 18, 0, 2, 0, 1.
- (a) Why does he object to talking about the mode and median of the number of fish caught?

6.6

- (b) What are the mean and range of the data?
- (c) Richard's friend, Najir, also goes fishing. The mode of the number of fish he has caught is also 0 and his range is 15.  
What is the largest number of fish that Najir has caught?

8. A garage owner records the number of cars which visit his garage on 10 days. The numbers are:

204, 310, 279, 314, 257, 302, 232, 261, 308, 217.

- (a) Find the mean number of cars per day.
- (b) The owner hopes that the mean will increase if he includes the number of cars on the next day. If 252 cars use the garage on the next day, will the mean increase or decrease?

9. The children in a class state how many children there are in their family. The numbers they state are given below.

1, 2, 1, 3, 2, 1, 2, 4, 2, 2, 1, 3, 1, 2,  
2, 2, 1, 1, 7, 3, 1, 2, 1, 2, 2, 1, 2, 3

- (a) Find the mean, median and mode for this data.
- (b) Which is the most sensible average to use in this case?

10. The mean number of people visiting Jane each day over a five-day period is 8. If 10 people visit Jane the next day, what happens to the mean?

11. The table shows the maximum and minimum temperatures recorded in six cities one day last year.

<i>City</i>	<i>Maximum</i>	<i>Minimum</i>
Los Angeles	22° C	12° C
Boston	22° C	−3° C
Moscow	18° C	−9° C
Atlanta	27° C	8° C
Archangel	13° C	−15° C
Cairo	28° C	13° C

- (a) Work out the range of temperature for Atlanta.
- (b) Which city in the table had the lowest temperature?
- (c) Work out the difference between the maximum temperature and the minimum temperature for Moscow.

(LON)

12. The weights, in grams, of seven potatoes are

260, 225, 205, 240, 232, 205, 214.

What is the median weight?

(SEG)



6.6

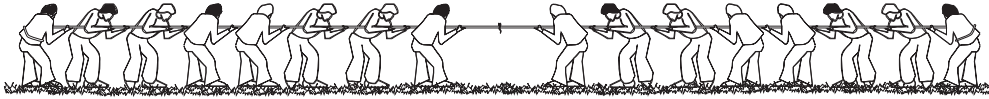
13. Here are the number of goals scored by a school football team in their matches this term.

3, 2, 0, 1, 2, 0, 3, 4, 3, 2

- (a) Work out the mean number of goals.
- (b) Work out the range of the number of goals scored.

(LON)

- 14.



- (a) The weights, in kilograms, of the 8 members of *Hereward House* tug of war team at a school sports are:

75, 73, 77, 76, 84, 76, 77, 78.

Calculate the mean weight of the team.

- (b) The 8 members of *Nelson House* tug of war team have a mean weight of 64 kilograms.

Which team do you think will win a tug of war between *Hereward House* and *Nelson House*? Give a reason for your answer.

(MEG)

15. Pupils in Year 8 are arranged in eleven classes. The class sizes are

23, 24, 24, 26, 27, 28, 30, 24, 29, 24, 27.

- (a) What is the modal class size?
- (b) Calculate the mean class size.

The range of the class sizes for Year 9 is 3.

- (c) What does this tell you about the class sizes in Year 9 compared with those in Year 8?

(SEG)

16. A school has to select one pupil to take part in a General Knowledge Quiz.

Kim and Pat took part in six trial quizzes. The following lists show their scores.

<i>Kim</i>	28	24	21	27	24	26
<i>Pat</i>	33	19	16	32	34	18

Kim had a mean score of 25 with a range of 7.

- (a) Calculate Pat's mean score and range.
- (b) Which pupil would you choose to represent the school? Explain the reason for your choice, referring to the mean scores and ranges.

(MEG)



Information

*The study of statistics was begun by an English mathematician, John Graunt (1620–1674). He collected and studied the death records in various cities in Britain and, despite the fact that people die randomly, he was fascinated by the patterns he found.*

## 6.6

17. Eight judges each give a mark out of 6 in an ice-skating competition.

Oksana is given the following marks.

5.3, 5.7, 5.9, 5.4, 4.5, 5.7, 5.8, 5.7

The mean of these marks is 5.5, and the range is 1.4.

The rules say that the highest mark and the lowest mark are to be deleted.

5.3, 5.7, ~~5.9~~, 5.4, ~~4.5~~, 5.7, 5.8, 5.7

- (a) (i) Find the mean of the six remaining marks.
- (ii) Find the range of the six remaining marks.
- (b) Do you think it is better to count all eight marks, or to count only the six remaining marks? Use the means and the ranges to explain your answer.
- (c) The eight marks obtained by Tonya in the same competition have a mean of 5.2 and a range of 0.6. Explain why none of her marks could be as high as 5.9. (MEG)

## 6.7 Finding the Mean from Tables and Tally Charts

Often data are collected into tables or tally charts. This section considers how to find the mean in such cases.



### Worked Example 1

A football team keep records of the number of goals it scores per match during a season.

<i>No. of Goals</i>	<i>Frequency</i>
0	8
1	10
2	12
3	3
4	5
5	2

Find the mean number of goals per match.



### Solution

The table above can be used, with a third column added.

The mean can now be calculated.

$$\begin{aligned} \text{Mean} &= \frac{73}{40} \\ &= 1.825. \end{aligned}$$

<i>No. of Goals</i>	<i>Frequency</i>	<i>No. of Goals × Frequency</i>
0	8	$0 \times 8 = 0$
1	10	$1 \times 10 = 10$
2	12	$2 \times 12 = 24$
3	3	$3 \times 3 = 9$
4	5	$4 \times 5 = 20$
5	2	$5 \times 2 = 10$
<b>TOTALS</b>	<b>40</b>	<b>73</b>

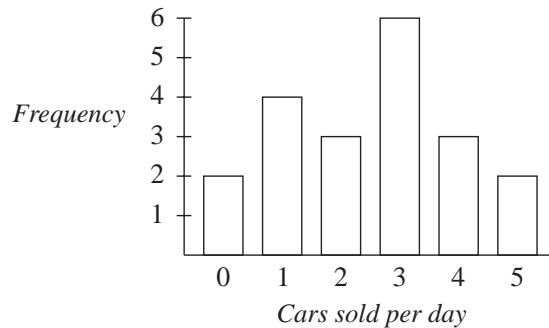
(Total matches) (Total goals)

6.7



### Worked Example 2

The bar chart shows how many cars were sold by a salesman over a period of time.



Find the mean number of cars sold per day.



### Solution

The data can be transferred to a table and a third column included as shown.

<i>Cars sold daily</i>	<i>Frequency</i>	<i>Cars sold × Frequency</i>
0	2	$0 \times 2 = 0$
1	4	$1 \times 4 = 4$
2	3	$2 \times 3 = 6$
3	6	$3 \times 6 = 18$
4	3	$4 \times 3 = 12$
5	2	$5 \times 2 = 10$
<b>TOTALS</b>	<b>20</b>	<b>50</b>

(Total days)                      (Total number of cars sold)

$$\begin{aligned} \text{Mean} &= \frac{50}{20} \\ &= 2.5 \end{aligned}$$



### Worked Example 3

A police station kept records of the number of road traffic accidents in their area each day for 100 days. The figures below give the number of accidents per day.

