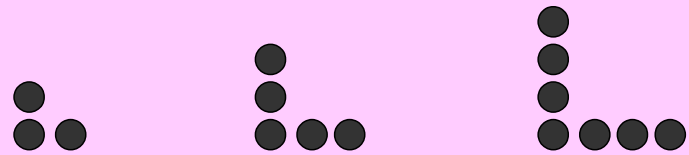
What do you know about sequences?

Look at this repeating pattern.



Describe the pattern as clearly as possible to someone who hasn't seen it .

Look at this growing pattern.



Shape 1 Shape 2 Shape 3

Describe Shape 10 to someone who hasn't seen it.

Look at this number sequence.

2, ☐, 16, ☐, 30, 37, ...

What are the missing numbers?

Look at this number sequence.

4, 13, 22, 31, 40, ...

How many terms would you need to exceed 100?

What happens when we add 3 consecutive integers?

1 What do we mean by “consecutive integers”?


Alice

1, 2, 3, ...


Evie

43, 44, 45, ...


Jon

7, 8, 9, ...


Matthew

98, 99, 100, ...


Sarah

16, 17, 18, ...

Adding up consecutive integers

2 For each of the suggestions above, add up the 3 consecutive integers. What do you notice?


Sarah

1 + 2 +3 = 6
7 + 8 + 9 = 24
43 + 44 + 45 = 132
Wow, they all divide by 6!


Matthew

No they don't!
16 + 17 + 18 = 51
98 + 99 + 100 = 297
But they do all divide by 3!


Amber

In general, if you start with an odd number, the answer is divisible by 6 ...


Evie

... but if you start with an even number, the answer is divisible by 3 not 6.

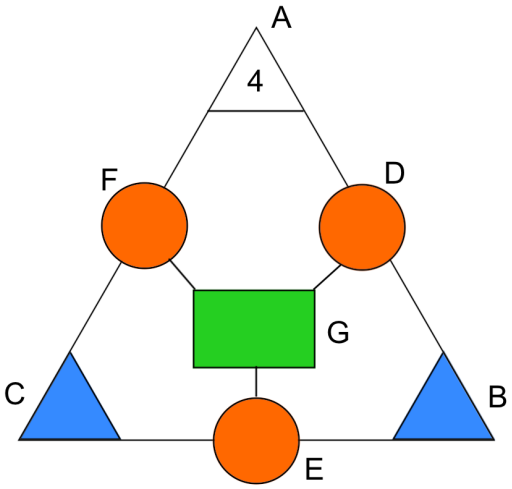

Alice

I've noticed something else. The answer is always 3 times the middle number. That makes finding the total really easy!

3 Using Alice's idea, can you find 3 consecutive integers that add up to 240?

But what goes in G?

This is a consecutive number puzzle.





Alice

Well, I think 5 should go at B. Consecutive numbers and consecutive letters.

Agreed. So 6 should go at C.



Jon



Matthew

But what about D? Should it be 7?

That's too obvious for a puzzle. What else could it be?



Sarah

4 Well, what else could belong at D?



Evie

It could be 4.5. That's halfway between the numbers at A and B.

Or it could be 9. I just added the numbers at A and B together.

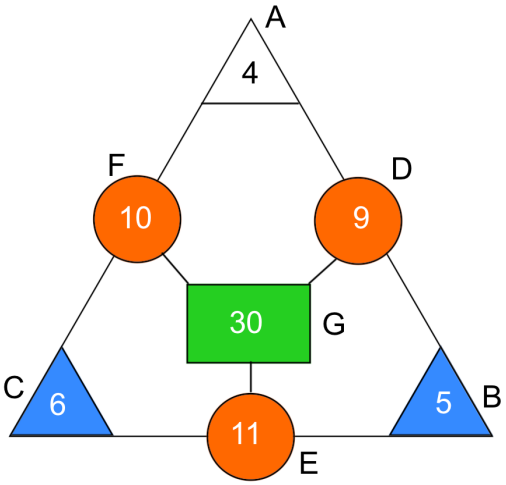


Amber

5 If Amber is correct, what goes at E and F? And then what goes at G?

But what goes in G?

