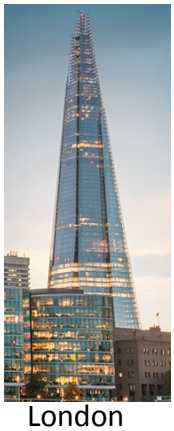
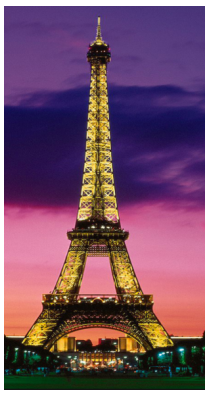
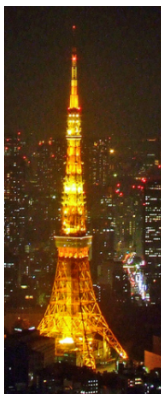
Shopping for a new phone?



London
£719



Paris
€939



Tokyo
96 800JPY

Recently a well known mobile phone manufacturer released their latest phone simultaneously across the world. Not surprisingly, the price looked very different in each city.

1 Which of these deals offers the best value?

Exchange rates



Alice: You need to know the exchange rate.
I think £1 is about 1.2 euros.



Jon: … and £1 is 140 Japanese yen.



Amber: How does that help?



Sarah: You can use a function machine!

Input 719 → [x 1.2] → 868.8 output



Evie: So £719 is equivalent to €868.9, so it is cheaper to buy it in London than in Paris.



Matthew: Input 719 → [x 140] → 100 660 output

So £719 is equivalent to 100 660JPY, so it is cheaper to buy it in Tokyo than London.

Function machines

A function machine acts as a simple computer.
It just follows your instructions.
You input a number, and the function machine carries out the calculations and outputs the answer. It is very useful for carrying out repetitive tasks.

Going on holiday?

Three friends decided to go on a holiday of a lifetime but, unfortunately, they couldn't agree on where to go, so they ended up in different places. They were each given £1000 spending money which they converted into local currency. They used their function machines to keep track of their spending in pounds.

Evie went to Australia (Adollars).

Input (Adollars) → [x 0.58] → (pounds) output

Jon went to India (Rupees).

Input (rupees) → [x 0.012] → (pounds) output

Alice went to New York (Dollars).

Input (dollars) → [x 0.78] → (pounds) output

Comparing prices

Every day they would contact each other on Facebook to compare how much they had spent.

After 2 days



Evie

I've spent 258 Adollars



Jon

I've spent 10 000 rupees



Alice

I've spent 180 dollars

2 How much has each spent so far in pounds?

After 4 days



Evie

I've spent 532 Adollars



Jon

I've spent 23 300 rupees



Alice

I've spent 410 dollars

3 How much has each spent so far in pounds?

After 8 days, Evie had spent $1680, Jon had spent 82 000 rupees and Alice had spent A$1350.

4 Who had overspent?

Two step function machines

This function machine is used to convert degrees Centigrade (°C) into degrees Fahrenheit (°F)

Input (degrees °C) → [x 1.8] → [+ 32] → (degrees °F) output

Use this function machine to convert:

a) 0°C into °F (Freezing point of water).
b) 100°C into °F (Boiling point of water).
c) 20°C into °F (A warm day).
d) 30°C into °F (A hot day).
e) 356.7°C into °F (Boiling point of mercury).
f) -38.8°C into °F (Freezing point of mercury).
g) -173°C into °F (Night time temperature on the Moon).
h) 36.9C into °F (Normal body temperature).

More function machines

Look at this function machine

Input 3 → [x 2] → [+ 4] → 10 output

5

Can you explain what is happening?



Amber

Amber's working

input (3) x 2 = 6
6 + 4 = (10) output ✓

Fitness check

What are the outputs in these function machines?



1. Input 3 → [x 4] → [-7] → ? output
2. Input 10 → [- 6] → [x 3] → ? output
3. Input 8 → [- 5] → [x 5] → ? output
4. Input -2 → [x 7] → [+ 8] → ? output
5. Input 7 → [+ 9] → [⊟ 8] → ? output
6. Input 5 → [x 6] → [÷ 10] → ? output
7. Input 5 → [x 6] → [+ 4] → [÷ 2] → ? output
8. Input 64 → [÷ 4] → [- 3] → [x 7] → ? output

The function game

In this game, you have to work out what two-step function machine fits these results. Unfortunately, some of the results are wrong! Can you generalise your result?

