"Let's think about writing maths sentences."

1 How much does it cost?

Look at this jar of sweets. Use the information on the label to answer the following questions.



1. What is the cost of:

   a) 3 bags of sweets?

   b) 10 bags of sweets?

   c) 100 bags of sweets?

   d) Complete the formula below:

   Cost (in pence) = ☐ x number of bags

2 How much will the journey cost?



Look at this advertisement for British Railway's weekend specials.

2. Use the information to find the cost of a weekend special for:

   a) 2 adults

   b) 2 adults and 3 children

   c) 4 adults and 8 children

   d) Now copy and complete the formula below:

Price = ☐ x number of adults + ☐ x number of children

In general ...

Look at these repeating patterns

Pattern 1  . . .

Pattern 2  . . .

Pattern 3  . . .

3 What do you think ... means?



Matthew

I think it means 'and so on'. The pattern keeps on repeating in the same way.

1. Look at Pattern 1. Describe the pattern as clearly as you can to someone who hasn't seen it. Would they be able to draw it from your description?
2. What colour would the 20th ball be?
3. How did you know?
4. Now, generalise you answer. Start “In general ...”
5. What colour would the 45th ball be?
6. How did you know?
7. Generalise your answer.



Sarah

In general, even number balls are red.

And in general, odd number balls are blue.



Jon

8. Now look at Pattern 2 and answer the same seven questions as you did for Pattern 1.
9. Finally, explain the difference between Pattern 2 and Pattern 3

4 How much will you pay for calls?



Look at this mobile phone offer...

1. Use the information to answer the following questions.

   How much would it cost to...

   a) Make a 1 minute call to a mobile?

   b) Make a 7 minute call to a mobile?

   c) Make an 80 minute call to a mobile?

   d) Copy and complete the formula below:

Cost (in pence) = ☐ x number of minutes

5 How much will you pay for football tickets?





To see a home match at Portsmouth Football Club, it costs £20 for an adult and £10 for a child (15 and under).

2. Use the information to find the cost of a match for:

   a) 6 adults

   b) 2 adults and 4 children

   c) 9 adults and 11 children

   d) Complete the formula below:

Price = ☐ x number of adults + ☐ x number of children

What comes next?



Amber

How can we extend this pattern?



Shape 1 Shape 2 Shape 3

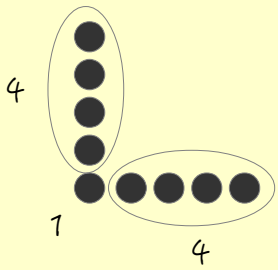
1. Can you draw the next shape in the sequence?
2. Can you describe in words what the tenth shape would look like?
3. How many counters would it have?
4. Can you describe shape 100?
5. How many counters would it have?
6. Can you generalise your answer?



Matthew

Matthew's solution

Shape 4 looks like this...



So shape 10 will have one dot in the corner and two arms of 10 counters, so 21 counters altogether.
Shape 100 will have one in the corner and two arms of 100 each, so 201 counters altogether.
In general there is always one counter in the corner and two arms with the same number of dots in each as the shape number.

6 Using Matthew's idea, how many counters will be in shape 1000?

Question Creation



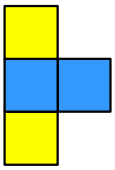
Question creation - Have fun creating problems for your friends to solve. Make up your own growing shape or pattern. Describe it to your friend in words and see if they can draw it.
Ask them to decide if your description was accurate.

Generalising

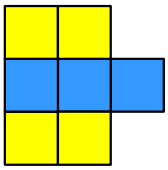


Jon

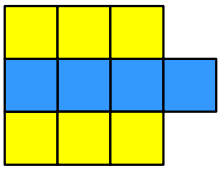
My growing shape looks like this



Shape 1



Shape 2



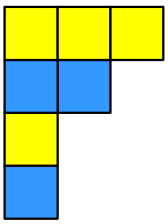
Shape 3

1. Can you describe what the tenth shape would look like?
2. How many blue boxes would it have?
3. How many yellow boxes would it have?
4. Can you describe shape 100?
5. How many blue boxes would it have?
6. How many yellow boxes would it have
7. Can you generalise your answers?