This is the solution to our consecutive number puzzle.



6 So where did the 30 at G come from?


Alice

… by adding up the numbers at D, E and F.

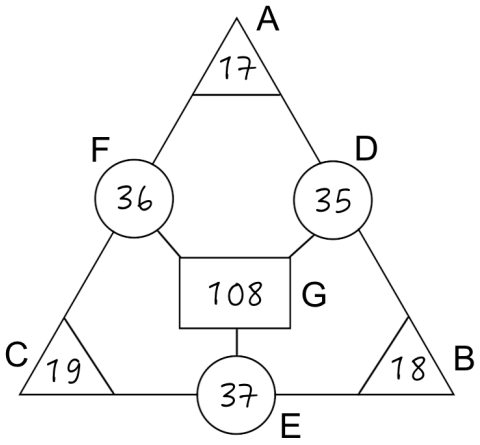
What happens if we start with a different number at A?


Jon

7 Try different starting numbers for yourself.

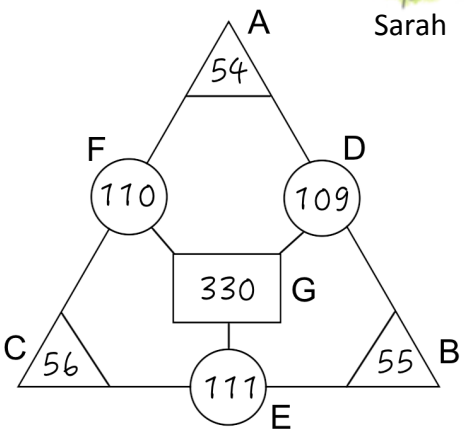

Evie

Evie's solution



Sarah's solution


Sarah

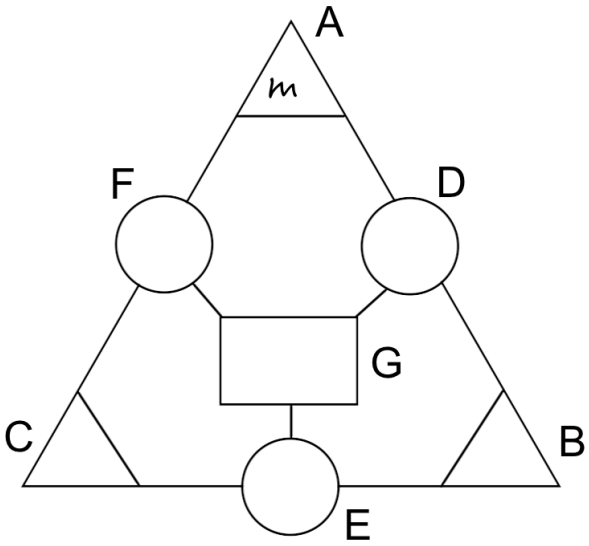


But what goes in G?



Matthew

I wanted to start with one million, but it wouldn't fit. So I wrote *m* for million.