2 → 5 

4 → 7 

4 → 9 

4 → 11 

5 → 14 

10 → 29 

n → ???



I've definitely got it.

Sarah

7 What do you think Sarah might be going to say?

Can you spot the rule in each of these questions?

1	2	3	4
3 → 7	1 → 2	5 → 22	4 → 19
6 → 13	4 → 11	8 → 34	7 → 37
9 → 19	7 → 20	12 → 50	10 → 55
20 → 41	15 → 44	25 → 102	20 → 115
n → ??	n → ??	n → ??	n → ??

8X were playing the function game, but Miss Arnold stopped after writing down the first line. Can you think of what her rule might have been? How many different rules can you think of that would fit the first line?

2 → 7 

Question Creation



Have fun creating problems for your friends to solve. Make up your own function and challenge your friends to find out what it is.

Function machines

What is the input on this function machine?

Input ? → x 2 → 16 output



Sarah

That's easy. I just worked right to left!

8

Can you explain Sarah's thinking?

What are the inputs on these function machines?

1. Input ? → - 5 → 12 output
2. Input ? → x 6 → 12 output
3. Input ? → + 9 → 12 output
4. Input ? → ÷ 6 → 12 output
5. Input ? → - 8 → 12 output



Matthew

What if it's a two-step machine?

For example, What is the input on this function machine?

Input ? → x 2 → - 7 → 11 output



Sarah

Still easy. Just work backwards!

9

Can you explain Sarah's thinking?



Evie

Evie's explanation

$11 + 7 = 18$

$18 \div 2 = 9$

Check $9 \times 2 = 18$

$18 - 7 = 11$ ✓

What are the inputs in these function machines?

1. Input ? → [x 3] → [+ 6] → 27 output
2. Input ? → [- 4] → [x 6] → 42 output
3. Input ? → [+ 9] → [x 9] → 54 output
4. Input ? → [÷ 4] → [- 5] → 43 output
5. Input ? → [+ 6] → [□ 4] → 7 output
6. Input ? → [x 8] → [÷ 10] → 20 output
7. Input ? → [x5] → [- 4] → [÷ 3] → 32 output
8. Input ? → [÷ 4] → [+ 7] → [x 6] → 72 output

Creating a formula

Function machines can help to work with formulae.

For example, suppose the cost of hiring the hall at your community centre is £20 plus £15 per hour. How much would it cost to hire the hall for 3 hours?



Alice

That's easy. 3 x 15 = 45
45 + 20 = 65
So £65 altogether.



Matthew

We could use a two step function machine. Then we could find the price for any number of hours.

Input → [x 15] → [+ 20] → ? output



Evie

Or we could put h for the number of hours and get a formula.

Input → [x 15] → [+ 20] → ? output

h $15h$ $15h + 20$ $15h + 20$

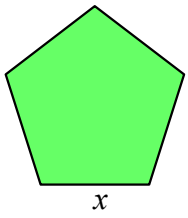
So if we do, we could write down a formula for the cost, £C, of hiring the hall for h hours is

$$C = 15h + 20$$

One step equations

Creating a formula

1. Write down a formula for the perimeter, P cms, of this regular pentagon.

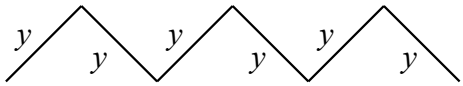




Amber

The perimeter is $x + x + x + x + x$,
So $P = 5x$

2. Write down a formula for the length, L cm, of this piece of wire.



If the length of wire is 84cms in total, how long is each short piece?

3. When you buy more than 3 tracks from the High Tunes Music Club, you receive some extra free bonus tracks. For example, if you buy 4 tracks, you receive 1 bonus tracks. If you buy 5 tracks, you receive 2 bonus tracks. If you buy 6 tracks, you receive 3 bonus tracks.
   a) Write down a formula that tells you the number of bonus tracks, B, if you buy n tracks if $n > 2$.
   b) How many tracks did you buy if you received 6 bonus tracks?

Solving one step equations

In Q1 above, the perimeter, P cms, can be calculated using the formula, $P = 5x$.
We can write this as a one step function machine.

Input (x) → | x 5 | → output (P)

If we know the perimeter and want to find the side length, x cms, then we work backwards using inverse operations.
For example, what is the side length if the perimeter is 65cms?

x → | x 5 | → 65 (P)

13 ← ÷ 5 ← 65

And so the side length is 13 cm

Inverse functions

In Q3, the formula for the number of bonus tracks was given by $B = n - 3$.

