

<p>Codes and Ciphers</p>	<p>UNIT 19 <i>Lorenz Cipher Machine</i> Lesson Plan 1</p>																																																																																	
<p>Activity 1</p>	<p>Introduction</p> <p>T: In this first lesson we'll look at the principles of the Lorenz cipher; in the next lesson we'll learn how the Lorenz cipher machine was used to break the code.</p> <p>T: We start with the enciphering of letters. Step 1 is to convert letters to binary numbers.</p> <p>T: How many codes are used for the letters of the alphabet? (26)</p> <p>T: Using 5 bits for the digits, how many codes are available?(32)</p> <p>T: Why do you say '32' ? (<i>Because $2 \times 2 \times 2 \times 2 \times 2 = 2^5$</i>)</p> <p>T: So how many are left to assign? ($32 - 26 = 6$)</p> <p>T: The sheet shows how these codes are used; only '9' is used in messages and it represents a space between words.</p> <table border="1" data-bbox="512 909 836 983"> <tr> <td>$0 + 0 = 0$</td> <td>$0 + 1 = 1$</td> </tr> <tr> <td>$1 + 0 = 1$</td> <td>$1 + 1 = 0$</td> </tr> </table> <p>T: We add a 'key' to any message in order to make it difficult to break, so we need to define the type of key (or additive) used.</p> <p>T: Let's look at an example. We want to send the letter J using the key letter B, so we actually send J + B. Who can do this?</p> <p>P (on board):</p> <table data-bbox="480 1189 692 1301"> <tr> <td>J</td> <td>\Rightarrow</td> <td>1 1 0 1 0</td> </tr> <tr> <td>+</td> <td>B</td> <td>\Rightarrow</td> <td>1 0 0 1 1</td> </tr> <tr> <td colspan="2"></td> <td><hr/></td> <td>0 1 0 0 1</td> </tr> </table> <p>T: Well done. Which letter is this? (<i>L</i>)</p> <p>T: So the letter L is sent. Now its time for you to do some examples.</p> <table border="1" data-bbox="341 1417 943 1473"> <tr> <td>Encipher A, B, C, D and E using the key letter B.</td> </tr> </table> <p>T: Who is going to show their answer?</p> <p>5 Ps (on board):</p> <table border="1" data-bbox="467 1659 986 2063"> <tr> <td>A</td> <td>\Rightarrow</td> <td>1 1 0 0 0</td> <td>B</td> <td>\Rightarrow</td> <td>1 0 0 1 1</td> </tr> <tr> <td>+</td> <td>B</td> <td>\Rightarrow</td> <td>1 0 0 1 1</td> <td>+</td> <td>B</td> <td>\Rightarrow</td> <td>1 0 0 1 1</td> </tr> <tr> <td colspan="2"></td> <td><hr/></td> <td>0 1 0 1 1</td> <td colspan="2"></td> <td><hr/></td> <td>0 0 0 0 0</td> </tr> <tr> <td>C</td> <td>\Rightarrow</td> <td>0 1 1 1 0</td> <td>D</td> <td>\Rightarrow</td> <td>1 0 0 1 0</td> </tr> <tr> <td>+</td> <td>B</td> <td>\Rightarrow</td> <td>1 0 0 1 1</td> <td>+</td> <td>B</td> <td>\Rightarrow</td> <td>1 0 0 1 1</td> </tr> <tr> <td colspan="2"></td> <td><hr/></td> <td>1 1 1 0 1</td> <td colspan="2"></td> <td><hr/></td> <td>0 0 0 0 1</td> </tr> <tr> <td>E</td> <td>\Rightarrow</td> <td>1 0 0 0 0</td> <td colspan="3"></td> </tr> <tr> <td>+</td> <td>B</td> <td>\Rightarrow</td> <td>1 0 0 1 1</td> <td colspan="3"></td> </tr> <tr> <td colspan="2"></td> <td><hr/></td> <td>0 0 0 1 1</td> <td colspan="3"></td> </tr> </table>	$0 + 0 = 0$	$0 + 1 = 1$	$1 + 0 = 1$	$1 + 1 = 0$	J	\Rightarrow	1 1 0 1 0	+	B	\Rightarrow	1 0 0 1 1			<hr/>	0 1 0 0 1	Encipher A, B, C, D and E using the key letter B.	A	\Rightarrow	1 1 0 0 0	B	\Rightarrow	1 0 0 1 1	+	B	\Rightarrow	1 0 0 1 1	+	B	\Rightarrow	1 0 0 1 1			<hr/>	0 1 0 1 1			<hr/>	0 0 0 0 0	C	\Rightarrow	0 1 1 1 0	D	\Rightarrow	1 0 0 1 0	+	B	\Rightarrow	1 0 0 1 1	+	B	\Rightarrow	1 0 0 1 1			<hr/>	1 1 1 0 1			<hr/>	0 0 0 0 1	E	\Rightarrow	1 0 0 0 0				+	B	\Rightarrow	1 0 0 1 1						<hr/>	0 0 0 1 1				<p style="text-align: center;">Notes</p> <p>T: Teacher P: Pupil Ex.B: Exercise Book</p> <p>Interactive introduction; T will find out how much the Ps know about code-breaking in the Second World War.</p> <p>Ps are each given a copy of OS 19.1, or it is shown on OHP.</p> <p>T writes on board, and makes sure all Ps are familiar with this (could refer back to Unit 18 where this type of addition is also used).</p> <p>T gives Ps a few minutes for this; monitors their progress and intervenes if necessary, They should each have a copy of OS 19.1 to refer to.</p> <p>Volunteer Ps work simultaneously at board. Other Ps watch and then T and whole class review the answers, correcting if necessary.</p> <p>T should ensure that all Ps understand this form of addition of these binary codes.</p>
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<p>Activity</p> <p>1</p> <p><i>(continued)</i></p>	<p>T: What do you notice about B+B <i>(It is '/')</i></p> <p>T: What about A+A or C+C, etc.? <i>(Also '/')</i></p> <p>T: So the '/' symbol is a really important one and was a crucial factor in the original breaking of the code.</p> <p style="text-align:right"><i>10 mins</i></p>	<p style="text-align:center">Notes</p> <p>Interactive discussion about importance of the '/' symbol.</p>																											