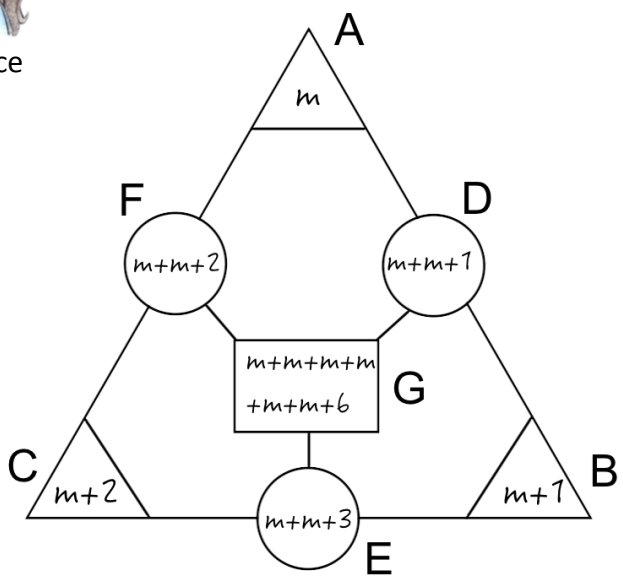


8 What did Matthew write in the other shapes?



Alice

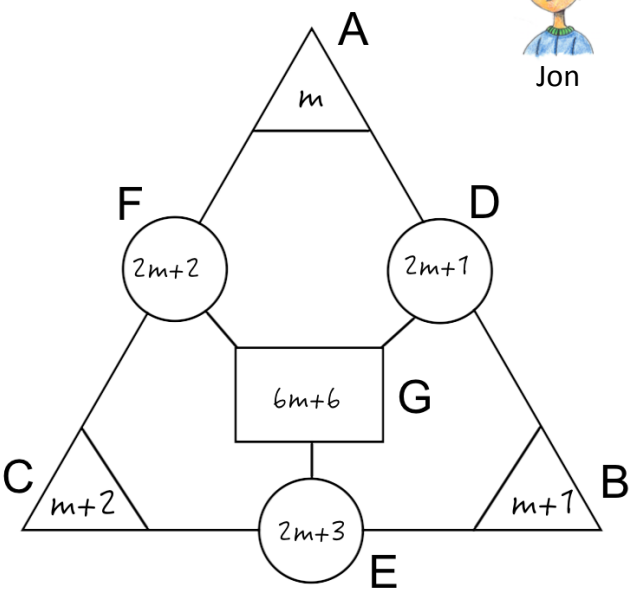
Alice's solution



Jon's solution



Jon





Sarah

That's really clever.
So 2*m* + 1 really means 2 million and 1.
Neat!

And so ...
6*m* + 6 means 6 million and 6. Very neat indeed!



Evie

Generalising the result



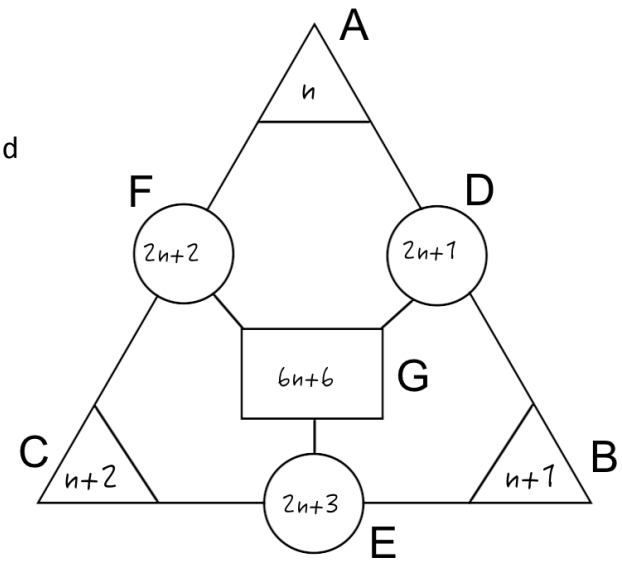
Amber

So if I wanted to start with any number, I could write n, and n could be anything!



Sarah

So 2n + 1 means multiply your number by 2 and then add 1.



9 If we use Amber's idea, if n = 1000, what goes in C, D and G?

Fitness check



1. If n = 50, what goes in E? What goes in G?
2. If n = 90, what goes in C? What goes in G?
3. If n = 250, what goes in D? What goes in G?
4. If n = 500, what goes in B? What goes in G?

The language of algebra

Algebra is firstly a language. It is a concise way of communicating your ideas to other people. It underpins any study of mathematics, science, or engineering, and such applications as medicine and economics. It differs from arithmetic in the use of abstractions, such as using letters to stand for numbers that are either unknown or allowed to take on many values.
Consequently it is very useful in generalising.