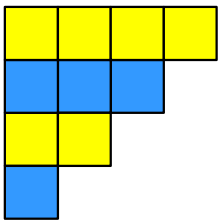


Sarah

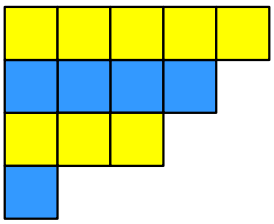
My growing shape looks a bit like Jon's



Shape 1



Shape 2



Shape 3

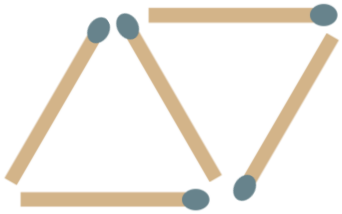
7

Answer Jon's questions but using Sarah's pattern.

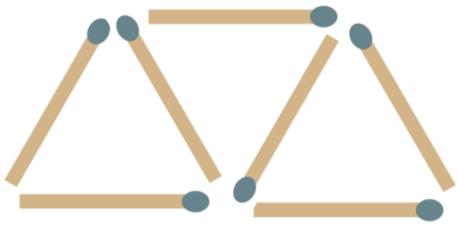
How many matches do you need?



Shape 1



Shape 2



Shape 3

1. Can you draw the next shape in this sequence?
2. How many matches are added each time?
3. Can you describe what the tenth shape would look like?
4. How many matches would it have?
5. Can you describe shape 100?
6. How many matches would it have?
7. Can you generalise your answer?



Alice

Sometimes it helps me to put my results in a table and see if I can spot any patterns.

Shape number	1	2	3	4	5
Number of matches	3	5	7	9	11

+ 2 + 2 + 2 + 2



Alice

So the Shape 10 will have 3 + (9 x 2) = 3 + 18 = 21 matches.
And Shape 100 will have 3 + (99 x 2) = 3 + 198 = 201 matches.

Well done, Alice. That is a great help!



Jon

Number sequences

The number of matches required in the previous question formed a **number sequence**.

3, 5, 7, 9, 11, ...

A **number sequence**, in mathematics, is a string of numbers that follow a particular pattern. The individual elements in the sequence are called terms.

In the sequence above, the first **term** is 3; the second term is 5; the third term is 7 and so on.

The next term can be generated by adding 2 each time.

This is called a **term to term rule**.

Quite often, we want to generate number sequences without drawing pictures.

In which case, you may be given a starting term and a rule to generate the additional terms.

There are two ways of doing this but, for now, we will concentrate on the **term to term** approach.

Term to term rules

8 The first term of a sequence is 5. The term to term rule is add 4. Write down the first 5 terms of this sequence.



Alice: That's easy. 5, 9, 13, 17, 21, ...



Sarah: I see, the terms just go up by 4 each time.

Fitness check



In each question, describe the sequence if possible and write down the term to term rule.

1. 6, 12, 18, 24, 30, …
2. 1, 3, 5, 7, 9, …
3. 2, 4, 6, 8, 10, …
4. 1, 2, 4, 8, 16, …
5. 1, 3, 7, 15, 31, ...



Evie

Evie's work

1. 6, 12, 18, 24, 30, ··· six times table, term to term rule add 6
2. 1, 3, 5, 7, 9, ··· odd numbers, term to term rule add 2
3. 2, 4, 6, 8, 10, ··· even numbers, term to term rule add 2
4. 1, 2, 4, 8, 16, ··· term to term rule multiply by 2
5. 1, 3, 7, 15, 31, ··· term to term rule multiply by 2 and add 1.

9 The first term of a sequence is 4. The term to term rule is to multiply by 2 and then take away 3. Write down the first 5 terms of this sequence.



Jon

That's easy. 4, 5, 7, 11, 19, ...

I noticed something else very interesting about how the numbers increase.



Sarah

10 What do you think that Sarah noticed?



Sarah

Sarah's work

4, 5, 7, 11, 19, 35

+1 +2 +4 +8 +16

The difference doubles each time. So the next number should be 19 + 16 = 35

Check

(19 x 2) − 3 = 38 − 3 = 35. I was right.

Fitness check

In each of the questions, write down the first 5 terms.

1. Start number 7, term to term rule add 6.
2. Start number 20, term to term rule take away 3.
3. Start number 6, term to term rule multiply by 3, take away 9.
4. Start number 144, term to term rule divide by 2.
5. Start number 16, term to term rule divide by 2, add 8.