1	4	3	5	5	2	5	4	3	2	0	3	1	2	2	3	0	5	2	1
3	3	2	6	2	1	6	1	2	2	3	2	2	2	2	5	4	4	2	3
3	1	4	1	7	3	3	0	2	5	4	3	3	4	3	4	5	3	5	2
4	4	6	5	2	4	5	5	3	2	0	3	3	4	5	2	3	3	4	4
1	3	5	1	1	2	2	5	6	6	4	6	5	8	2	5	3	3	5	4

Find the mean number of accidents per day.

6.7



**Solution**

The first step is to draw out and complete a tally chart. The final column shown below can then be added and completed.

<i>Number of Accidents</i>	<i>Tally</i>	<i>Frequency</i>	<i>No. of Accidents × Frequency</i>
0		4	0 × 4 = 0
1		10	1 × 10 = 10
2		22	2 × 22 = 44
3		23	3 × 23 = 69
4		16	4 × 16 = 64
5		17	5 × 17 = 85
6		6	6 × 6 = 36
7		1	7 × 1 = 7
8		1	8 × 1 = 8
<b>TOTALS</b>		<b>100</b>	<b>323</b>

$$\text{Mean number of accidents per day} = \frac{323}{100} = 3.23.$$



**Exercises**

1. A survey of 100 households asked how many cars there were in each household. The results are given below.

<i>No. of Cars</i>	<i>Frequency</i>
0	5
1	70
2	21
3	3
4	1

Calculate the mean number of cars per household.

2. The survey of question 1 also asked how many TV sets there were in each household. The results are given below.

<i>No. of TV Sets</i>	<i>Frequency</i>
0	2
1	30
2	52
3	8
4	5
5	3

Calculate the mean number of TV sets per household.

6.7

3. A manager keeps a record of the number of calls she makes each day on her mobile phone.

<i>Number of calls per day</i>	0	1	2	3	4	5	6	7	8
<i>Frequency</i>	3	4	7	8	12	10	14	3	1

Calculate the mean number of calls per day.

4. A cricket team keeps a record of the number of runs scored in each over.

<i>No. of Runs</i>	<i>Frequency</i>
0	3
1	2
2	1
3	6
4	5
5	4
6	2
7	1
8	1

Calculate the mean number of runs per over.

5. A class conduct an experiment in biology. They place a number of 1m by 1m square grids on the playing field and count the number of worms which appear when they pour water on the ground. The results obtained are given below.

6	3	2	1	3	2	1	3	0	1
0	3	2	1	1	4	0	1	2	0
1	1	2	2	2	4	3	1	1	1
2	3	3	1	2	2	2	1	7	1

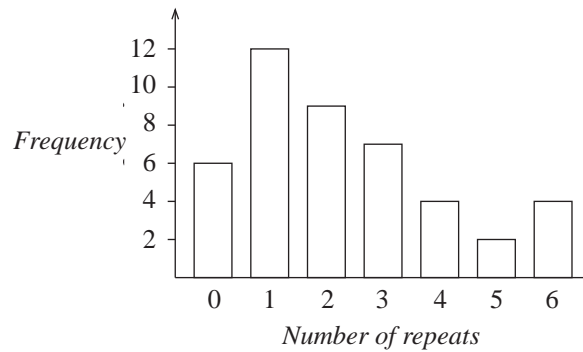
- (a) Calculate the mean number of worms.  
 (b) How many times was the number of worms seen greater than the mean?
6. As part of a survey, a station recorded the number of trains which were late each day. The results are listed below.

0	1	2	4	1	0	2	1	1	0
1	2	1	3	1	0	0	0	0	5
2	1	3	2	0	1	0	1	2	1
1	0	0	3	0	1	2	1	0	0

Construct a table and calculate the mean number of trains which were late each day.

6.7

7. Hannah drew this bar chart to show the number of repeated cards she got when she opened packets of football stickers.



Calculate the mean number of repeats per packet.

8. In a season a football team scored a total of 55 goals. The table below gives a summary of the number of goals per match.

Goals per Match	Frequency
0	4
1	6
2	
3	8
4	2
5	1

- (a) In how many matches did they score 2 goals?  
 (b) Calculate the mean number of goals per match.
9. A traffic warden is trying to work out the mean number of parking tickets he has issued per day. He produced the table below, but has accidentally rubbed out some of the numbers.

Tickets per day	Frequency	No. of Tickets $\times$ Frequency
0	1	●
1	●	1
2	10	●
3	7	●
4	●	20
5	2	●
6	●	●
<b>TOTALS</b>	<b>26</b>	<b>72</b>

Fill in the missing numbers and calculate the mean.

## 6.7

10. Here are the weights, in kg, of 30 students.

45, 52, 56, 65, 34, 45, 67, 65, 34, 45, 65, 87, 45, 34, 56,  
54, 45, 67, 84, 45, 67, 45, 56, 76, 57, 84, 35, 64, 58, 60

- (a) Copy and complete the frequency table below using a class interval of 10 and starting at 30.

<i>Weight Range (w)</i>	<i>Tally</i>	<i>Frequency</i>
$30 \leq w < 40$		

- (b) Which class interval has the highest frequency?

(LON)

11. The number of children per family in a recent survey of 21 families is shown.

1	2	3	2	2	4	2	2
3	2	2	2	3	2	2	2
4	1	2	3	2			

- (a) What is the range in the number of children per family?  
 (b) Calculate the mean number of children per family. *Show your working.*

A similar survey was taken in 1960.

In 1960 the range in the number of children per family was 7 and the mean was 2.7.

- (c) Describe **two** changes that have occurred in the number of children per family since 1960.

(SEG)

## 6.8 Calculations with the Mean

This section considers calculations concerned with the mean, which is usually taken to be the most important measure of the average of a set of data.



### Worked Example 1

The mean of a sample of 6 numbers is 3.2. An extra value of 3.9 is included in the sample. What is the new mean?

6.8



## Solution

$$\begin{aligned}\text{Total of original numbers} &= 6 \times 3.2 \\ &= 19.2\end{aligned}$$

$$\begin{aligned}\text{New total} &= 19.2 + 3.9 \\ &= 23.1\end{aligned}$$

$$\begin{aligned}\text{New mean} &= \frac{23.1}{7} \\ &= 3.3\end{aligned}$$



## Worked Example 2

The mean number of a set of 5 numbers is 12.7. What extra number must be added to bring the mean up to 13.1?



## Solution

$$\begin{aligned}\text{Total of the original numbers} &= 5 \times 12.7 \\ &= 63.5\end{aligned}$$

$$\begin{aligned}\text{Total of the new numbers} &= 6 \times 13.1 \\ &= 78.6\end{aligned}$$

$$\begin{aligned}\text{Difference} &= 78.6 - 63.5 \\ &= 15.1\end{aligned}$$

So the extra number is 15.1.