Conventions

Of course, other people need to understand your work, and so there are some **universal conventions** that we all have to observe.

Firstly, we always write $6n$ to mean 6 x n. But, as multiplication is really the same as repeated addition; it also means $n + n + n + n + n + n$.

In the product of a number and a letter, the number is always written in the front. There is one exception we do not write $1n$; we simply write n as we did above.

If we want to represent the product of two letters a and b, we don't write a x b; we simply write ab.

Because the letters act just like integers, addition and multiplication are **commutative**. That means the order doesn't matter.

So $a + b = b + a$ and a x $b = b$ x $a = ab$. We usually write products of letters in alphabetical order.

Algebraic notation

Fitness check

Write these expressions concisely using the universal conventions.



1. $a + a + a + a$
2. 3 x b
3. c x 5
4. $2d + d + 3d$
5. $4e + e$ x 3
6. 4 more than n
7. 3 less than n
8. Twice as big as n

Substitution

Replace the letters with the number below.

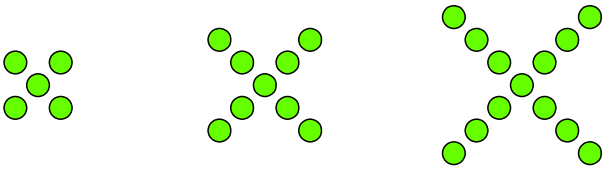
If $a = 2$, $b = 6$, $c = 10$ and $d = 3$, calculate:

1)	$a + b$	5)	$c - b$	9)	$d + 7$
2)	$3a + d$	6)	$4a$	10)	ad
3)	$3b$	7)	$2c$	11)	$3c - b$
4)	$6a + b$	8)	$3a + 2b$	12)	$4a - d$

Match these algebra cards:

2 more than n	2 x n	2 less than n	$n + n$	$2n + 2$
$2n + 1$	$n + 2$	n x 2	$n - 2$	$2n$
Double n	$n + n - 1$	3 x $n + 2 - n$	$n - 1 + n$	$3n - n + 1$

How many dots do you need?



Shape 1 Shape 2 Shape 3

1. Can you draw the next shape in this sequence?
2. How many dots are added each time?
3. Can you describe what the tenth shape would look like?
4. How many dots would it have?
5. Can you describe shape 100?
6. How many dots would it have?
7. Can you generalise your answer in words?
8. Can you generalise using algebra?



Alice

Alice's work

1



Shape 4

2. You add 4 dots each time
3. The 10th shape has a dot in the centre, and each arm has 10 dots
4. So it has 41 dots!
5. Shape 100 would have 1 dot in the middle and then each arm has 100 dots
6. so it has 401 dots.
7. You always have a dot in the middle then 4 arms each with the shape number of dots – so 1 plus 4 x shape number.
8. For shape n, you would have 1 + 4n

Question Creation



Have fun creating problems for your friends to solve. Make up your own growing shape or pattern. Describe it to your friend in words and see if they can draw it. Try to generalise your results using algebra. Ask your friends to do the same.
Check to see if you end up with the same result.

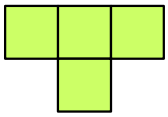
Growing patterns



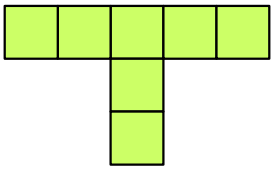
Jon

My growing shape looks like this. In general, how many boxes do I need?

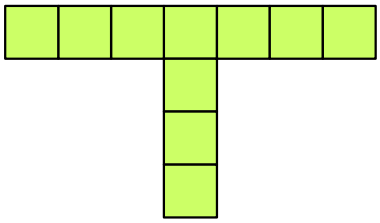
Shape 1



Shape 2



Shape 3

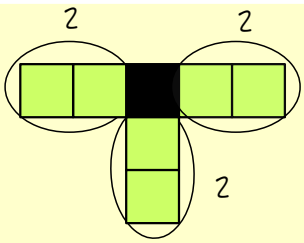




Evie

Evie's work

Jon always has one box in the middle of the top row and then adds on the shape number of boxes three times.