We can write this as a one-step function machine.

Input (n) → | - 3 | → output (B)

To find how many times you order, we can work backwards again using inverse operations.

For example: suppose you received 9 bonus tracks, how many tracks did you pay for?

n → | - 3 | → 9 (B)

12 ← + 3 ← 9

And so you paid for 12 tracks.

Inverse functions

When solving equations, we make use of inverse functions. These are functions that reverse another function.

The inverse of addition is subtraction. The inverse of multiplication is division.

Function	Inverse Function
+	-
-	+
x	÷
÷	x



Jon

So, to solve the equation $\frac{x}{7} = 15$

I just think of the function machine and work backwards using inverse functions.



Sarah

Sarah's working

x → | ÷ 7 | → 15

105 ← x 7 ← 15

So $x = 105$

Solving equations

Finding the solution to an equation is referred to as solving the equations. To solve an equation, we need to change the original equation into the form, x = ☐. Generally, we do this one step at a time.

Think about solving the equation $x + 7 = 15$.

The "equals" sign means that both sides have the same value.

In order to maintain this equality, whatever we do to one side, we must also do the same to the other side.

So let's see how this works in practice.

1. Solve $x + 7 = 15$

$x + 7 = 15$ (isolate the unknown)

$x + 7 - 7 = 15 - 7$ (take away 7 from both sides)

$x = 8$

Properties of equalities

1. If we add the same number or expression to both sides of an equality, then it remains an equality.
2. If we subtract the same number or expression from both sides of an equality, then it remains an equality.
3. If we multiply both sides of an equality by the same number, then it remains an equality.
4. If we divide both sides of an equality by the same number, then it remains an equality.
5. If both sides of an equality are switched, it remains an equality.

One Step Equations

In the examples on the previous page, we were solving one step equations as only one operation was involved.
For example

2. Solve $3n = -15$

$\frac{3n}{3} = \frac{-15}{3}$ (Divide both sides by 3)

$\frac{\cancel{3}^{1}n}{\cancel{3}_{1}} = \frac{\cancel{15}^{5}}{\cancel{3}_{1}}$ (Cancelling by 3)

$n = -5$

Fitness check

Solve the following equations:

1. $7p = 21$
2. $q + 4 = 17$
3. $3r = 36$
4. $s - 8 = 14$
5. $\frac{t}{8} = 9$
6. $12u = 78$
7. $v + 9 = 5$
8. $4.5w = 22.5$
9. $x - 5 = -3$
10. $\frac{y}{12} = 2.5$

11. This rule gives the approximate temperature in °F for a temperature in °C.

    "Double the number of °C, add 30"

    Roughly, how many °F is:

    a) 5°C b) 20°C c)100°C ?

12. Paula gives judo lessons at the local sports centre.

    This rule gives the amount she earns for each session:

    Amount = (Number of students x 6) – 10

    How much does she earn when there are:

    a) 6 students b) 10 students c) 14 students?

13. This formula gives the cost of a ferry crossing:

    Cost = £50 per car + £20 per passenger

    Find the cost for:

    a) a car and 2 passengers b) a car and 4 passengers

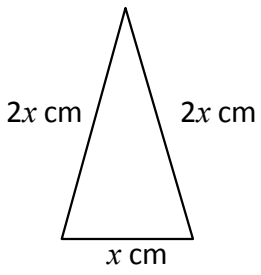
14. The perimeter of an isosceles triangle is P cm.

    a) Write down a formula, P = …, connecting P and x.

    b) What is the perimeter if x = 2.5cm?

    c) What are the side lengths if P = 55cm?

    d) Write down a formula x = …, connecting x and P.



15. This formula tells you how many eggs (n) to use in an omelette for T people:

    $n = T + 2$

    a) How many eggs would you need for 5 people?

    b) Matthew makes an omelette with 10 eggs. For how many people is this?

    c) Write down a formula T = …, connecting T and n.