## Exercises

1. The mean height of a class of 28 students is 162 cm. A new girl of height 149 cm joins the class. What is the mean height of the class now?
2. After 5 matches the mean number of goals scored by a football team per match is 1.8. If they score 3 goals in their 6th match, what is the mean after the 6th match?
3. The mean number of children ill at a school is 3.8 per day, for the first 20 school days of a term. On the 21st day 8 children are ill. What is the mean after 21 days?
4. The mean weight of 25 children in a class is 58 kg. The mean weight of a second class of 29 children is 62 kg. Find the mean weight of all the children.
5. A salesman sells a mean of 4.6 conservatories per day for 5 days. How many must he sell on the sixth day to increase his mean to 5 sales per day?
6. Adrian's mean score for four tests is 64%. He wants to increase his mean to 68% after the fifth test. What does he need to score in the fifth test?
7. The mean salary of the 8 people who work for a small company is £15 000. When an extra worker is taken on this mean drops to £14 000. How much does the new worker earn?



6.8

8. The mean of 6 numbers is 12.3. When an extra number is added, the mean changes to 11.9. What is the extra number?
9. When 5 is added to a set of 3 numbers the mean increases to 4.6. What was the mean of the original 3 numbers?
10. Three numbers have a mean of 64. When a fourth number is included the mean is doubled. What is the fourth number?

## 6.9 Mean, Median and Mode for Grouped Data

The mean and median can be estimated from tables of *grouped* data.

The class interval which contains the most values is known as the *modal class*.



### Worked Example 1

The table below gives data on the heights, in cm, of 51 children.

<i>Class Interval</i>	$140 \leq h < 150$	$150 \leq h < 160$	$160 \leq h < 170$	$170 \leq h < 180$
<i>Frequency</i>	6	16	21	8

- (a) Estimate the mean height.
- (b) Estimate the median height.
- (c) Find the modal class.



### Solution

- (a) To estimate the mean, the mid-point of each interval should be used.

<i>Class Interval</i>	<i>Mid-point</i>	<i>Frequency</i>	<i>Mid-point <math>\times</math> Frequency</i>
$140 \leq h < 150$	145	6	$145 \times 6 = 870$
$150 \leq h < 160$	155	16	$155 \times 16 = 2480$
$160 \leq h < 170$	165	21	$165 \times 21 = 3465$
$170 \leq h < 180$	175	8	$175 \times 8 = 1400$
<b>Totals</b>		<b>51</b>	<b>8215</b>

$$\begin{aligned} \text{Mean} &= \frac{8215}{51} \\ &= 161 \text{ (to the nearest cm)} \end{aligned}$$

- (b) The median is the 26th value. In this case it lies in the  $160 \leq h < 170$  class interval. The 4th value in the interval is needed. It is estimated as

$$160 + \frac{4}{21} \times 10 = 162 \text{ (to the nearest cm)}$$

- (c) The modal class is  $160 \leq h < 170$  as it contains the most values.

## 6.9

Also note that when we speak of someone by age, say 8, then the person could be any age from 8 years 0 days up to 8 years 364 days (365 in a leap year!). You will see how this is tackled in the following example.



### Worked Example 2

The age of children in a primary school were recorded in the table below.

<i>Age</i>	5 – 6	7 – 8	9 – 10
<i>Frequency</i>	29	40	38

- (a) Estimate the mean.      (b) Estimate the median.      (c) Find the modal age.



### Solution

- (a) To estimate the mean, we must use the mid-point of each interval; so, for example for '5 – 6', which really means

$$5 \leq \text{age} < 7,$$

the mid-point is taken as 6.

<i>Class Interval</i>	<i>Mid-point</i>	<i>Frequency</i>	<i>Mid-point</i> $\times$ <i>Frequency</i>
5 – 6	6	29	$6 \times 29 = 174$
7 – 8	8	40	$8 \times 40 = 320$
9 – 10	10	38	$10 \times 38 = 380$
<b>Totals</b>		<b>107</b>	<b>874</b>

$$\begin{aligned} \text{Mean} &= \frac{874}{107} \\ &= 8.2 \text{ (to 1 decimal place)} \end{aligned}$$

- (b) The median is given by the 54th value, which we have to estimate. There are 29 values in the first interval, so we need to estimate the 25th value in the second interval. As there are 40 values in the second interval, the median is estimated as being

$$\frac{25}{40}$$

of the way along the second interval. This has width  $9 - 7 = 2$  years, so the median is estimated by

$$\frac{25}{40} \times 2 = 1.25$$

from the start of the interval. Therefore the median is estimated as

$$7 + 1.25 = 8.25 \text{ years.}$$

- (c) The modal age is the 7 – 8 age group.

6.9

Worked Example 1 uses what are called *continuous data*, since height can be of any value. (Other examples of continuous data are weight, temperature, area, volume and time.)

The next example uses *discrete data*, that is, data which can take only a particular value, such as the integers 1, 2, 3, 4, . . . in this case.

The calculations for mean and mode are not affected but estimation of the median requires replacing the *discrete* grouped data with an approximate *continuous* interval.



### Worked Example 3

The number of days that children were missing from school due to sickness in one year was recorded.

<i>Number of days off sick</i>	1 – 5	6 – 10	11 – 15	16 – 20	21 – 25
<i>Frequency</i>	12	11	10	4	3

- (a) Estimate the mean      (b) Estimate the median.      (c) Find the modal class.



### Solution

- (a) The estimate is made by assuming that all the values in a class interval are equal to the midpoint of the class interval.

<i>Class Interval</i>	<i>Mid-point</i>	<i>Frequency</i>	<i>Mid-point × Frequency</i>
1–5	3	12	$3 \times 12 = 36$
6–10	8	11	$8 \times 11 = 88$
11–15	13	10	$13 \times 10 = 130$
16–20	18	4	$18 \times 4 = 72$
21–25	23	3	$23 \times 3 = 69$
<b>Totals</b>		<b>40</b>	<b>395</b>

$$\begin{aligned} \text{Mean} &= \frac{395}{40} \\ &= 9.925 \text{ days.} \end{aligned}$$

- (b) As there are 40 pupils, we need to consider the mean of the 20th and 21st values. These both lie in the 6–10 class interval, which is really the 5.5–10.5 class interval, so this interval contains the median.

As there are 12 values in the first class interval, the median is found by considering the 8th and 9th values of the second interval.

As there are 11 values in the second interval, the median is estimated as being

$$\frac{8.5}{11}$$

of the way along the second interval.

6.9

But the length of the second interval is  $10.5 - 5.5 = 5$ , so the median is estimated by

$$\frac{8.5}{11} \times 5 = 3.86$$

from the start of this interval. Therefore the median is estimated as

$$5.5 + 3.86 = 9.36.$$

- (c) The modal class is 1–5, as this class contains the most entries.



## Exercises

1. A door to door salesman keeps a record of the number of homes he visits each day.

<i>Homes visited</i>	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49
<i>Frequency</i>	3	8	24	60	21

- (a) Estimate the mean number of homes visited.  
 (b) Estimate the median.  
 (c) What is the modal class?
2. The weights of a number of students were recorded in kg.

