Topical Applications of Mathematics

Genetic Fingerprinting TEACHER INFORMATION

Key Stage 3 or 4

Target High ability KS3 pupils and Higher level GCSE students

MEP references GCSE Unit 5

Teaching notes This resource is highly relevant to real-life situations and is, at its heart, based on straightforward probability. It could be used with KS3 pupils but the range of the applications (for example, not just paternity but many high profile murder and rape cases; it played a major part in the prosecution and conviction in 2008 of the murderer of five prostitutes in Ipswich) is wide and instances where genetic fingerprinting is used are extensive and of interest to all. The material presented here attempts only to explain the part played by probability. You might consider working with your Biology teachers in order to cover in more detail the background to DNA.

Useful references include those at

http://www.parliament.uk/documents/upload/postpn258.pdf

and

http://www.forensic.gov.uk/forensic_t/inside/news/fact_sheets.htm

An interesting lecture by the originator of the method, Professor Sir Alec Jeffreys, can be found at

http://www.ntu.ac.uk/news/events/57767gp.html

Another avenue to explore would be to invite the forensic officer from your local police force to talk to your students about this application (and indeed other techniques used to solve crimes).

Solutions and Notes for material in the Pupil Text

Activity 1

There is about a 1 in 1 million chance of 10 out of 10 bands matching.

Activity 2

There is about a 1 in 1000 chance of 10 out of 10 bands matching.

Activity 3

p	5	10	15	20
0.2	1 in 3125	1 in 9.8 million	1 in 30 thousand million	1 in 95 million million
0.25	1 in 1024	1 in 1 million	1 in thousand million	1 in 1.1 million million
0.5	1 in 32	1 in 1024	1 in 32 768	1 in 1 million

Activity 4

We want $\left(\frac{1}{4}\right)^n = \frac{1}{60\,000\,000}$

You can use trial and improvement or, if familiar with logarithms, then

$$\begin{aligned}
\ln\left(\frac{1}{4}\right)^n &= \ln\left(\frac{1}{6\times 10^7}\right) \\
\ln\ 4^{-n} &= \ln\ 6^{-1}\times 10^{-7} \\
-n\ln 4 &= -\ln 6 - 7\ln 10 \\
-2n\ln 2 &= -\ln 6 - 7\ln 10 \\
n &\approx 12.92
\end{aligned}$$

So we take $n = 13$.

Topical Applications of Mathematics

Genetic Fingerprinting SAMPLE LESSON PLAN

Activity		***Notes***
		T: Teacher P: Pupil
1	**Introduction** T: Who knows anything about DNA? *(Ps volunteers their knowledge and understanding).* T: What is the mathematical basis for Genetic Fingerprinting? Look at *Data Sheet 1*. T: The 'bands' of your unique DNA are made up of 50% from your mother and 50% from your father. Is your DNA unique? *(Yes, unless you have an identical twin!)* T: It is estimated that the chance of having one band in common is $\frac{1}{4}$. This is though an experimental value and has been the subject of recent debate in court cases. T: So what is the probability of matching 2 out of 2 bands compared? P: $\left(\frac{1}{4}\right)^2 = \frac{1}{16} = 0.0625$ T: Can you express this as a '1 in ?' chance? P: 1 in 16 T: That's right. This probability is not sufficient to avoid error so we need to look at matching more bands. *10 mins*	It is best if this topic has been introduced recently in Biology lessons. If not, you need to capture the key points, moving on to the mathematical underpinning rather than some of the moral aspects. Put Data Sheet 1 on OHP to clarify the concept of matching bands.
2 *(continued)*	**Matching more bands** T: Work in pairs for 2 minutes to find the probability of matching (a) 10 out of 10 bands (b) 20 out of 20 bands T: Who will show us their answer on the board? P (at board): $\left(\frac{1}{4}\right)^{10} = \frac{1}{1048576}$ About 1 in 1 million chance	Make sure that Ps understand the problem; intervene if necessary. Choose Ps to show solution on the board.

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Activity		*Notes*
2 *(continued)*	P (at board): $= \ln\ 6^{-1} \times 10^{-7}\ \left(\frac{1}{4}\right)^{20} \approx \frac{1}{1.09 \times 10^{12}}$ About 1 in 1.1 million million chance. *20 mins*	Some discussion may be needed on accuracy and how best to present the answers.
3	**Different probabilities** T: The value of $\frac{1}{4}$ for matching one band at random is only experimental. Investigate the effect that varying this value has on the results. T: How can we cope with this? T (after about 5 minutes): What can you conclude? P: The technique is very sensitive to the value assumed for *p*. *30 mins*	You can make this as open as you like, but if you want to tie it down, then use Data Sheet 2, in which $p = 0.2$ and 0.5 is compared with $p = 0.25$ for 5, 10, 15 and 20 bands. Give Ps time to analyse the results and ensure that they all take part in the discussions.
4 *(continued)*	**Extensions** T: Assume that it is safe to take $p = \frac{1}{4}$. The population of the UK is about 60 million. What is the number of bands that need to be compared to ensure that it is safe to convict on DNA evidence alone? You have 10 minutes to find your answer. T (after about 10 minutes): Who has a method? What do we need to do? P: We need to solve $\left(\frac{1}{4}\right)^{n} = \frac{1}{60\,000\,000}$ to obtain the value of *n*. T: How can we solve this? P: Trial and improvement. T: Yes, but if we use logarithms we can get the answer quickly. We know that $\ln\left(\frac{1}{4}\right)^{n} = \ln\left(\frac{1}{6 \times 10^{7}}\right)$ $\ln\ 4^{-n} = \ln\ 6^{-1} \times 10^{-7}$ $-n \ln 4 = -\ln 6 - 7 \ln 10$	You might want to give more help or discuss the approach here. Monitor progress, intervening if necessary; working in pairs should be encouraged. Help Ps to find a method of solution. Use this method if they have covered logs; otherwise intelligent use of trial and improvement is needed.

Activity **4** *(continued)*	$-2n\ln 2 = -\ln 6 - 7\ln 10$ $n = \dfrac{1}{2\ln 2}\left(\ln 6 + 7\ln 10\right)$ ≈ 12.92 So we take $n = 13$ *45 mins*	***Notes*** You need to stress that rounding up is needed to ensure the result is valid.