

Topical Applications of Mathematics

Happy Planet Index

TEACHER INFORMATION

Key Stage 3 or 4

Target Extension work for Y8/9 and mainstream for GCSE

MEP references GCSE Unit 2 Formulae

Teaching notes This is a multi-disciplinary resource that raises awareness of the balance between economic success and environmental impact. The mathematics used is straightforward but the terms and definitions will need careful handling. In particular, it should be stressed that this is just one of a number of models that could be used for this situation. It has a particular aim: to make countries aware that they can achieve well-being without destroying the planet. As such, it is an excellent vehicle for discussion of an example of how mathematics can help to make people more aware of the issues. The level of discussion will depend on the age and ability of the classes but even Key Stage 3 pupils should be aware of environmental concerns. Remember to praise pupils and encourage them, building their confidence to contribute to class discussions.

You could also take the themes (of life expectancy, life satisfaction and carbon emissions) and let students, at GCSE level, discuss how they would develop a model of happiness, and how they could collect data.

Solutions and Notes for material in the Pupil Text

Activity 1

This is a discussion activity where some of the key issues (for example, pay, housing and food costs, entertainment, transport, etc.) should be brought out. Work has been done on finding the best UK cities/towns to live in: this will feature in a separate resource.

Activity 2

The UK is in the middle of the table, but the life expectancy is only about 2 years below the top country although it is 8 years more than the country at the bottom of the table. So the data is *skewed*. Although the questioning in this activity is more focused on explaining the differences, this example would be a good practical exercise for comparing measures of central tendency, mean and median, as well as measures of variations, inter-quartile range and standard deviation. It is worth noting that the countries at the bottom are states of the former Soviet Union; these countries have been moving from communist state control to democracy with free markets.

Activity 3

You might want to set this activity as a preparation for the lesson. You might also get students to suggest how *they* would measure life satisfaction; for example, a more detailed questionnaire that asked participants to note satisfaction on a series of topics (e.g. education, transport, food costs, housing costs, holidays, entertainment, etc.).

Activity 4

The obvious winners here are the Scandinavian countries, with the losers being the Eastern European countries (formerly part of the Soviet Union). The UK, along with other Western European countries, is mid-table.

Perhaps the most interesting information that can be gleaned is that the differences are quite marked. On a 0 – 10 scale, the range is 4, showing considerable variation.

Activity 5

- (a) WBI = life expectancy ; that is, there is no reduction as life expectation is perfect.
- (b) WBI = 0 as here life satisfaction is non-existent.

Activity 6

See *Data Sheet 5* for complete table.

Activity 7

Again, there are considerable differences, although Luxembourg (at 6.88) can be regarded as an outlier. Nevertheless, there is a factor of almost 8 between the second (Estonia at 3.54) and last (Latvia at 0.45).

Activity 8

- (a) The carbon footprint does have an impact on the HPI. Take, for example, Latvia, with the lowest carbon footprint. It is in the top half of the HPI table whereas it is almost bottom in the WBI table.

The important point to note is that countries like Sweden can be very high on the well-being index yet remain high on the Happy Planet Index as they have a low carbon footprint per capita.

- (b) See *Data Sheet 6* for complete table.

Activity 9

See *Data Sheet 7*.

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SAMPLE LESSON PLAN

Note: *this lesson plan is designed for GCSE classes. It is an example of the use of functional mathematics: that is, mathematics used in a meaningful context.*

<i>Activity</i>		<i>Notes</i>
Before lesson (previous homework)	<p>Undertake a survey of 20 people you know to find out how satisfied they are in their life. Use the question,</p> <p><i>If you consider your life overall, how satisfied would you say you are nowadays?</i></p> <p><i>Give your response on a 0 – 10 scale, from 'Not at all satisfied' to 'Extremely satisfied'.</i></p> <p>Calculate the mean response to this question.</p>	<p>T should make sure everyone uses the same question and same scale. Interesting to also collect the age of each respondent.</p>
1	<p>Life satisfaction</p> <p>T: You should all have data about how satisfied people are in the UK.</p> <p>What results have you obtained?</p> <p><i>Ps give mean value for their samples.</i></p> <p>T: How shall we calculate the overall mean value for all the people in the samples?</p> <p><i>Ps explain and all do the calculation.</i></p> <p>T: Does this final mean value give a good representation for the UK?</p> <p><i>Discussion on representative and biased samples.</i></p> <p>T: Fortunately others have already done this for us and there is agreement that the value for the UK is approximately 7.2.</p> <p>T: What value for life satisfaction do you expect for Ireland, Switzerland, Latvia?</p> <p>(Values are 7.7, 8.2 and 5.1)</p> <p>T: The complete set of this data for 30 European countries is on this sheet</p> <p>T: What do you notice about this data?</p> <p>Ps: Large variation, range of 4.</p> <p>T: What other factors might affect well-being?</p> <p>Ps: Housing, pay, costs, transport, health services, etc.</p> <p style="text-align: right;"><i>15 mins</i></p>	<p>It might be helpful (and interesting) for T to have previously tested a sample of 20 people.</p> <p>T writes data on board or gets each P to write in their mean value.</p> <p>T must ensure all Ps understand what to do.</p> <p>Answers checked around the class.</p> <p>Hopefully the class will recognise the deficiency of this method, but the discussion should be turned to,</p> <p>“How can you obtain a representative sample?”</p> <p>T hands a copy of Data Sheet 2 to each P.</p> <p>T gives Ps enough time to look at data. Perhaps working in pairs, Ps should write down interesting facts and then review with whole class.</p> <p>Any sensible suggestions considered by whole class.</p>