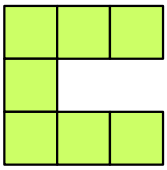


For example, shape 2 has 1 + (3 x 2) = 7 boxes.
So for shape 10, he would need 1 + (3 x 10) = 31 boxes.
For shape 100, he would need 1 + (3 x 100) = 301 boxes.
In general he needs 1 + (3 x the shape number).
Using algebra, for shape n, he needs 1 + 3n boxes.

10 Using Evie's formula, how many boxes does Jon need if n = 300, 1000 or 100 000?

The idea of generalising from the shape number is a very powerful tool to have in your maths toolbox.

For example, look at the diagram below for 30 seconds.



Shape 3

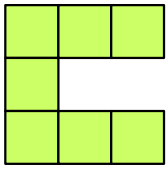
1. Tell your partner exactly what you **see**
2. Why do you **think** this is called shape 3?
3. What does this make you **wonder**?

Growing patterns



Sarah

If this is shape 3, I wonder what shape 4 looks like?



Shape 3



Matthew

And I wonder what shape 2 looks like?

Well, in that case, I wonder what shape 1 looks like?



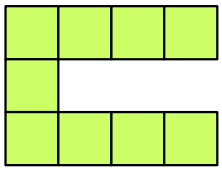
Amber

11 What diagrams would you draw for shape 4, shape 2 and shape 1?



Alice

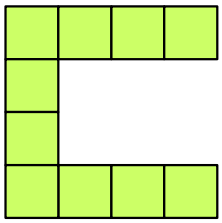
This is what I think shape 4 would look like.





Evie

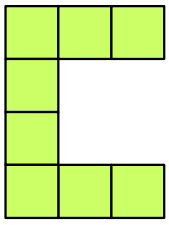
Oh, I thought it would look like this.





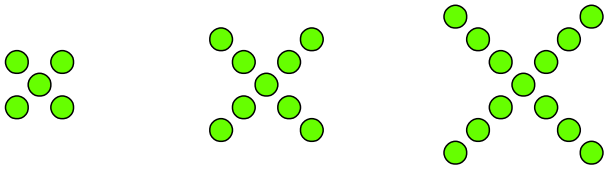
Jon

I think it should look like this.



12 For each of these ideas, explain how the shape number fits into their pattern.

How many dots do you need?



Shape 1 Shape 2 Shape 3



Alice

Sometimes it helps me to put my results in a table and see if I can spot any patterns.

Shape number	1	2	3	4	5
Number of dots	5	9	13	17	21





Alice

So the Shape 10 will have 5 + (9 x 4) = 5 + 36 = 41 dots.
And Shape 100 will have 5 + (99 x 4) = 5 + 396 = 401 dots.



Jon

I've got an easier way of working it out. Use your general result!
We know we have $1 + 4n$ dots for shape n, so for shape 10 we have 1 + (4 x 10) = 1 + 40 = 41 dots.



Matthew

So if we know the rule, we can easily work out the answer!

13

If the rule for the number of dots is $3n + 1$, write down how many boxes you would need for shapes 1, 2, 3, 4, 5, 6, ...

Generating number sequences

In the last question, the rule for the number of dots was $1 + 3n$.

This rule generates a number sequence as we choose different values of n.



Amber

Amber's work

n	1	2	3	4	5	6
dots	4	7	10	13	16	19

Some important terminology

A **number sequence**, in mathematics, is a string of numbers that follow a particular pattern. The individual elements in a sequence are called terms. Some of the simplest sequences can be found in multiplication tables.

Quite often, we want to generate number sequences without drawing pictures. You need to be familiar with two different ways this might arise.