<i>Mean (kg)</i>	$30 \leq w < 35$	$35 \leq w < 40$	$40 \leq w < 45$	$45 \leq w < 50$	$50 \leq w < 55$
<i>Frequency</i>	10	11	15	7	4

- (a) Estimate the mean weight.                      (b) Estimate the median.  
 (c) What is the modal class?
3. A stopwatch was used to find the time that it took a group of children to run 100 m.

<i>Time (seconds)</i>	$10 \leq t < 15$	$15 \leq t < 20$	$20 \leq t < 25$	$25 \leq t < 30$
<i>Frequency</i>	6	16	21	8

- (a) Is the median in the modal class?      (b) Estimate the mean.  
 (c) Estimate the median.  
 (d) Is the median greater or less than the mean?
4. The distances that children in a year group travelled to school is recorded.

<i>Distance (km)</i>	$0 \leq d < 0.5$	$0.5 \leq d < 1.0$	$1.0 \leq d < 1.5$	$1.5 \leq d < 2.0$
<i>Frequency</i>	30	22	19	8

- (a) Does the modal class contain the median?  
 (b) Estimate the median and the mean.  
 (c) Which is the largest, the median or the mean?

## 6.9

5. The ages of the children at a youth camp are summarised in the table below.

<i>Age (years)</i>	6 – 8	9 – 11	12 – 14	15 – 17
<i>Frequency</i>	8	22	29	5

Estimate the mean age of the children.

6. The lengths of a number of leaves collected for a project are recorded.

<i>Length (cm)</i>	2 – 5	6 – 10	11 – 15	16 – 25
<i>Frequency</i>	8	20	42	12

Estimate (a) the mean (b) the median length of a leaf.

7. The table shows how many nights people spend at a campsite.

<i>Number of nights</i>	1 – 5	6 – 10	11 – 15	16 – 20	21 – 25
<i>Frequency</i>	20	26	32	5	2

- (a) Estimate the mean. (b) Estimate the median.  
 (c) What is the modal class?
8. (a) A teacher notes the number of correct answers given by a class on a multiple-choice test.

<i>Correct answers</i>	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50
<i>Frequency</i>	2	8	15	11	3

- (i) Estimate the mean. (ii) Estimate the median.  
 (iii) What is the modal class?
- (b) Another class took the same test. Their results are given below.

<i>Correct answers</i>	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50
<i>Frequency</i>	3	14	20	2	1

- (i) Estimate the mean. (ii) Estimate the median.  
 (iii) What is the modal class?
- (c) How do the results for the two classes compare?



### Information

A **quartile** is one of 3 values (lower quartile, median and upper quartile) which divides data into 4 equal groups.

A **percentile** is one of 99 values which divides data into 100 equal groups.

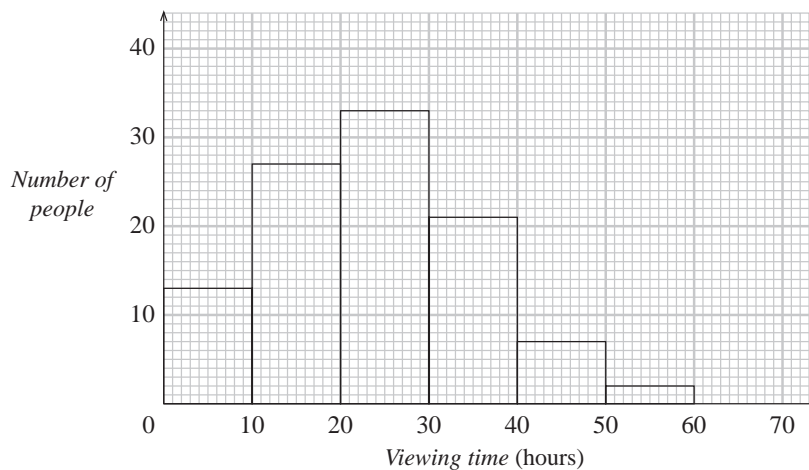
The **lower quartile** corresponds to the 25th percentile. The **median** corresponds to the 50th percentile. The **upper quartile** corresponds to the 75th percentile.

6.9

9. 29 children are asked how much pocket money they were given last week. Their replies are shown in this frequency table.

<i>Pocket money</i> £	<i>Frequency</i> <i>f</i>
0 – £1.00	12
£1.01 – £2.00	9
£2.01 – £3.00	6
£3.01 – £4.00	2

- (a) Which is the modal class?  
 (b) Calculate an estimate of the mean amount of pocket money received per child. (NEAB)
10. The graph shows the number of hours a sample of people spent viewing television one week during the summer.



- (a) Copy and complete the frequency table for this sample.

<i>Viewing time</i> ( <i>h</i> hours)	<i>Number of</i> <i>people</i>
$0 \leq h < 10$	13
$10 \leq h < 20$	27
$20 \leq h < 30$	33
$30 \leq h < 40$	
$40 \leq h < 50$	
$50 \leq h < 60$	

- (b) Another survey is carried out during the winter. State **one** difference you would expect to see in the data.  
 (c) Use the mid-points of the class intervals to calculate the mean viewing time for these people. You may find it helpful to use the table below.

6.9

<i>Viewing time (h hours)</i>	<i>Mid-point</i>	<i>Frequency</i>	<i>Mid-point × Frequency</i>
$0 \leq h < 10$	5	13	65
$10 \leq h < 20$	15	27	405
$20 \leq h < 30$	25	33	825
$30 \leq h < 40$	35		
$40 \leq h < 50$	45		
$50 \leq h < 60$	55		

(SEG)

11. In an experiment, 50 people were asked to estimate the length of a rod to the nearest centimetre. The results were recorded.

<i>Length (cm)</i>	20	21	22	23	24	25	26	27	28	29
<i>Frequency</i>	0	4	6	7	9	10	7	5	2	0

- (a) Find the value of the median.                      (b) Calculate the mean length.
- (c) In a second experiment another 50 people were asked to estimate the length of the same rod. The most common estimate was 23 cm. The range of the estimates was 13 cm.

Make two comparisons between the results of the two experiments.

(SEG)

12. The following list shows the maximum daily temperature, in °F, throughout the month of April.

56.1	49.4	63.7	56.7	55.3	53.5	52.4	57.6	59.8	52.1
45.8	55.1	42.6	61.0	61.9	60.2	57.1	48.9	63.2	68.4
55.5	65.2	47.3	59.1	53.6	52.3	46.9	51.3	56.7	64.3

- (a) Copy and complete the grouped frequency table below.

<i>Temperature, T</i>	<i>Frequency</i>
$40 < T \leq 50$	
$50 < T \leq 54$	
$54 < T \leq 58$	
$58 < T \leq 62$	
$62 < T \leq 70$	

- (b) Use the table of values in part (a) to calculate an estimate of the mean of this distribution. *You must show your working clearly.*
- (c) Draw a histogram to represent your distribution in part (a).

