

You might be surprised to learn that most countries charge for planes to fly through their airspace, even if they do not land. The charges levied vary considerably. A plane flying from Singapore to London can take a number of routes: the costs levied on each possible route are calculated just before the plane takes off. In theory, the airline will choose the cheapest route for the journey, provided this does not take much longer than its scheduled time of 13 hours 35 minutes.

Formula 1 Eurocontrol

In Europe, the following formula is used to calculate flying charges:

$$\text{charge} = T \times D \sqrt{\frac{W}{50}}$$

where T = unit rate (currently £63.96 for the UK),

W = takeoff weight of plane in metric tonnes,

D = distance flown, in hundreds of km, taken to 2 decimal places.

So, for example, a Singapore Airlines Boeing 747, flying from Frankfurt to New York, with a takeoff weight of 395 tonnes, and flying 765 km across UK airspace, incurs a charge of

$$£63.96 \times 7.65 \times \sqrt{\frac{395}{50}} \approx £1375 \quad (\text{to the nearest } £)$$

You can see that flying charges are not insignificant!

Problem 1

Find the flying charges across the UK in each of the following cases:

- (a) Boeing 747 of takeoff weight 415 tonnes, flying 823 km,
- (b) Boeing 737 of takeoff weight 233 tonnes, flying 4237 km.

Problem 2

If the charge for flying 572 km is £1022, what was the takeoff weight of the plane?

Formula 2 ASECNA

This is another formula used by some countries.

Here

$\text{charge} = T \times \text{coefficient}$

where T is the unit rate and the *coefficient* depends on the distance travelled and takeoff weight of the plane, according to the table below.

<i>Weight in tonnes</i>	<i>Distance in kilometres</i>			
	<i>0 - 750</i>	<i>750 - 2000</i>	<i>2000 - 3500</i>	<i>Over 3500</i>
14 - 20	1	5	12	20
20 - 50	1.2	6	14.4	24
50 - 90	1.4	7	16.8	28
90 - 140	1.6	8	19.2	32
140 - 200	1.8	9	21.6	36
200 - 270	2	10	24	40
270 - 350	2.15	10.75	25.8	43
350 - 440	2.3	11.5	27.6	46
440 - 540	2.45	12.25	29.4	49
540 - 650	2.6	13	31.2	52

As an example, suppose a plane with takeoff weight 250 tonnes, travels 1950 km across a country where the unit rate is £134, then

$$\begin{aligned}
 \text{charge} &= \text{£}134 \times \text{coefficient} \\
 &= \text{£}134 \times 10 \\
 &= \text{£}1340
 \end{aligned}$$

Problem 3

For a unit rate of £126.51, calculate the charge when:

- (a) takeoff weight is 223 tonnes, and distance flown is 1860 km,
- (b) takeoff weight is 455 tonnes, and distance flown is 2370 km,
- (c) takeoff weight is 455 tonnes, and distance flown is 3423 km.

Problem 4

- (a) For a Boeing 747 of takeoff weight 395 tonnes, illustrate on a graph the charges for flying a distance of up to 5000 km when the unit rate is £156.64.
- (b) On the same graph, illustrate the UK charge given by formula 1.
- (c) Interpret the graph drawn. What are your main conclusions from the graph?