2, 5, 8, 11, ...

Look carefully at this number sequence for 30 seconds.

Now tell your partner exactly what you see.

What do you think '...' means?

What does this make you wonder?



Matthew

Well, it starts with a 2 and then goes up in threes.
I know the '...' means 'and so on'.
I wonder what the 20th term might be?

11 What is the 20th term?



Sarah

If you use Alice's method it will be
2 + 19 x 3 = 2 + 57 = 59

I was wondering how many terms you would need to get over 100?



Evie

12 How many terms would you need before you got over 100?



Jon

Well, it's more than 20. I think about 30.
The 30th term will be 2 + 29 x 3 = 2 + 87 = 89
Not enough, so it's more than 30.
Now I know the answer, it must 34!

13 How did Jon know it must be 34?



Amber

Jon knew he needed at least another 11.
And, as it goes up in 3's, he would need 4 more terms.
Check it out for yourself!

What's missing?

Look at this number sequence

3, 7, ☐ , 15, 19, ☐ , ...

1. What are the missing numbers?
2. Describe your sequence in words to someone who hasn't seen it before.
3. What is the 15^{th} term?
4. How many terms do you need before you pass 150?



Alice

Alice's work

The missing numbers are 11 and 23.
The sequence starts with 3 and goes up in 4's.
The 15^{th} term is $3 + 14 \times 4 = 3 + 56 = 59$
First guess 35 terms $3 + 34 \times 4 = 3 + 136 = 139$
I need another 11, so 3 more terms
Check 38^{th} term is $3 + 37 \times 4 = 3 + 148 = 152$
I was right!

Fitness check



In each of the following questions, find the missing numbers, describe the sequence in words and determine how many terms you would need before you passed 200.

1. 7, ☐ , 17, 22, ☐ , ...
2. ☐ , 8, 15, ☐ , 29, ☐ , ...
3. 8, 14, 20, ☐ , ☐ , 38, ...
4. 3, 11, ☐ , ☐ , ☐ , 43, ...

Question Creation



Have fun creating problems for your friends to solve. Make up your own number sequence and ask your friends some questions of your own.

14 The first term of a sequence is 18. The term to term rule is if the number is even halve it; if it is odd add one. Investigate what happens to the number sequence.


Jon

Well it starts 18, 9, 10, ...

... 5, 6, 3, 4, 2, 1, ...


Amber


Matthew

... 2, 1, 2, 1, ... and so on.


Sarah

I wonder what happens if you start with a different number?

15 Investigate what does happen if you start with different numbers.

Position to term rule

Earlier, we said there are two ways of generating number sequences. In the matches problem, Matthew used a completely different approach.


Matthew

Matthew's work

shape	1	2	3	4	5	6
matches	3	5	7	9	11	13

I've spotted another pattern. If you multiply the shape number by 2 and then add 1, you get the number of matches needed. So for shape 100, you need (100 x 2) + 1 = 201 matches.


Alice

You could write this as a formula
Number of matches = (2 x shape number) + 1

Position to term rules

Alice's idea is called a position to term rule or **general rule**.
It means you can find a specific term in a sequence without finding all of the other terms.
Generally, this is a much quicker approach.

16 The general rule for a sequence is to multiply the term number by 4 and add 2. Write down the first 5 terms, the 10^{th} term and the 100^{th} term.


Evie

Evie's work

First term = (1 x 4) + 2 = 6
Second term =(2 x 4) + 2 = 10
Third term = (3 x 4) + 2 = 14
Fourth term =(4 x 4) + 2 = 18
Fifth term = (5 x 4) + 2 = 22
10^{th} term = (10 x 4) + 2 = 42
100^{th} term = (100 x 4) + 2 = 402

Fitness check



In each of the questions, you are given the general rule. Generate the first 4 terms, the 12^{th} term and the 50^{th} term, from the general rule.

1. Multiply by 5, add 3
2. Multiply by 4, take away 3
3. Multiply by 10, take away 7
4. Multiply by 5, take away 1
5. Multiply by 4, add 2
6. Multiply by 10, add 5


Sarah

I think I have noticed something quite interesting ...