<p><i>Activity</i></p>		<p><i>Notes</i></p>	
<p>2</p>	<p>Life expectancy</p> <p>T: One important factor, which is at least in part dependent on all the types of factors that have been suggested, is that of life expectancy.</p> <p>Do you know the current value in the UK?</p> <p>Is it the same for males and females?</p> <p style="text-align: right;"><i>(Ps will guess!)</i></p> <p>T: In fact, the overall expectation is 78.4 years (76.4 years for males and 80.4 years for females).</p> <p>T: How can countries estimate their population’s life expectancy?</p> <p style="text-align: center;"><i>(Clearly this calculation is based on past data but also uses trends in data over time; for example, restrictions on smoking will improve life expectancy)</i></p> <p>T: Would you expect the same value in other countries? ... other European countries?</p> <p>T: What does the table show?</p> <p>Ps: UK in middle of table but only 2 years behind highest value; range of about 10 years; data skewed.</p> <p style="text-align: right;"><i>25 mins</i></p>	<p>Useful if T can get Ps to come up with this factor – they might need help.</p> <p>Discussion on methods of estimating life expectancy.</p> <p>T hands a copy of Data Sheet 1 to each P, for discussion.</p> <p>Discussion on data, which could be extended by T asking about the average and variation in the values.</p> <p>T could also pose the question, “Why is there variation?”, but this is about social policy and trends (for example, health, housing, and education are all key factors).</p>	
<p>3</p>	<p>Well-being index</p> <p>T: Well-being includes both</p> <ul style="list-style-type: none"> ▪ life satisfaction and ▪ life expectancy <p>but how can we bring the two factors together?</p> <p>T: Remember that life satisfaction is a 0 – 10 range and think how this could affect life expectancy.</p> <p>T: <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td style="padding: 5px;"> Well-being index (WBI) $= \frac{\text{(life satisfaction)}}{10} \times \text{life expectancy}$ </td> </tr> </table></p> <p>T: Let’s calculate its value, to one decimal place, for the UK and for Switzerland.</p> <p style="text-align: right;"><i>(56.4, 66.0)</i></p> <p>T: Now calculate the WBI for the other countries.</p> <p style="text-align: right;"><i>35 mins</i></p>	Well-being index (WBI) $= \frac{\text{(life satisfaction)}}{10} \times \text{life expectancy}$	<p>It is best if Ps can suggest the formula rather than it being provided for them. It could also be valuable to pursue other ideas from Ps.</p> <p>Discuss what this means. For example, what happens if life satisfaction is (a) zero, (b) 10?</p> <p>Opportunity for Ps to work at board with the whole class checking and T intervening if problems occur.</p> <p>Each pair of Ps could be given 2 countries to complete and values given to whole class.</p>
Well-being index (WBI) $= \frac{\text{(life satisfaction)}}{10} \times \text{life expectancy}$			

<i>Activity</i>		<i>Notes</i>
<p>4</p>	<p>Happy Planet Index (HPI)</p> <p>T: We need now to balance well-being with the damage done to the planet by each country. How might we measure this?</p> <p>T: We'll use the carbon footprints of each country. What does the data show?</p> <p>Ps: Considerable variation between countries; UK high in table.</p> <p>T: Now we are in a position to bring all our data together in a Happy Planet Index (HPI). We have added in some numbers so that an idealized country with</p> <p style="padding-left: 40px;">life expectancy = 85</p> <p style="padding-left: 40px;">life satisfaction = 10</p> <p style="padding-left: 40px;">carbon emission = 0.95</p> <p>would have HPI = 100.</p> <p>Here is the formula:</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $\text{HPI} = \frac{\text{WBI} \times 5}{(\text{carbon footprint} + 3.3)}$ </div> <p>T: Calculate the HPI for UK and for Switzerland.</p> <p style="text-align: right;">(42.7, 52.1)</p> <p>T: Now calculate the HPI for other counties.</p> <p style="text-align: right;"><i>45 mins</i></p>	<p>Discussion about the balance between well-being and damage to the environment. For example, holidays abroad are enjoyable but CO₂ emissions from planes, trains, could damage the planet.</p> <p>T gives out Data Sheet 3.</p> <p>T can either give the formula and then show that its value is 100 for the idealised country</p> <p><i>or</i></p> <p>start with</p> $\text{HPI} = \frac{\text{WBI} \times \alpha}{(\text{c.f.} + \beta)}$ <p>with α, β numbers to be determined with discussion on the characteristics.</p> <p>T chooses 2 Ps (or volunteers) to work at board; class monitor and check.</p> <p>Countries could be divided up between Ps, working in pairs.</p>
<p>5</p>	<p>Homework</p> <ol style="list-style-type: none"> 1. Complete the HPI for all 30 countries and put in rank order. 2. Plot the countries on Data Sheet 4 and identify those with a good balance between well-being and carbon footprint. 	<p>Discussion of data at start of next lesson, with the emphasis on the balance between well-being and carbon emissions.</p>