Sometimes you will be given the rule as a **mapping**. For example, the rule above could be written as

$$n \to 1 + 3n \text{ or } n \to 3n + 1$$

But sometimes you will be given a **general rule** for the **n^{th} term**. For example, the rule above could be written as

$$n^{th} \text{ term} = 1 + 3n \text{ or } n^{th} \text{ term} = 3n + 1$$

14 If the n^{th} term of a sequence is $4n + 3$, write down the first 6 terms.



Alice

So the first term is 7

7, 11, 15, ...



Sarah



Matthew

..., 19, 23, 27, ...

I think I have noticed something...



Jon

15 What do you think Jon noticed?



Jon

Did you notice that sequence went up in 4's?

I wonder if that has anything to do with the n^{th} term being $4n + 3$?

16 Check out Jon's idea with the mapping $n \rightarrow 3n + 2$



Alice

The sequence is 5, 8, 11, 14, …

It goes up in 3's – so Jon is right.

Fitness check



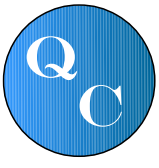
In each of these questions, write down the first 3 terms of the sequences, the 10^{th} term 50^{th} terms.

1. $n \rightarrow 2n$
2. $n \rightarrow 3n + 4$
3. $n \rightarrow 5n - 3$
4. n^{th} term is $6n$
5. n^{th} term is $7n + 4$
6. n^{th} term is $9n - 2$
7. $n \rightarrow 10 - 2n$
8. $n \rightarrow 18 - 5n$

17 Match these sequences with their generating function

1, 5, 9, 13, ...	$n \rightarrow 6n - 1$
2, 3, 4, 5, ...	$n \rightarrow 4n - 3$
5, 11, 17, 23, ...	$n \rightarrow 3n - 1$
5, 9, 13, 17, ...	$n \rightarrow n + 1$
2, 5, 8, 11, ...	$n \rightarrow 4n + 1$

Question Creation



Have fun creating problems for your friends to solve. Make up your own number sequences and challenge your friends to find your rule.

More terminology

So far, the sequences we have looked at have been going up by the same amount each time. Such sequences are called **linear sequences**. They can be uniquely defined by the **initial term** (that's the first term) and the **common difference** (that's the amount it goes up by each time or the difference between successive terms).

For example, suppose the initial term is 5, and the common difference is 3, then the sequence is

5, 8, 11, 14, 17, ...

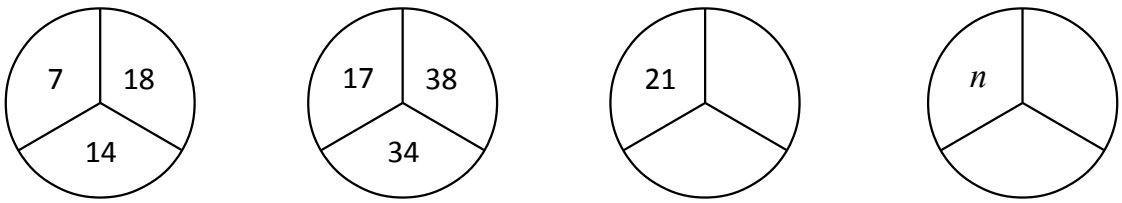
Fitness check



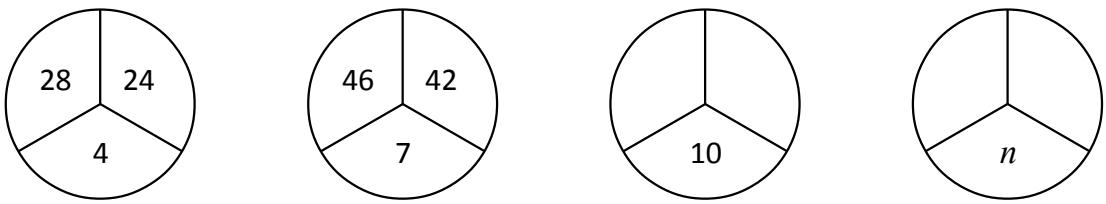
1. Write down the first 4 terms and the 10^{th} term of the following linear sequences.
   a) Initial term = 2, common difference = 4
   b) Initial term = 4, common difference = 2
   c) Initial term = 3, common difference = 7
   d) Initial term 99, common difference = -10
2. In these sequences, write down the initial term and the common difference.
   a) 6, 11, 16, 21, …
   b) -3, 1, 5, 9, …
   c) 21, 19, 17, 15, …
   d) 51, 42, 33, 24, ...
3. Which of these are not linear sequences?