<p>2</p>	<p>Enciphering</p> <p>T: How can this code be made more secure? <i>(By using a sequence of key letters which is kept secret)</i></p> <p>T: Yes, that's right. We encipher HELP using the key sequence ABCD. Who will show us?</p> <p>P (on board):</p> <table style="margin-left: 40px; border-collapse: collapse;"> <tr> <td style="padding-right: 10px;">H E L P</td> <td style="padding-right: 10px;">⇒</td> <td style="padding-right: 10px;">00101</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">10000</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">01001</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">01101</td> </tr> <tr> <td style="padding-right: 10px;">A B C D</td> <td style="padding-right: 10px;">+</td> <td style="padding-right: 10px;">11000</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">10011</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">01110</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">10010</td> </tr> <tr> <td style="padding-right: 10px;">Q O M 8</td> <td style="padding-right: 10px;">⇐</td> <td style="padding-right: 10px;">11101</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">00011</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">00111</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">11111</td> </tr> </table> <p>T: Well done. How can the message be made more secure? <i>(By using a key sequence which is not obvious)</i></p> <p>T: Yes. Now try this one. You have 5 minutes to come up with the answer.</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Encipher LONDON using the key sequence HBVQZM.</p> </div> <p style="text-align:right"><i>20 mins</i></p>	H E L P	⇒	00101		10000		01001		01101	A B C D	+	11000		10011		01110		10010	Q O M 8	⇐	11101		00011		00111		11111	<p>Whole class interactive discussion.</p> <p>OS 19.3 will speed up the process here.</p> <p>T must make sure that the class are understanding this and paying attention – they can take it in turns to do the addition and to identify the letters.</p> <p>Further whole class interactive discussion about ways of making the code more difficult to break.</p> <p>Review answers – volunteer (or chosen by T) Ps can work at board and the class then agree/disagree with their answers until correct solutions are given.</p>
H E L P	⇒	00101		10000		01001		01101																					
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<p>3</p> <p><i>(continued)</i></p>	<p>Decipher</p> <p>T: How can we decipher messages we have been sent? <i>(We will need to know the key sequence)</i></p> <p>T: If we have the key sequence, what do we do? <i>(Reverse the operations for enciphering)</i></p> <p>T: Yes, but the reverse if doing the same thing again! Go back to our message QOM8. What do you write?</p> <p>P (at board):</p> <table style="margin-left: 40px; border-collapse: collapse;"> <tr> <td style="padding-right: 10px;">H E L P</td> <td style="padding-right: 10px;">⇒</td> <td style="padding-right: 10px;">11101</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">00011</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">00111</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">11111</td> </tr> <tr> <td style="padding-right: 10px;">A B C D</td> <td style="padding-right: 10px;">+</td> <td style="padding-right: 10px;">11000</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">10011</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">01110</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">10010</td> </tr> <tr> <td style="padding-right: 10px;">Q O M 8</td> <td style="padding-right: 10px;">⇐</td> <td style="padding-right: 10px;">00101</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">10000</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">01001</td> <td style="padding-right: 10px;"> </td> <td style="padding-right: 10px;">01101</td> </tr> </table> <p>T: Good. Now I'll give you a few minutes to retrieve LONDON from your last message.</p>	H E L P	⇒	11101		00011		00111		11111	A B C D	+	11000		10011		01110		10010	Q O M 8	⇐	00101		10000		01001		01101	<p>Interactive discussion about the procedures.</p> <p>OS 19.3 will help the process here.</p> <p>T gives Ps a few minutes; monitors their progress, intervening if there are problems. Answers are checked interactively.</p>
H E L P	⇒	11101		00011		00111		11111																					
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<p>Activity</p> <p>3 <i>(continued)</i></p>	<p>T: What takes time in this? <i>(The addition)</i></p> <p>T: The Bletchley Park experts soon memorised each additive; we can make it easier by using a table.</p> <p style="text-align: right;"><i>30 mins</i></p>	<p style="text-align: center;">Notes</p> <p>Each pair of Ps has a copy of OS 19.2 and time is given for them to familiarise themselves with how it is used.</p>
<p>4</p>	<p>Simplified Lorenz cipher machine</p> <p>T: Here is a very simplified Lorenz cipher machine; do you see what it does? <i>(Uses the code wheels; they change after each turn)</i></p> <p>T: Let's see what happens if we send the message THE. First the letter T. With the starting positions shown what happens to it? <i>(T + A + B)</i></p> <p>T: Use your table to work this out. <i>(R)</i></p> <p>T: Each wheel now turns one position. What are they now on? <i>(B and A)</i></p> <p>T: So what is the output for 'H' ? <i>(H + B + A)</i></p> <p>T: And that is ...? <i>(C)</i></p> <p>T: And for 'E' ? <i>(E + C + A = N)</i></p> <p>T: Well done. So the message would be sent as RCN.</p> <p>T: Now you can encipher a message.</p> <p>With starting positions K = 5 and S = 2 encipher</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px auto;">SECRET MESSAGE</div> <p>What must you remember to do? <i>(Put 9 for the space; remember that the wheels move on one position each time)</i></p> <p>T: And the message is? <i>(UYFX9 4LFVT 8BQZ)</i></p> <p style="text-align: right;"><i>45 mins</i></p>	<p>Interactive discussion. The real Lorenz machine had twelve wheels and operated in a much more complex way than the simplified version. OS 19.4 can be shown or each pair of Ps given a copy.</p> <p>Ps will need at least 5 minutes for this. T should monitor progress and intervene if necessary.</p> <p>T and Ps review the answers together with T making sure that all Ps understand the way the cipher works.</p>
	<p>Homework</p> <p>Decipher the coded message</p> <p style="text-align: center;">UYFX9 4LFVT 8BQZ</p>	