(MEG)

# Answers to Exercises

## 6.1 Tables and Timetables

1. (a) 0800 (b) 0923  
 (c) He should catch the next train and get off at Exeter St. Davids and walk.  
 (d) For each journey - catch the 0723 from Paignton and change at either Newton Abbot or Exeter St. Davids - arriving at Bristol at 0932 or London at 1110.
2. (a) 2106 (b) 1 hr, 39 mins. (c) Yes  
 (d) The 2027 train from Reading.
3. (a) No (b) 1235 (c) 1141
4. (a) (i) 109 miles (ii) 34 miles (iii) 78 miles (b) (i) 149 miles  
 (ii) The travelling distance would be reduced by 29 miles because Manchester is "en route" travelling from Birmingham to Leeds.  
 (c) The route via Sheffield.
5. (a) St. Malo (b) 3984 km (c) 2781 km  
 (d) Quimper is closest to St. Malo, and Ile de Re is closest to Calais.
6. (a) 9 (b) 11 (c) "D" grade.
7. (a) 7 (b) 6 (c) Years 7, 8, 9 and 11  
 (d) (i) Year 10 (ii) 43 students
8. (a) £305 (b) £156 (c) £276 (d) (i) £260 (ii) £45

9. (a)

	<i>Male</i>	<i>Female</i>	<i>Total</i>
<i>Standard</i>	64	48	112
<i>Senior</i>	20	8	28
<i>Total</i>	84	56	140

- (b) Men are more likely to become senior conductors than women.
10. (a) 51 (b) 11  
 (c) 52 outdoor and 50 indoor shows a roughly even split (only marginally in favour of outdoor sports)
  11. (a) 4 (b) 19  
 (c) People seem to achieve better English results than French.
  12. (a) 31 (b) 90 (c) 30%

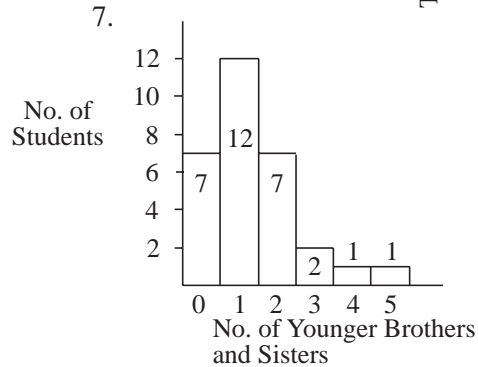
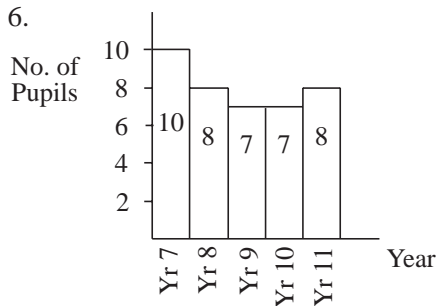
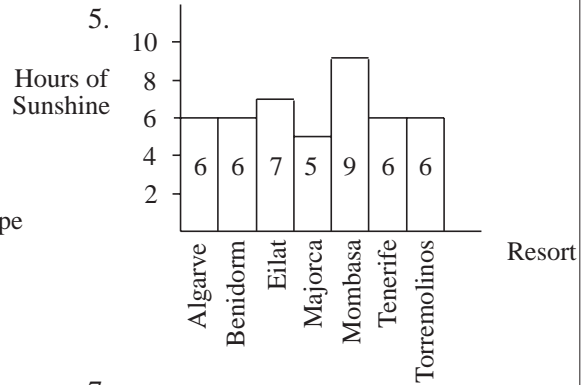
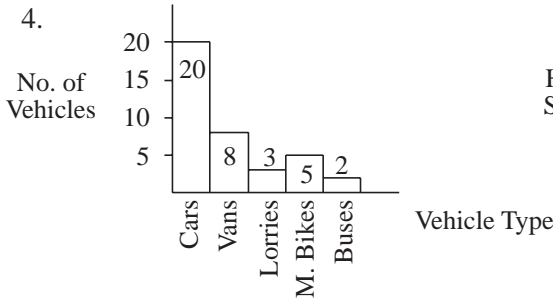
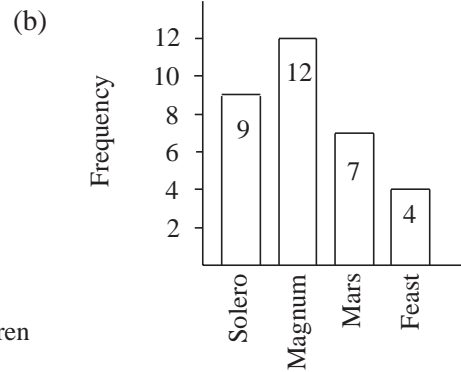
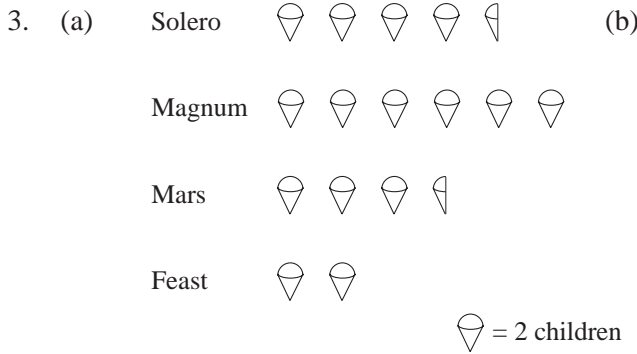
## 6.2 Pictograms and Bar Charts

1. (a) 1996 (b) (i) 10 (ii) 7 (iii) 12 (c) 1995
2. (a) 400 (b) 250 (c) 700 (d)  $5\frac{1}{2}$  (e) 3300



# Answers

6.2



8. (a) (i) 90% (ii) About 97% (b) (i) 0% (ii) About 22%  
 (c) The percentage of households with some sort of TV hasn't changed very much, but there has been an increase in the number of homes receiving satellite.

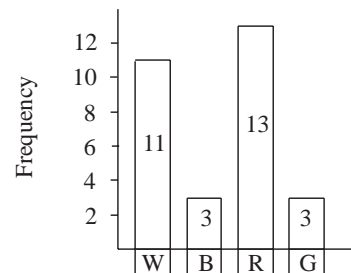
9. (a) 50 (b) 35

10. (a) 23 (b) We would expect there to be many more boys with shoe sizes around 8 and 9 than for 5 or 12, so the results are surprising.

11. (a) £14 (b) £375 (c) 16 coins  
 (d) Only whole coins are used, and so the number is rounded up/down.

12.

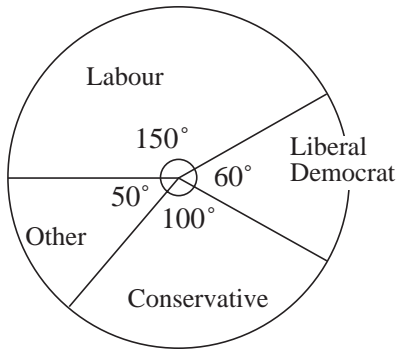
Colour	Tally	Frequency
White	<del>    </del>     1	11
Blue		3
Red	<del>    </del> <del>    </del>	13
Green		3



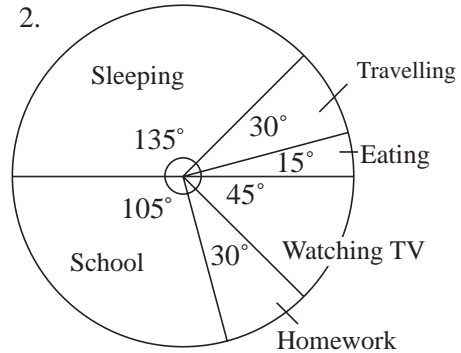
# Answers

## 6.3 Pie Charts

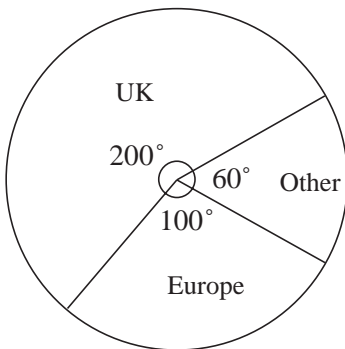
1.