   Write down the next term in each case.
   a) 1, 7, 13, 19, 25, …
   b) 4, 9, 15, 22, 30, …
   c) 2, 6, 18, 54, 162, …
4. What are missing numbers in the number patterns below?

   a)



   b)



Writing Formulas

What is the (P) Perimeter of this rectangle?





Alice

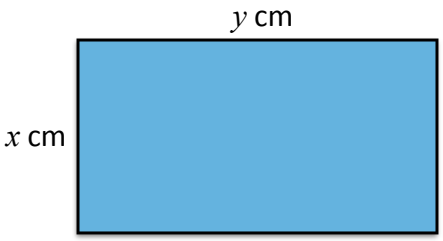
What happens if we do not have the numbers?

I think we can still write a formula without them...



Jon

What is the (P) Perimeter of this rectangle?





Evie

I would write the formula as
P = $x + y + x + y$

I would write the formula as
P = $2x + 2y$



Matthew

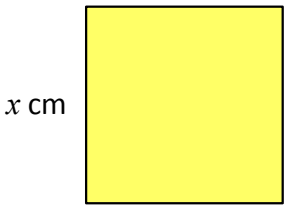


Sarah

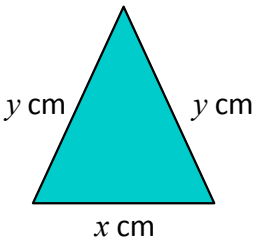
Well I would write the formula as
P = $2(x + y)$

18 How would each of the students write the formula for the perimeter of these shapes?

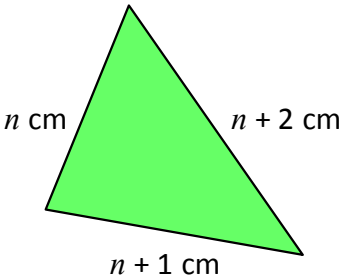
Square



Isosceles triangle



Scalene triangle



Fitness check



Simplify the following expressions, where possible:

1) $2a + 3b$

2) $6c - 4c$

3) $6e + 9e - 5e$

4) $9g + 7g - 8g - 2g - 6g$

5) $3a + 4b - 2a$

6) $8t - 6t + 7s - 2s$

7) $5b + 8b$

8) $5d + 4d + 7d$

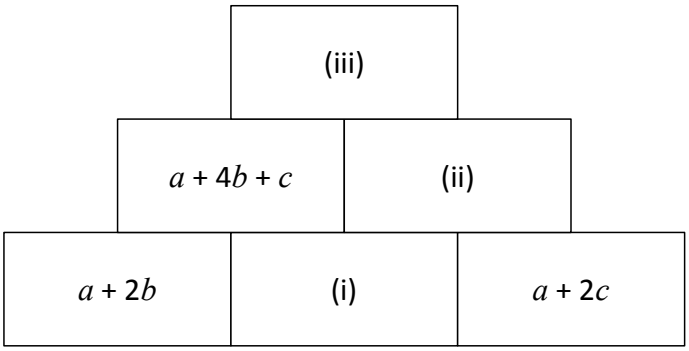
9) $8f + 6f - 13d$

10) $5p + 2h$

11) $11m + 3n - 5p + 2q - 2n + 9q - 8m + 14p$

Algebra walls

Each brick in this algebra wall is the sum of the expressions in the two bricks underneath it.



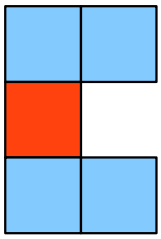
What expressions are missing in (i), (ii) and (iii)?