<p><i>Codes and Ciphers</i></p>	<p>UNIT 19 <i>Lorenz Cipher Machine</i> Lesson Plan 2</p>	<p><i>Breaking the Cipher</i></p>
<p>Activity</p> <p>1</p>	<p>Breaking the cipher</p> <p>T: The Wartime success at Bletchley Park in breaking this cipher depended upon the fact that most of the German plaintext messages contained many pairs of repeated characters. (There were certain technical reasons why the Germans adopted this practice.)</p> <p>T: We'll illustrate the technique with the following short message in which the words are separated by double spaces represented by pairs of 9s.</p> <div style="border: 1px solid black; padding: 5px; text-align: center;"> <p>99HERE99IS99A99TEST99MESSAGE99FOR99YOU99TO99TRY99OUT99</p> </div> <p>T: What is the first coded character? (9 + G + B = U)</p> <p>T: Now you code the next 7 characters. (9HERE99)</p> <p>T: Well done. In fact, the complete coded message is</p> <div style="border: 1px solid black; padding: 5px; text-align: center;"> <p>UDZMR+JMSDC+TXUVQMYEDE8LWOKUD3TMK+G4UDC3NXWKOB YEFURWH</p> </div> <p>T: The codebreakers at Bletchley Park devised this process. We will not show here why it works (you can read an account if you are interested), but just show that it does work.</p> <p>T: We'll follow through the process for the first 8 letters of the message.</p> <div style="border: 1px solid black; padding: 10px;"> <p>1. $Z = \underbrace{U D Z D M R + J}_{\Delta Z = C O O Y P Z T}$</p> <p>2. Here $K = 1$, so we first find the K sequence.</p> <p>$K = \underbrace{A B C D E F G H}_{\Delta K = G Q U 3 N Q C}$</p> <p>3. $\Delta Z = C O O Y P Z T$ $\Delta K = G Q U 3 N Q C$ $\Delta Z + \Delta K = H K 8 X G I V$</p> <p>4. There are no 's in this sequence for $\Delta Z + \Delta K$.</p> </div> <p>T: So as there are no 's, it is unlikely that $K=1$ was the starting position of the K wheel.</p> <p>T: Now you work through the process using $K=7$ and see what happens.</p> <p>T: How many 's did you get? (2)</p> <p>T: Using the complete message, you actually get 7 's.</p> <p style="text-align: center;">_____ 25 mins _____</p>	<p style="text-align: center;">Notes</p> <p>This needs careful handling; Ps need to work through the process but T should not allow it to become too long and tedious. The printout will help with this. Ps with IT skills could perhaps write a program to complete the process.</p> <p>The class need a few minutes for this; some Ps could work at the board, each tackling a letter, in turn.</p> <p>T reviews answers with Ps, praising when deserved.</p> <p>Each P is given a copy of OS 19.5 and it is shown on OHP.</p> <p>It is probably best if this is done interactively; Ps either give answers aloud or write them on the board.</p> <p>T will need to monitor work closely to check that Ps have understood what is required.</p>
<p>2</p> <p><i>(continued)</i></p>	<p>K wheel positions</p> <p>T: By hand, this process is tedious, so here is the printout for each position of the K wheel.</p> <p>T: Quickly check the number of 's in each line of the printout. What do you get? (3, 1, 1, 2, 1, 0, 7, 1, 1, 2, 1, 6, 2, 2)</p>	<p>Each P has a copy of OS 19.6.</p> <p>T chooses Ps to give answers and the rest of the class agree/disagree.</p>

