Topical Applications of Mathematics

Athletics Track Layout TEACHER INFORMATION

Key Stage 3

Target Mainstream Y9

***MEP* references** MEP, Y9, Unit 5

Teaching notes This is a very topical resource in the light of LONDON hosting the 2012 Olympic Games. It will also be of interest in the build-up to the 2008 BEIJING Olympic Games when there will be extensive media publicity.

It is an excellent example of mathematics used in solving practical problems. It would be helpful for you to work with the PE department and better still if you have a local real-life context for this.

The mathematics needed is really not high level but the way it has to be used makes it more challenging.

Solutions and Notes for material in the Pupil Text

Activity 1

398.1163 m: this is less than the required 400 m. Also, it would be impossible to run on the very edge of the inside lane.

Activity 2

$$2\pi\ \ 3650 + x\ \ + 16878 = 40000$$

$$40704 \text{ cm}$$

$$x = 29.98 \text{ cm}$$

i.e. $x \approx 30$ cm

Activity 3

Distance covered $= 2\pi\ \ 3650 + 122 + 20\ \ + 16878$

$= 40704$ cm

So the athlete in lane 2 runs an extra 7.04 m.

Activity 4

Lane	*Stagger* (m)
3	14.70
4	22.23
5	30.03
6	37.70
7	45.37
8	53.03

Activity 6

Clearly lane 1 has the advantage of leading at the start of the race but has several disadvantages, e.g. radius of curve is smallest of all runners; difficult to see competitors behind you.

The athlete in lane 8 has the distinct advantage of seeing exactly where all the other athletes are and knows what is needed to win!

Topical Applications of Mathematics

Athletics Track Layout — SAMPLE LESSON PLAN

Activity		*Notes*
		T: Teacher P: Pupil
1	**Introduction to topic**	
	T: What needs to be considered when an athletics track is designed? *(400 m in circumference; equal distance for each runner, etc.)*	Discuss with class the layout of a 400 m athletics track.
	T: How many lanes are there on a running track? *(Usually between 6 and 10 lanes)*	Use Ps' ideas. (The need for space for long jump, high jump, discus, javelin etc. which might be accommodated within the track is of interest although not directly relevant to the problem here.)
	T: What athletics events are run in lanes? *(Sprints, middle distance and long distance races, steeplechases, hurdling, relays)*	Show Data Sheet 1 or put up OHP.
	T: This is a typical layout for the three inner lanes of a professional running track. What do you notice? *(Shape, lanes, inside measurement, etc.)*	
	T: What length should the inside perimeter be? *(400 m)*	
	T: Let's check it! How can we calculate it? *(Formula for circumference of circle plus two straight lines)*	
	T: Work in pairs to calculate the inside perimeter. You have 4 minutes!	Monitor work, giving help if needed.
	T: Who can show us the calculation?	One pair of Ps give solution at board.
	P: at board $\approx 704 \text{ cm} \quad 2\pi \times 3650 + 2 \times 8439$ $= 39800 \text{ cm}$	Make sure that the whole class understands this calculation (perimeter of two semi-circles equivalent to perimeter of circle).
	T: In metres? *(398 metres)*	
	T: Why is this less than 400 m? *(You cannot run on the very edge of the track)*	Discussion with class on how we can find out the assumptions made.
	T: Let's see how we can work out the model used when designing the track.	
	T: Assume the average distance in from the inner edge is x cm. What is the distance run? *(* $2\pi \times \ 3650 + x \ + 16878$ *cm)*	
	T: What should this equal? *(* 40000 cm *)*	
	T: So what is the equation that must be satisfied? *(* $40000 = 2\pi \ \ 3650 + x \ + 16878$ *)*	

Activity		*Notes*
	T: How do we solve this for x ? We'll go through the method together and then you can work in pairs to find the solution. T: I'll give you 5 minutes. $2\pi \quad 3650 + x = 40000 - 16878$ $= 23122$ $3650 + x = \frac{23122}{2\pi}$ $x = \frac{23122}{2\pi} - 3650$ $x \approx 29.98 \approx 30$ cm T: Is this reasonable? *(Yes, as runner has to be careful not to hit the edge)* *20 mins*	Go through method with Ps, using their ideas where possible. Make sure they understand the method. Check work, helping if needed. Revise and praise after 5 minutes.
2	**Stagger for Lane 2** T: Why don't all the runners start in a straight line? *(Not fair; distances not equal)* T: What can be done to compensate for this? *(Stagger the starting positions)* T: Let's work out the stagger for the runner in lane 2, assuming that they run at an average distance of 20 cm from the inside lane marking. What other information do we need? *(Width of each lane)* T: We'll use a width of 122 cm. T: Who can tell us how to proceed? *(Stagger = distance round track − 400 m)* T: Show us the calculation on the board. $($Stagger $= 2\pi \quad 3650 + 122 + 20$ $+ 16878 - 40000$ ≈ 704 cm *30 mins*	Again, use the experiences and knowledge of Ps to develop this important practical concept. For some Ps, further explanation of a 'staggered start' might be needed. In fact, showing video of an actual 400 m race would be helpful. Encourage Ps to take over the calculation. It might be helpful to have a copy of Data Sheet 1 and add in the extra information.

Activity		*Notes*
3	**Staggers for other lanes** T: Now it's your turn to find the stagger for lanes 3 to 8. Work in pairs. T: Complete this table. ***Lane*** | ***Stagger*** 3 | (*14.70*) 4 | (*22.23*) 5 | (*30.03*) 6 | (*37.70*) 7 | (*45.37*) 8 | (*53.02*) T: You can illustrate all this on an accurate sketch of the track. See how you get on. T: What are the advantages/disadvantages of lanes 1 and 8 ? *45 mins*	Give Ps about 7 minutes for this activity, and review with whole class. Take their answers and check them with the rest of the class. Give each P a copy of Data Sheet 2. Allow 5 minutes to complete the illustration. Discussion with class on which lane is best.
4	**Extensions** 1. The area available for a school running track is $90\text{ m} \times 173\text{ m}$. How many lanes could the track have? 2. Design a smaller running track with lanes to fit an area $40\text{ m} \times 90\text{ m}$. $40\text{ m} \times 90\text{ m}$	