Algebra 2a: Mastery questions

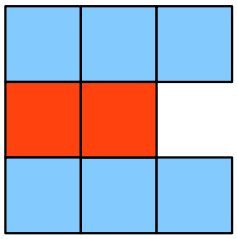
1. The nth term of a sequence is $2n + 5$. What is the 10th term of this sequence?

A	25	B	70	C	52	D	30

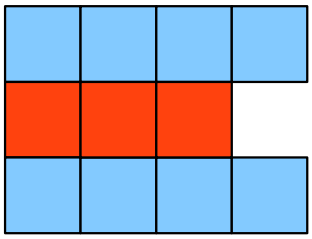
2. Here is a pattern made of red and blue tiles.



Shape 1



Shape 2



Shape 3

A shape has 12 red tiles. How many blue tiles does it have?

A	13	B	24	C	26	D	38

3. The first three terms of a linear sequence are

12, 17, 22, …

What is the rule for the nth term of this sequence?

A	$5n + 12$	B	$5n + 7$	C	$12n + 5$	D	$5n$

4. A linear sequence has an initial term of 3 and a common difference of 7.
What is the 4th term of this sequence?

A	28	B	16	C	31	D	24

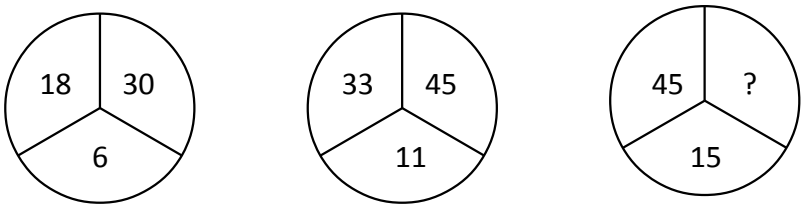
5. A linear sequence has a common difference of 9. The 5th term of the sequence is 49.
What is the first term of this sequence?

A	4	B	13	C	9	D	14

6. Which of the following statements are true?

A	$4x + x = 4x$	B	$4x + 2x = 8x$
C	$2x + 3y = 5xy$	D	$4x - 2x = 2x$

7. What is the missing number in this number pattern?

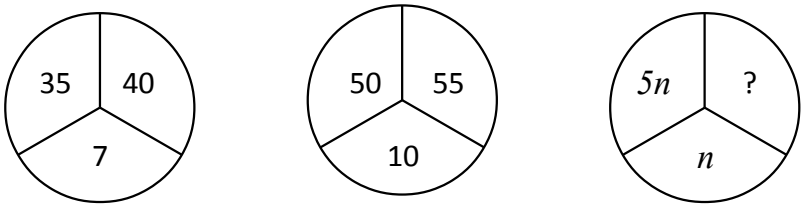


A	33	B	65	C	57	D	61

8. Which of these is not a linear sequence?

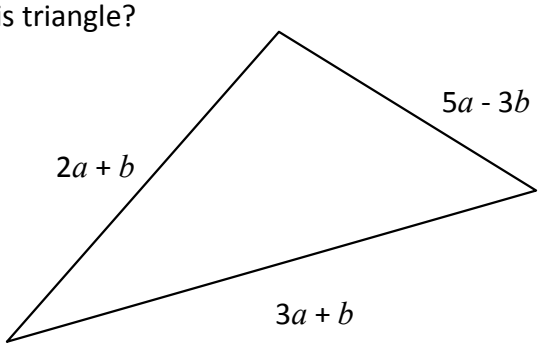
A	3, 6, 12, 24, ...	B	2, 5, 8, 11, ...
C	26, 23, 20, 17, ...	D	3, 6, 9, 12, ...

9. What is the missing expression in this number pattern?



A	$6n$	B	$4n$	C	$5n + 10$	D	$5n + 5$

10. What is the perimeter of this triangle?



A	$8a + 5b$	B	$9ab$	C	$10a + b$	D	$10a - b$