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Activity 2 <i>(continued)</i>	T: So what was the position of the K wheel? <i>(7 or 12)</i> T: In practice, both will need to be checked although we do know in this case that it is 7. <div style="text-align: right;"><i>30 mins</i></div>	Notes																																				
3	S wheel position T: Taking $K=7$, we now need to find the position of the S wheel. How can we do this? <i>(Try each value in turn)</i> T: OK – starting with $S=1$ and using just the first 8 letters, what do we get? P (at board): <div style="border: 1px solid black; padding: 10px; width: fit-content; margin: 10px auto;"> <table style="border-collapse: collapse; text-align: center; font-family: monospace;"> <tr> <td style="padding: 2px 10px;">U</td> <td style="padding: 2px 10px;">D</td> <td style="padding: 2px 10px;">Z</td> <td style="padding: 2px 10px;">D</td> <td style="padding: 2px 10px;">M</td> <td style="padding: 2px 10px;">R</td> <td style="padding: 2px 10px;">+</td> <td style="padding: 2px 10px;">J</td> <td style="padding: 2px 10px;"></td> </tr> <tr> <td style="padding: 2px 10px;">+</td> <td style="padding: 2px 10px;">G</td> <td style="padding: 2px 10px;">H</td> <td style="padding: 2px 10px;">I</td> <td style="padding: 2px 10px;">J</td> <td style="padding: 2px 10px;">K</td> <td style="padding: 2px 10px;">L</td> <td style="padding: 2px 10px;">M</td> <td style="padding: 2px 10px;">N</td> </tr> <tr> <td style="padding: 2px 10px;">+</td> <td style="padding: 2px 10px;">A</td> <td style="padding: 2px 10px;">A</td> <td style="padding: 2px 10px;">B</td> <td style="padding: 2px 10px;">B</td> <td style="padding: 2px 10px;">A</td> <td style="padding: 2px 10px;">A</td> <td style="padding: 2px 10px;">B</td> <td style="padding: 2px 10px;">B</td> </tr> <tr style="border-top: 1px solid black;"> <td style="padding: 2px 10px;"></td> <td style="padding: 2px 10px;">V</td> <td style="padding: 2px 10px;">V</td> <td style="padding: 2px 10px;">C</td> <td style="padding: 2px 10px;">+</td> <td style="padding: 2px 10px;">T</td> <td style="padding: 2px 10px;">+</td> <td style="padding: 2px 10px;">V</td> <td style="padding: 2px 10px;">V</td> </tr> </table> <div style="text-align: right; margin-top: 5px;"> <i>(K wheel)</i> <i>(S wheel)</i> </div> </div> T: Does this message make sense? <i>(No!)</i> T: Your turn nowtry $S=2$ and then $S=3$. T: So we can confirm that $K=7$ and $S=3$ are the correct starting positions. What complicates the real problem? <i>(The starting position of all twelve wheels had to be determined and the number of possible starting positions of the wheels were greater. (They were all within the range 23 to 61.))</i> T: Yes – this provided the motivation for the development, at Bletchley Park, of 'Colossus', the world's first programable electronic computer. <div style="text-align: right;"><i>45 mins</i></div>	U	D	Z	D	M	R	+	J		+	G	H	I	J	K	L	M	N	+	A	A	B	B	A	A	B	B		V	V	C	+	T	+	V	V	Interactive discussion; Ps should be able to suggest the method. One P at board; other Ps can help with the calculations, etc. and check what is being written on board. T will need to give Ps sufficient time to complete this calculation and to confirm that $S=3$ gives the start of a meaningful message.
U	D	Z	D	M	R	+	J																															
+	G	H	I	J	K	L	M	N																														
+	A	A	B	B	A	A	B	B																														
	V	V	C	+	T	+	V	V																														
	Homework Go through the same process again with a new short message and confirm that this method of deciphering works.																																					

