Topical Applications of Mathematics

Heptathlon PUPIL TEXT

The 2008 Olympic Games, in Beijing, give rise to renewed interest in many events where the UK stands a good chance of winning medals.

One of these events is the **heptathlon**, which is an athletics contest made up of seven separate events (from the Greek *hepta* (seven) and *athlon* (contest)). The heptathlon is contested outdoors by women and consists of the following events over 2 days, in the order shown below.

100 m hurdle High jump Shot put 200 m	DAY 1
Long jump Javelin throw 800 m	DAY 2

Activity 1

Find out all the athletics events that exist for women in the Olympic Games. If you had to choose 7 of them to best represent the complete range of events, which ones would you choose?

Are your choices in line with those that are actually in the heptathlon?

Scoring is based on adding the points scored for each event. A formula is used that gives a score of 1000 points for a very good performance.

The current world record is 7291 points by the USA athlete Jackie Joyner-Kersee in the Seoul, South Korea, Olympic Games in September 1988. The table gives her performances and points for each event.

Event	Performance	Points
100 m hurdles	12.69 secs	1172
High jump	1.86 m	1054
Shot put	15.80 m	915
200 m	22.56 secs	1123
Long jump	7.27 m	1264
Javelin	45.66	776
800m	2 mins 08.51 secs	987

Activity 2

Which were the weakest and strongest performances by Jackie Joyner-Kersee?

Why is her average score per event only just over 1020?

This table gives the top 10 performances of all time (accurate as of August 2007).

Mark	Athlete	Nationality	Venue	Date
7291	Jackie Joyner-Kersee	United States	Seoul	September 24, 1988
7032	Carolina Klüft	Sweden	Osaka	August 26, 2007
7007	Larisa Turchinskaya	Soviet Union	Bryansk	June 11, 1989
6985	Sabine Braun	Germany	Götzis	May 31, 1992
6946	Sabine John	East Germany	Potsdam	May 6, 1984
6942	Ghada Shouaa	Syria	Götzis	May 26, 1996
6935	Ramona Neubert	East Germany	Moscow	June 19, 1983
6889	Eunice Barber	France	Arles	June 5, 2005
6859	Natalya Shubenkova	Soviet Union	Kiev	June 21, 1984
6858	Anke Vater-Behmer	East Germany	Seoul	September 24, 1988

Activity 3

What significant results are shown by this data?

One disappointment with this data is the lack of any UK athletes in the list. We do indeed have one famous heptathlete, **Denise Lewis**, who won the event in the Sydney Olympics in 2000 with a total of 6584 points.

We now have two potential winners for the 2008 Olympic Games: **Kelly Sotherton** (personal best score of 6510 points, was third in the World Athletics Championships in Osaka, Japan, in August 2007) and **Jessica Ennis** (personal best of 6469, was fourth place in the Osaka Championships). They will face stiff competition from other world-class athletes!

In order to understand their chance of winning medals it is helpful to know about the points system used to calculate the final scores. We can then see which of their events are most in need of improvement.

For track events, the points are given by the formula

$$P = a\ b - M^{\ c}$$

where a, b c are constants, given in the table below, and M is the actual time (in seconds) taken by the competitor.

Event	***a***	***b***	***c***
100 m hurdles	9.23076	26.70	1.835
200 m	4.99087	42.50	1.81
800 m	0.11193	254.00	1.88

We will first see how this formula works. Here are the results for the first 4 athletes in the Osaka World Championships in Athletics.

Name	**100 m hurdles** (s)	**High jump** (m)	**Shot put** (m)	**200 m** (s)	**Long jump** (m)	**Javelin** (m)	**800 m** (s)
Caroline Klüft (SWE)	13.15	1.95	14.81	23.38	6.85	47.98	132.56
Lydmila Blonska (Ukr)	13.25	1.92	14.44	24.09	6.88	47.77	136.68
Kelly Sotherton (UK)	13.21	1.86	14.14	23.40	6.68	31.90	131.58
Jessica Ennis (UK)	12.97	1.89	11.93	23.15	6.33	38.07	131.39

Example 1

Calculate the points (P) scored by Kelly Sotherton in the 100 m hurdles.