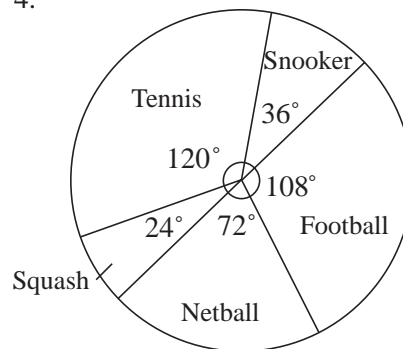
2.



3.



4.



5. (a) 2 hours (b) 5 hours

6. (a) £10 (b) £15 (c) £35

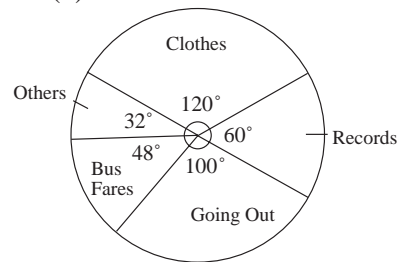
7. (a) 96° (b) 6 (c) 30 (d) 13

8. Airmail = 50, 1st class = 320, 2nd class = 350

9. (a)

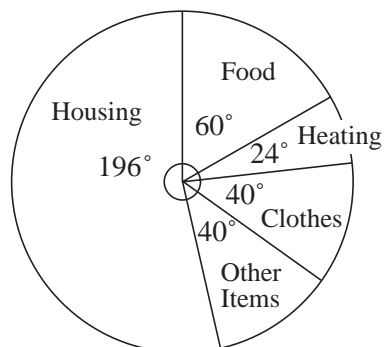
Items	Angle of Sector
Bus fares	48°
Going out	100°
Clothes	120°
Records	60°
Others	32°
Total of angles	360°

(b)



(c)  $\frac{1}{3}$

10. (a)



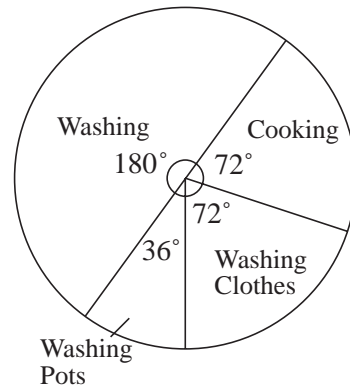
(b) Arthur spends more money on housing.

## Answers

### 6.3

11. (a) (i) 81 litres (ii)  $\frac{15}{100} = \frac{3}{20}$

(b)



12. (a) (i) Five hundred thousand (ii) 200 000

(b) (i) 39% (ii) Over 21 year olds. (iii) 0.61 or  $\frac{61}{100}$

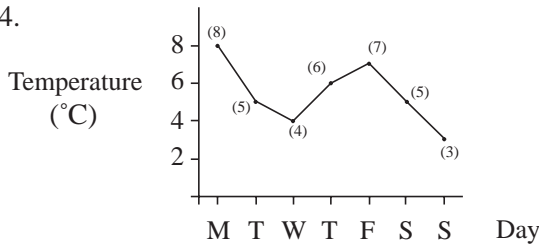
### 6.4 Line Graphs

1. (a) 4 cm (b) April (c) February & December (d) July & August

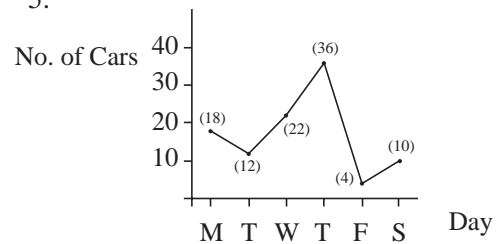
2. (a) 40°C (b) 80°C (c) 20 mins after filling the mug. (d) 25 mins

3. (a) 8 cm (b) 22 cm (c) 84 cm (d) 3 weeks

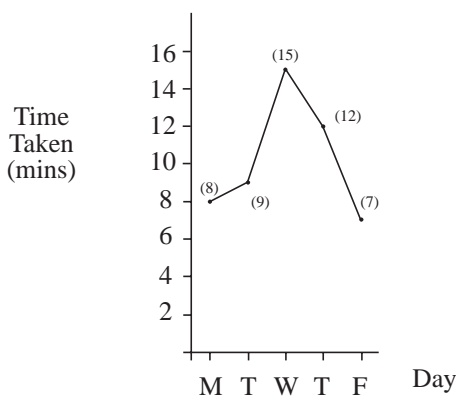
4.



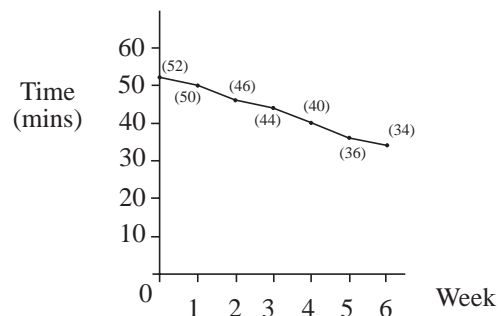
5.



6.



7.