UNIT 19 *Lorenz Cipher Machine* Teacher Resource Material

Key Stage: 4 / A-level

Target: Gifted and talented students

This is a simplified model for the Lorenz cipher machine – but it is still quite complex. Although messages can be enciphered by following the instructions, deciphering is much more complicated. The method illustrated here simulates what actually happened at Bletchley Park in the Second World War, when the breaking of the Lorenz Code was a very significant breakthrough for the Allies.

We are particularly grateful to Frank Carter (of Bletchley Park) for providing a first version of this resource.

Solutions and Notes

- Exercise 1* $A + B \Rightarrow 11000 + 10011 = 01011 \Rightarrow G$
 $B + B \Rightarrow /$
 $C + B \Rightarrow 01110 + 10011 = 11101 \Rightarrow Q$
 $D + B \Rightarrow 10010 + 10011 = 00001 \Rightarrow T$
 $E + B \Rightarrow 10000 + 10011 = 00011 \Rightarrow O$

- Exercise 2* LONDON \Rightarrow 01001 | 00011 | 00110 | 10010 | 00011 | 00110
 HBVQZM + 00101 | 10011 | 01111 | 11101 | 10001 | 00111
 IELVDT \Leftarrow 01100 | 10000 | 01001 | 01111 | 10010 | 00001

- Exercise 3* IELVDT \Rightarrow 01100 | 10000 | 01001 | 01111 | 10010 | 00001
 HBVQZM + 00101 | 10011 | 01111 | 11101 | 10001 | 00111
 LONDON \Leftarrow 01001 | 00011 | 00110 | 10010 | 00011 | 00110

- Activity 1* $2^5 = 32$ codes.
 All codes are needed as adding two codes might give a code that is not used.