Solution

$$P = 9.23076\ (26.70 - 13.21)^{1.835}$$

$$= 1093.476...$$

$$P = 1093$$

*(Note that 'points scored' are always the actual value, rounded **down**.)*

Activity 4

Work out the points scored by the other three competitors in the 100 m hurdles and the rank order of these four competitors after this first event.

We can see that it is the overall performances in **all** the events that wins the day. In fact, the Gold medal winner, Caroline Klüft, won only one single event, the high jump.

Activity 5

(a) *Use a spreadsheet (or any other method) to complete the points scored in each running event. Also work out the ranking for these three events.*

(b) *Which events should the UK competitors concentrate on improving?*

The **Appendix** gives a justification for what might appear to be a very complicated way of assigning points to each event. To complete the total points scored in the heptathlon, we also need the scoring formula for field events. This is of the form

$$P = a\ (M - b)^{c}$$

when M is the distance thrown or jumped, in cm.

Event	***a***	***b***	***c***
High jump	1.84523	75.00	1.348
Long jump	0.188807	210.00	1.41
Shot put	56.0211	1.50	1.05
Javelin	15.9803	3.80	1.04

Activity 6

*Complete the points scored for the **field** events for the four competitors and calculate their total scores for the heptathlon.*

This scoring system works well for these high-performing athletes. Might it be possible to design a simpler system for use at school level?

Activity 7

Design a method of comparing athletics events in order to determine the overall best athlete in a school athletics competition.

APPENDIX: Design of Scoring System

Although it looks complicated, we will begin to understand more about the system by looking at one formula in particular, namely the points formula for the 800 m, which is

$$P = 0.11193\ (254.00 - M)^{1.88}$$

Worked Example 1

(a) What is the time M, in seconds, that gives $P = 0$?

(b) What is the time M that gives $P = 1000$?

(c) What points would you score if $M = 0$?

Solution

(a) $P = 0$ if $M = 254$; that is, anything more than about 4 minutes scores no points at all.

(b) $1000 = 0.11193\ (254 - M)^{1.88}$

$$(254 - M)^{1.88} = \left(\frac{1000}{0.11193}\right)$$

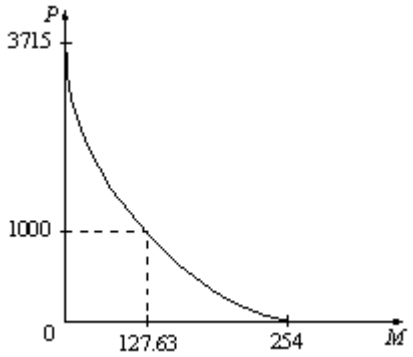
$$254 - M = \left(\frac{1000}{0.11193}\right)^{\frac{1}{1.88}}$$

$$M = 254 - \left(\frac{1000}{0.11193}\right)^{\frac{1}{1.88}}$$

$$M = 127.63 \text{ seconds} \quad \text{i.e. just over 2 minutes.}$$

(c) If $M = 0, \quad P = 3715$

With these values, we can sketch the form of this curve of P against M.



In fact, we can find $a, b\ c$ in the formula by defining three points on this graph, namely

$M = 254 \Rightarrow P = 0$ (very poor performance)

$M = 127.63 \Rightarrow P = 1000$ (good performance)

$M = 114.77 \Rightarrow P = 1200$ (excellent performance)

Worked Example 2

Find a, b, c for the 800 m event, using the three data points given.

Solution

$$P = a\ b - M^{\ c}$$

Using $M = 254$ when $P = 0$ immediately gives $b = 254$.

We also have

$$1000 = a\ 254 - 127.63^{\ c}$$

$$1200 = a\ 254 - 114.77^{\ c}$$

Dividing, we have

$$1.2 = \left(\frac{139.23}{126.37}\right)^c = 1.1018^c \left(\frac{139.23}{126.37}\right)^c = 1.1018^c$$

To find c, we can use trial and error, but, using logs

$$\ln 1.2 = c \ln 1.1018$$

giving

$$c \approx \frac{\ln 1.2}{\ln 1.1018}$$

$$c \approx 1.88$$

Finally we determine a from

$$1000 = a\ 126.37^{1.88}$$

giving

$$a \approx 0.1192$$ (rounding errors change the final decimal place)