

## 5: Dividing Whole Numbers by Fractions

**Question:** What is the value of  $3 \div \frac{1}{4}$ ?

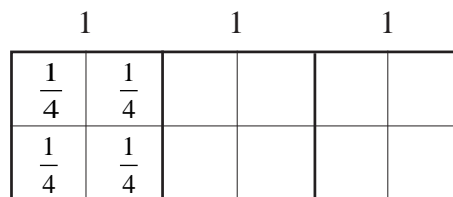
Misconception	Correct
<p>The value of <math>3 \div \frac{1}{4}</math> is equivalent to <math>3 \div 4</math> and hence has value <math>\frac{3}{4}</math> or 0.75</p> <p>Similarly <math>5 \div \frac{1}{2}</math> is equivalent to <math>5 \div 2</math> and hence has value 2.5 or <math>2\frac{1}{2}</math></p>	<p>The division <math>3 \div \frac{1}{4}</math> means how many <math>\frac{1}{4}</math> are there in the number 3. Clearly there are 4 quarters in 1 and hence <math>3 \times 4 (=12)</math> in 3.</p> <p>So <math>3 \div \frac{1}{4} = 12</math></p>

### Further Explanation

We must learn NEVER to be influenced by what things look like: the meaning of dividing by 2, dividing by 5, etc. is clear: the concept of dividing by a quarter is, however, less straightforward and requires more thought.

Think of  $3 \div \frac{1}{4}$  as 'the number of  $\frac{1}{4}$ 's that fit into 3'.

There are 4 quarters in 1, so in 3 there are  $3 \times 4$  quarters in 3 as can be seen in the diagram below.



So  $3 \div \frac{1}{4}$  (or  $\frac{3}{\frac{1}{4}}$ ) =  $3 \times 4 = 12$ .

Generally  $n \div \frac{1}{m} = n \times m$  or  $\frac{n}{(\frac{1}{m})} = n \times m$

Hence, for example,  $5 \div \frac{1}{2} = 5 \times 2 = 10$ .

### Misconception 5

There is another way to approach this task *logically* which we will demonstrate with  $6 \div \frac{3}{5}$ .

Use the problem solving method – 'if you are having difficulties, find something similar which you know you CAN do and work out the difference between this and the problem given'.

The difficult part here is *dividing by a fraction*.

Start with something similar which is straightforward: just divide the 6 by 3  $\left(\frac{6}{3}\right)$ . Now continue by examining the effect of the difference between what we did and what was given (using clearer terminology to refer to division, i.e. divide *between*).

We divided the 6 by 3 instead of by the given  $\frac{3}{5}$  (which is, of course, less than 3).

When a cake is divided between a certain number of people, each gets a certain portion. Dividing it between *fewer people* results in each one receiving a larger portion. How much larger? If it is divided between, say, 5 times fewer people, each portions would become 5 times larger.

We arrived at 2 by dividing the 6 by 3. We *should* have divided by something that is 5 times smaller than the 3, (by  $\frac{3}{5}$ ), so, the result should be 5 times *larger* than the  $\frac{6}{3}$ . Thus we

deduce that our  $6 \div \frac{3}{5}$  must mean  $\frac{6}{3} \times 5 (= 10)$ . Generalising,

$$a \div \frac{b}{c} \left( \text{or } \frac{a}{\frac{b}{c}} \right) = \frac{a}{b} \times c \left( = \frac{a \times c}{b} = a \times \frac{c}{b} \right)$$

Yet another way of determining  $\frac{3}{\frac{1}{4}}$  is to forget about the unclear meaning of *dividing by a fraction* and to do whatever yields a result which doesn't contradict other things that are already established.

Whatever we mean by  $\frac{p}{k}$ , we already know that its result,  $r$ , must be such that  $r \times k$  will be equal to  $p$ . i.e. in  $\frac{p}{k} = r$ ,  $r$  must be such that  $r \times k = p$ , (e.g.  $\frac{187}{11}$  is 17 because  $17 \times 11 = 187$ ).

Following this for  $\frac{3}{\frac{1}{4}}$ , we simply seek a result which gives 3 when multiplied by a  $\frac{1}{4}$ .

The question then becomes: "*what times a quarter is 3?*", or using a familiar rephrasing "*a quarter of what is 3?*" (The answer is of course 12.) In summary

$$\text{to determine the value of } r \text{ in } \frac{3}{\frac{1}{4}} = r, \text{ find which value of } r \text{ satisfies } r \times \frac{1}{4} = 3$$

**Follow-up Exercises**

1. Calculate the value of:

(a)  $4 \div \frac{1}{2}$

(b)  $3 \div \frac{1}{3}$

(c)  $6 \div \frac{1}{4}$

(d)  $10 \div \frac{1}{5}$

(e)  $4 \div \frac{1}{3}$

(f)  $5 \div \frac{1}{4}$

(g)  $20 \div \frac{1}{5}$

(h)  $6 \div \frac{1}{6}$

2. Calculate the value of:

(a)  $4 \div \frac{2}{5}$

(b)  $3 \div \frac{3}{4}$

(c)  $10 \div \frac{2}{3}$

(d)  $4 \div \frac{3}{4}$

(e)  $6 \div \frac{3}{5}$

(f)  $1 \div \frac{2}{5}$

(g)  $7 \div \frac{5}{8}$

(h)  $20 \div \frac{4}{5}$

**Answers**

1. (a) 8 (b) 9 (c) 24 (d) 50 (e) 12 (f) 20 (g) 100 (h) 36

2. (a) 10 (b) 4 (c) 15 (d)  $\frac{16}{3}$  (e) 10 (f)  $\frac{5}{2}$  (g)  $\frac{56}{5}$  (h) 25