- Exercise 4* $S + E + A = G + A = U$
 $E + F + B = N + B = Y$
 $C + G + B = H + B = F$
 $R + H + A = V + A = X$
 $E + I + A = U + A = 9$
 $T + J + B = + + B = 4$
 $9 + K + B = J + B = L$
 $M + L + A = C + A = F$
 $E + M + A = X + A = V$
 $S + N + B = D + B = T$
 $S + A + B = I + B = 8$
 $A + B + A = G + A = B$
 $G + C + A = H + A = Q$
 $E + D + B = 3 + B = Z$

Enciphered message UYFX9 4LFVT 8BQZ

UNIT 19 *Lorenz Cipher Machine* Teacher Resource Material (continued)

Exercise 5 $U + E + A = I + A = S$
 $Y + F + B = O + B = E$, etc.

Exercise 6 $9 + G + B = V + B = U$
 $9 + H + B = T + B = D$
 $H + I + A = L + A = Z$
 $E + J + A = R + A = D$
 $R + K + B = S + B = M$
 $E + L + B = W + B = R$
 $9 + M + A = 0 + A = +$
 $9 + N + A = 3 + A = J$

i.e. UDZDMR+J

- Exercise 7* 1. $\Delta Z = COOYPZT$
 2. For starting position 7,

$$\begin{array}{cccccccc} \mathbf{K} & = & \mathbf{G} & \mathbf{H} & \mathbf{I} & \mathbf{J} & \mathbf{K} & \mathbf{L} & \mathbf{M} & \mathbf{N} \\ \Delta \mathbf{K} & = & \underbrace{\mathbf{C} \ \mathbf{L} \ \mathbf{F}} & \underbrace{\mathbf{9} \ \mathbf{X}} & \underbrace{\mathbf{C} \ \mathbf{T}} & & & & & \end{array}$$

$$\begin{array}{cccccccc} 3. & \Delta \mathbf{Z} & = & \mathbf{C} & \mathbf{O} & \mathbf{O} & \mathbf{Y} & \mathbf{P} & \mathbf{Z} & \mathbf{T} \\ & \Delta \mathbf{K} & = & \mathbf{C} & \mathbf{L} & \mathbf{F} & \mathbf{9} & \mathbf{X} & \mathbf{C} & \mathbf{T} \\ \hline & \Delta \mathbf{Z} + \Delta \mathbf{K} & = & / & \mathbf{R} & \mathbf{Y} & \mathbf{Z} & \mathbf{J} & \mathbf{8} & / \end{array}$$

4. There are two '/'s in this sequence.

Activity 2 No. of '/'s: $K = 1, 3; K = 2, 1; K = 3, 1; K = 4, 2; K = 5, 1; K = 6, 0; K = 7, 7$
 $K = 8, 1; K = 9, 1; K = 10, 2; K = 11, 1; K = 12, 6; K = 13, 2; K = 14, 2$

The greatest number of '/'s occur when $K = 7$.

Activity 3

$$\begin{array}{cccccccc} & \mathbf{U} & \mathbf{D} & \mathbf{Z} & \mathbf{D} & \mathbf{M} & \mathbf{R} & + & \mathbf{J} \\ + & \mathbf{G} & \mathbf{H} & \mathbf{I} & \mathbf{J} & \mathbf{K} & \mathbf{L} & \mathbf{M} & \mathbf{N} \\ + & \mathbf{B} & \mathbf{B} & \mathbf{A} & \mathbf{A} & \mathbf{B} & \mathbf{B} & \mathbf{A} & \mathbf{A} \\ \hline & \mathbf{9} & \mathbf{9} & \mathbf{H} & \mathbf{E} & \mathbf{R} & \mathbf{E} & \mathbf{9} & \mathbf{9} \end{array}$$

Hence $S = 3$ will recover the message.