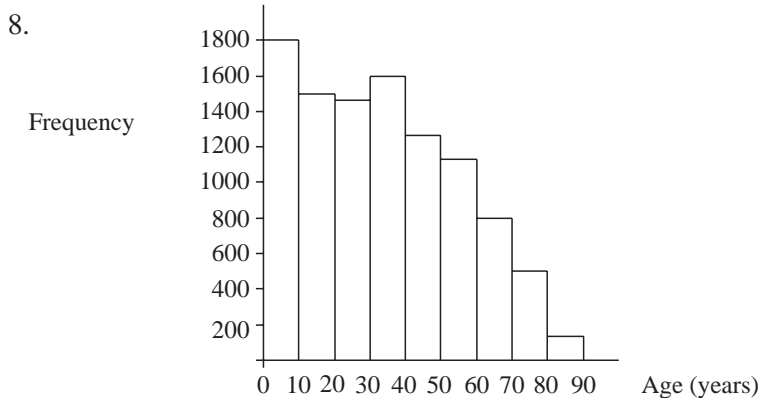
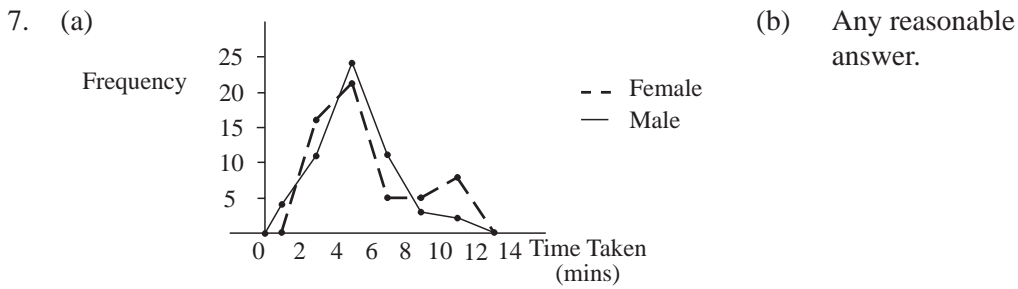
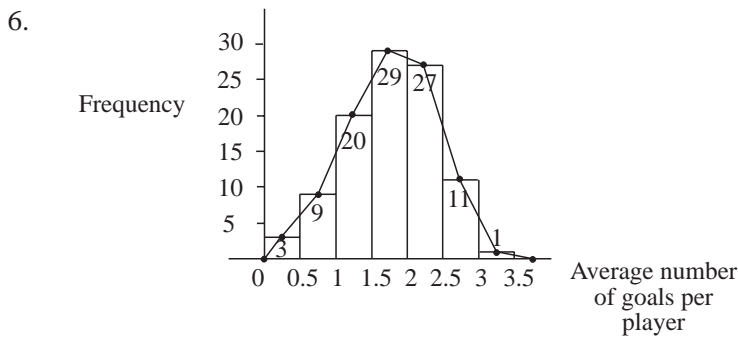
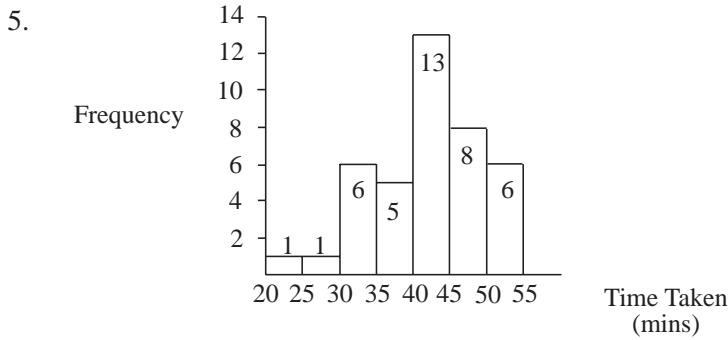
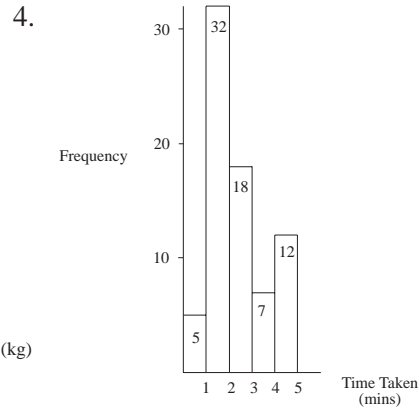
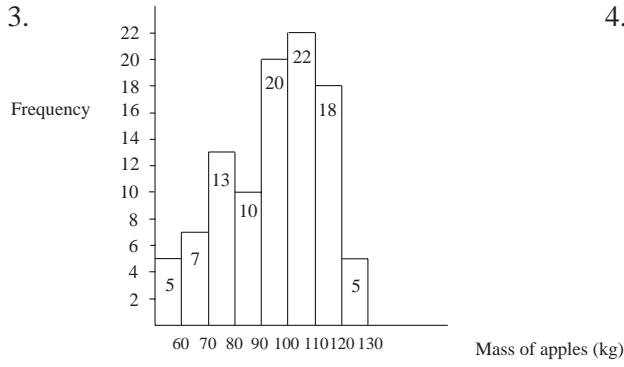
### 6.5 Frequency Graphs

1. (a) 5 (b) 55 (c) 15 (d) 81 pupils in year group.

2. (a) 10 (b) 49 (c) 73 (d) largest = £549.99, smallest = £50.

# Answers

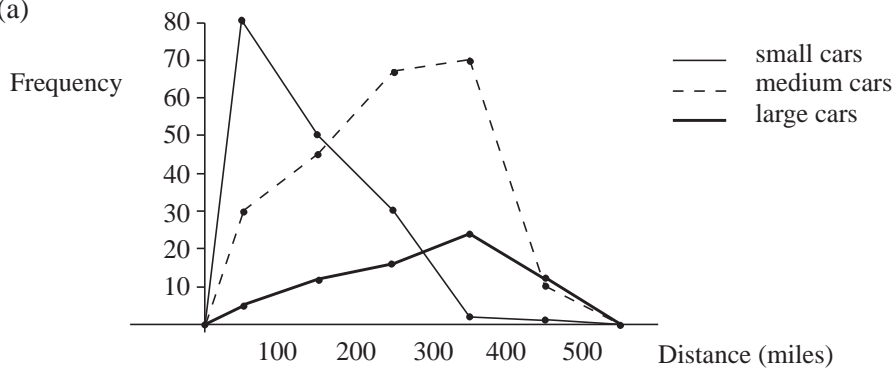
6.5



# Answers

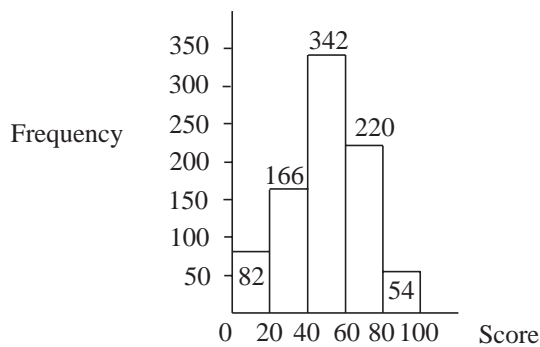
6.5

9. (a)



- (b) There is a downward trend in the small cars, and an upward trend in the medium and large cars.  
 i.e. People use small cars for short journeys and the larger ones for long distances.

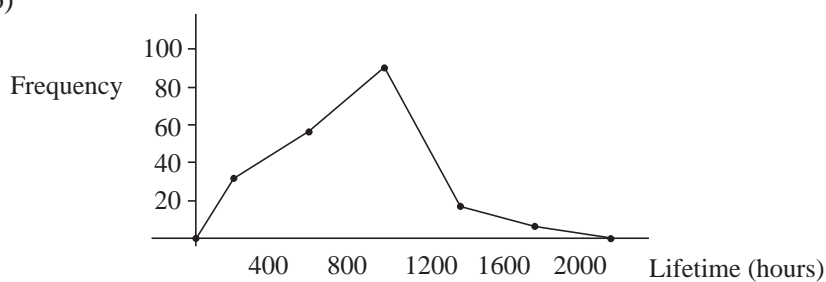
10.



11.  $62 + 70 = 132$  pupils.