17 What do you think Sarah noticed?

Tidying up

Giving the general rule for a sequence in words is time consuming, so we use a shorthand approach.
For example, if the general rule is multiply the term number by 6 and then add 3 we would simply write

$$n^{th} \text{ term} = 6n + 3$$

where n simply is a shorthand notation for the term number


Amber

Amber's work

First term = 1 x 6 + 3 = 9
Second term = 2 x 6 + 3 = 15
Third term = 3 x 6 + 3 = 21
Fourth term = 4 x 6 + 3 = 27
Fifth term = 5 x 6 + 3 = 33
12^{th} term = 12 x 6 + 3 = 75
50^{th} term = 50 x 6 + 3 = 303

Fitness check



Write these general rules in shorthand form.

1. Multiply by 7 add 5
2. Multiply by 9 take away 4
3. Multiply by 12 add 3

Write the shorthand form for the general terms in words.

4. n^{th} term = $5n - 4$
5. n^{th} term = $2n + 3$
6. n^{th} term = $n + 4$
7. What is the 10th term of the sequence if the n^{th} term is $3n + 1$
8. What is the 8th term of the sequence if the n^{th} term is $3n + 6$
9. What is the 5th term of the sequence if the n^{th} term is $4n + 1$
10. What is the 9th term of the sequence if the n^{th} term is $5n - 3$
11. Write down the first three terms, the tenth term and the 100th term, if the n^{th} term is $9n - 4$

Algebra 1a: Mastery questions

1. What are the next two terms in this sequence?

   4, 9, 14, 19, 24, ...

A	28, 33, ...	B	29, 34, ...
C	34, 39, ...	D	39, 44, ...

2. Look at this number sequence. What are the missing numbers?

   3, ☐, 17, ☐, 31, 38, ...

A	10, 25, ...	B	9, 25, ...
C	9, 24, ...	D	10, 24, ...

3. What is the term to term rule for this sequence?

   6, 11, 16, 21, 26, ...

A	The term increases by adding 6
B	The term is multiplied by five and add 1
C	The term increases by adding 5
D	The term is a multiple of 5

4. What are the next two terms in this sequence?

   3, 6, 12, 24, 48, ...

A	60, 72, ...	B	72, 96, ...
C	72, 84, ...	D	96, 192, ...

5. Look at this number sequence. How many terms would you need to exceed 100?

   6, 14, 22, 30, 38, ...

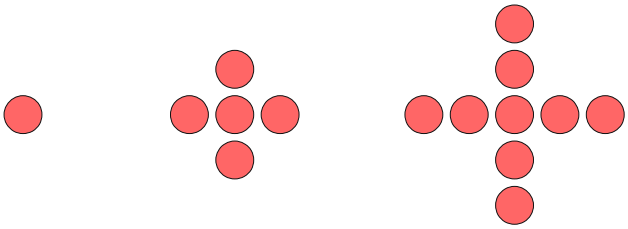
A	11	B	12	C	13	D	14

6. Look at this repeating pattern. Think about the 20th and 21st marbles. What colours would they be?



A	blue, red	B	blue, blue
C	red, blue.	D	red, red

7. Here are the first three patterns in a sequence. How many circles will there be in pattern 4?



Pattern 1 Pattern 2 Pattern 3

A	4	B	12	C	13	D	14

8. If the pattern continues, how many circles will there be in pattern 10?

A	37	B	39	C	41	D	43

9. This formula gives the cost of a ferry crossing:

   Cost = £50 per car + £20 per passenger.

   What is the cost for a car and 3 passengers?

A	£70	B	£170	C	£210	D	£110

10. This rule gives the next number in the number chain:

    Subtract 3 and then multiply by 2.

    7 Í 8 Í 10 Í 14 Í ...

    What is the next number in the chain?

A	36	B	22	C	25	D	33

11. The 'position to term' rule for a sequence is to 'multiply the term number by 4 and add 7'. What is the 5th term in this sequence?

A	5	B	20	C	27	D	39

12. The nth term of a sequence is $2n + 5$. What is the 8th term in this sequence?

A	21	B	8	C	13	D	26

13. Look at this sequence. What is the position to term rule?

8, 15, 22, 29, 36, ...

A	The term increases by adding 7
B	The term number is multiplied by seven, then add 1
C	The number is multiplied by 8, then take away 1
D	None of these

14. Look at this sequence. What is the nth term?

2, 11, 20, 29, 38, ...

A	+ 9	B	$2n + 9$
C	$9n - 7$	D	$2 + 9n$