2: Multiplication can Increase or Decrease a Number

Question: **Does multiplication always increase a number?**

Misconception

Yes it does; take the number 8, for example:

$2 \times 8 = 16$

$3 \times 8 = 24$

$4 \times 8 = 32$

etc.

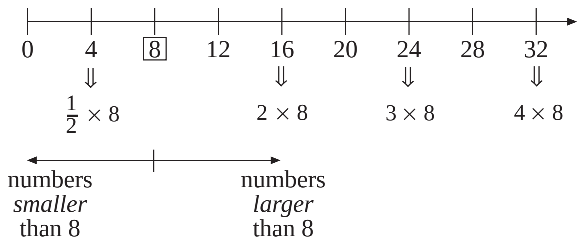
In each it is getting larger, so, yes, multiplication clearly increases a number.

Correct

No – it increases a number only under certain conditions.

Multiplying any positive number by a whole number greater than 1 will always increase its value – see the example opposite; but consider

$\frac{1}{2} \times 8 = 4$; here the number 8 is reduced.



Further Explanation

So, multiplying can have a *reducing* effect when multiplying a positive number by a fraction which is less than one. But this can still be confusing. While we accept the above, the concept of 'a number *times* 8' continues to be perceived as an increase. How then can we attach a meaning to $\frac{1}{2} \times 8$ so that this will be perceived as decreasing?

When multiplying by a whole positive number, e.g. 6 *times* 5, we understand this as being 5 added over and over again, how ever many *times* – six times in this example. But this interpretation of *times* does not quite work with fractions. If we ask *how many times*, the answer is *"not quite once"*.

Again we need to put the term *multiplying* into a context with which we can identify, and which will then make the situation meaningful.

We want to buy 30 roses which are sold in bunches of 5, so we ask for "6 *of* the 5-rose bunches". In this way, the word *times* also often means *of*. If we try using the word *of* when *times* appears to have an unclear meaning, we get $\frac{1}{2}$ *of* 8 rather than $\frac{1}{2}$ *times* 8. Indeed we know what $\frac{1}{2}$ *of* 8 means – namely 4.

So, by using *of* instead of *times* we are able to understand the concept of multiplying by a fraction and how this can have a reducing effect when the fraction is smaller than 1.

This also helps us to understand how we multiply by a fraction, and why the method works:

the 4 which results from $\frac{1}{2} \times 8$ (or $\frac{1}{2}$ *of* 8) can be reached by dividing 8 by 2;

similarly, the 5 which results from $\frac{1}{3} \times 15$ (or $\frac{1}{3}$ *of* 15), (or a third *of* fifteen) can be reached by dividing 15 by 3.

Generalising this result gives:

$$\frac{1}{d} \times n \text{ is the same as } \frac{n}{d}$$

Negative numbers

When your bank balance is +4 pounds you *have* £4.

When your bank balance is –4 pounds you *owe* £4.

Owing is the opposite of *having*, so we find that we can associate the concept of 'minus' with *'(the) opposite (of)'*. This also works in reverse.

Thus, $(-4) \times 8$ means "*owing* £4, eight times over" or "owing £32" which is $-£32$.

Now -32 is smaller than 8, so we have illustrated another case where *multiplying* has a *reducing* effect, i.e. when multiplying by a negative number.

Note that, using the method shown above, it follows that $-1 \times 8 = -8$, and vice versa.

Follow-up Exercises

1. Calculate:

   (a) 1×6 (b) 2×6 (c) $\frac{1}{2} \times 6$ (d) $\frac{1}{3} \times 6$

2. Calculate:

   (a) $\frac{1}{4} \times 12$ (b) $\frac{1}{5} \times 20$ (c) $\frac{1}{3} \times 18$

3. Are the following statements:

   always true | *sometimes false and sometimes true* | *always false*

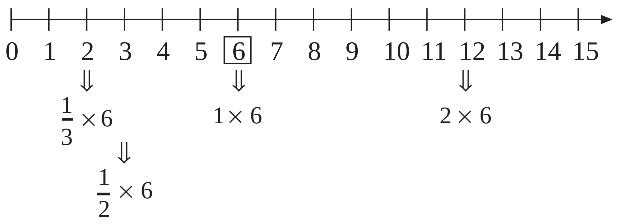
   (a) Multiplication of a positive number by a number greater than 1 always increases the number.

   (b) Multiplication of a positive number by a positive number between 0 and 1 always increases the number.

   (c) Multiplication of a negative number by a positive number always increases the first number.

Answers

1. (a) 6 (b) 12 (c) 3 (d) 2



2. (a) 3 (b) 4 (c) 6

3. (a) Always true

   (b) Always false, as multiplication of a positive number by a number between 0 and 1 will always reduce the number. (e.g. $\frac{1}{2} \times 12 = 6$, $\frac{1}{3} \times 12 = 4$, etc.)

   (c) Sometimes false and sometimes true; e.g. for the number -8, $2 \times (-8) = -16$, so the number is decreased, whereas the number increases in the example below:

$$\frac{1}{2} \times (-8) = -4$$