12. (a) (i)  $\text{prob.} = \frac{32}{200} = 0.16$  (ii)  $\text{prob.} = \frac{106}{200} = 0.53$

(b)



13. (a)

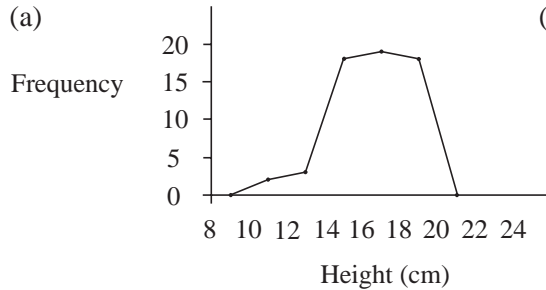
Height	Frequency
120 - 130	7
130 - 140	22
140 - 150	20
150 - 160	4

- (b) The columns should be adjacent (no gaps).  
 The second column ( $130 \leq h < 140$ ) should be 22 not 24.  
 (c) The last category.

## Answers

6.5

14. (a)



(b) Type B has an even spread of plants between mean = 5 and 22, whereas Type A has more taller plants.

The maximum number interval is 14 for Type B plants and 19 for Type A plants.

## 6.6 Mean, Median, Mode and Range

- |                  |               |            |            |
|------------------|---------------|------------|------------|
| (a) mean = 5     | median = 4.5  | mode = 3   | range = 8  |
| (b) mean = 9.9   | median = 9.5  | mode = 7   | range = 11 |
| (c) mean = 16.25 | median = 16.5 | mode = 17  | range = 10 |
| (d) mean = 107.6 | median = 108  | mode = 108 | range = 13 |
| (e) mean = 63    | median = 64   | mode = 61  | range = 10 |
| (f) mean = 21.75 | median = 21.5 | mode = 16  | range = 14 |
- |         |          |       |                    |
|---------|----------|-------|--------------------|
| (a) 6.8 | (b) 6.75 | (c) 6 | (d) $5\frac{1}{2}$ |
|---------|----------|-------|--------------------|
- |                 |           |          |
|-----------------|-----------|----------|
| (a) (i) 4.875   | (ii) 3.5  | (iii) £3 |
| (b) (i) mean    | (ii) mode |          |
| (c) range = £13 |           |          |
- |                                   |                                 |
|-----------------------------------|---------------------------------|
| (a) Fred: mean = 20.8, range = 2. | Harry: mean = 24.2, range = 24. |
| (b) Fred                          | (c) Harry                       |
- |  |                     |
|--|---------------------|
| (a) A: mean = 15.9, median = 20, mode = 20 |                     |
| B: mean = 16.9, median = 17, mode = 17     |                     |
| (b) Mode suggests A                        | (c) Mean suggests B |
| (d) Range: A = 15, B = 3                   |                     |
- |   |                                 |
|---|---------------------------------|
| (a) mean = 68, median = 68.5  | (b) His mean increases to 68.4. |
| (c) The median. It increases from 68.5 to 70, whereas the mean increases by only 0.4. |                                 |
- |   |                             |
|---|-----------------------------|
| (a) He objects because the mode = 0 = median. | (b) mean = 2.30, range = 18 |
| (c) 15 fish                                   |                             |
- |                |                         |
|----------------|-------------------------|
| (a) 268.4 cars | (b) The mean decreases. |
|----------------|-------------------------|
- |  |
|--|
| (a) mean = 2.035, median = 2, mode = 2.    |
| (b) Either median or mode (whole numbers). |
10. The mean will increase.
- |          |               |          |
|----------|---------------|----------|
| (a) 19°C | (b) Archangel | (c) 27°C |
|----------|---------------|----------|
12. 225 grams

## Answers

### 6.6

13. (a) mean = 2      (b) range = 4
14. (a) 77 kg      (b) *Hereward House*, because they have a much heavier team.
15. (a) modal class = 24 pupils      (b) mean class = 26 pupils  
 (c) there is more of an even spread of pupils in Year 9.
16. (a) Pat: mean = 25.3, range = 18  
 (b) They both have approximately the same mean, but Kim's scores are more consistent as shown by the smaller range; thus Kim should be selected.
17. (a) (i) 5.6      (ii) 0.5  
 (b) Leaving out the two extreme marks probably gives a less biased measure of performance.  
 (c) Mean + Range = 5.2 + 0.6 = 5.8, therefore no mark could possibly exceed 5.8.

### 6.7 Finding the Mean Using Tables and Tally Charts

1. mean = 1.25
2. mean = 1.93
3. mean = 4.08
4. mean = 3.56
5. (a) mean = 1.95      (b) 22 times
6. mean number = 1.15 trains
7. mean = 2.30
8. (a) 6      (b) 2.04
9. Missing frequencies are 1, 5, 1. Missing numbers of tickets are 0, 16, 21, 10, 6.  
 mean = 2.96

10. (a)

<i>Weight Range (w)</i>	<i>Tally</i>	<i>Frequency</i>
$30 \leq w < 40$		4
$40 \leq w < 50$		7
$50 \leq w < 60$		7
$60 \leq w < 70$		8
$70 \leq w < 80$		1
$80 \leq w < 90$		3

- (b) class  $60 \leq w < 70$
11. (a) 3      (b) frequency = 21, total = 48, mean = 2.29.  
 (c) the number of children per family has decreased on average ( $2.29 < 2.7$ ), and there is less variation from family to family (today's range is 3, whilst in 1960 it was 7).

## Answers

### 6.8 Calculations with the Mean

1. mean = 16155
2. mean = 1.32
3. mean = 4
4. mean = 60.15 kg
5. 7
6. 84%
7. £6000
8. 9.5
9. mean  $\approx$  4.47
10. 320

### 6.9 Mean, Median and Mode for Grouped Data

1. (a) 33.09                      (b) 33.42                      (c) 30 - 39
2. (a) 40.90                      (b) 41                          (c)  $40 \leq w < 45$
3. (a) Yes                          (b) 20.54                      (c) 20.95  
(d) Median is greater than the mean.
4. (a) No                              (b) median = 0.72 mean = 0.78  
(c) The mean is the largest.
5. 11.45 years
6. (a) 11.95                          (b) 10.92
7. (a) 9.65                          (b) 9.92                          (c) 11 - 15
8. (a) (i) 26.78                      (ii) 27.17                      (iii) 21 - 30  
(b) (i) 21.5                          (ii) 22.25                      (iii) 21 - 30  
(c) The second class have a lower mean but similar range.
9. (a) 0 - £1.00                      (b) £1.44
10. (a) 21, 7, 2  
(b) People would spend more time watching television than in summer.  
(c) 23.82 (24 hours)
11. (a) 24 cm                          (b) 24.32 cm
12. (a)

<i>Temperature, T</i>	<i>Mid-point</i>	<i>Frequency</i>
$40 < T \leq 50$	45	6
$50 < T \leq 54$	52	6
$54 < T \leq 58$	56	8
$58 < T \leq 62$	60	5
$62 < T \leq 70$	66	5



# Answers

6.9

(b)  $\frac{1660}{30} \approx 55.3$

<i>Interval</i>	<i>Freq. Density</i>
40-50	$\frac{6}{10} = 0.6$
50-54	$\frac{6}{4} = 1.5$
54-58	$\frac{8}{4} = 2$
62-70	$\frac{5}{8} = 0.625$