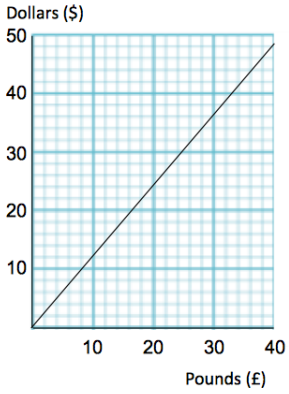
Exchange rates

1. This conversion graph graph can be used to change between pounds and dollars.

   Use the graph to convert:

   a) $30 into pounds

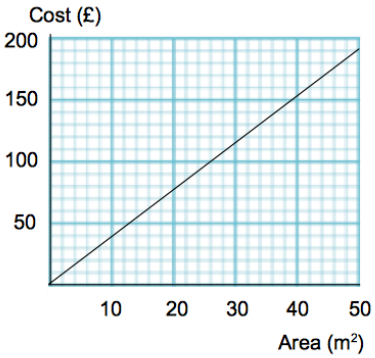
   b) £30 into dollars



2. James steam cleans carpets of different areas. He uses this graph to work out the cost of cleaning a carpet.

   a) Find the cost of a carpet that has an area of 34m^2.

   b) A carpet cost £170 to clean. What was the area of this carpet?



3. This conversion graph can be used to change between litres and gallons.

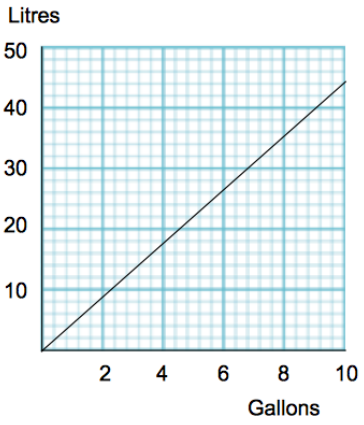
   Use the graph to convert:

   a) 8 gallons into litres

   b) 28 litres into gallons

   c) Fuel costs £1.20 per litre, work out the cost of 40 litres of fuel.

   d) Estimate the cost of 1 gallon of fuel.



Let's investigate proportionality



Sarah

I've found this recipe for making flapjacks.

Ingredients for 10 flapjacks

80g rolled oats

60g butter

30ml golden syrup

36g light brown sugar



Jon

But what happens if we want to make a different number of flapjacks?



Evie

We could make a table for different numbers of flapjacks.

Evie's table

Number of flapjacks	2	5	10	15	20	50
Oats			80g			
Butter			60g			
Golden syrup			30ml			
Sugar			36g			

10 Can you complete Evie's table?



Amber

What would I need if I wanted to make:

a) 8 flapjacks?
b) 12 flapjacks?
c) 18 flapjacks?
d) 13 flapjacks?
e) 33 flapjacks?

11 Can you answer Amber's questions?



Matthew

I've found a recipe for making gingerbread men.

What would I need to make 24 gingerbread men?

Ingredients for 16 gingerbread men

180g flour

40g ginger

110g butter

30g sugar



Sarah

Sarah's solution

Divide by 2 – that will give me the ingredients for 8 gingerbread men, and then add that onto the original recipe to get enough for 24 gingerbread men.

16 men	+	8 men	= 24 men
180g flour	+	90g flour	= 270g flour
40g ginger	+	20g ginger	= 60g ginger
110g butter	+	55g butter	= 165g butter
30g sugar	+	15g sugar	= 45g sugar



Jon

I did it a different way.
I multiplied everything by 1.5.

12 Can you explain Jon's thinking?



Evie

Evie's explanation

I think he used this function machine

Input	16 →	× 1.5	→	24	output
Flour	180 →	× 1.5	→	270	output
Ginger	40 →	× 1.5	→	60	output

And so on ...



Alice

I need to make a pear and almond crumble for 10 people using this recipe.

What do I need to buy?

Pear and almond crumble

Ingredients for 4 servings

250g plain flour

100g ground almonds

200g soft brown sugar

150g butter

8 ripe pears

13

Can you use Jon's idea to help Alice?



Amber

We just need to find the multiplier.

... and that is easy to do.



Matthew



Matthew

Matthew's explanation

Alice's recipe is for 4 – and so to find the unit cost, divide by 4.
That would be the recipe for 1.
Now multiply by 10, and we will have the recipe for 10 people.
To find the multiplier, simply divide the output needed by the input.
So, in this case it is $10 \div 4 = 2.5$



Sarah

So if y is proportional to x, then it means that y is a constant multiple of x

It also means that $y = m\,x$ where m is a constant



Jon

1. Look at this table. It shows the distance travelled (y km) by a bus travelling at a steady 50kph for a number of hours (x hours)

Time x (hour)	1	2	3	4	5	6
Distance y (km)	50		150			

   a) Copy and complete the table.

   b) How can you find y if you know x?

   c) Write down a formula that starts $y =$

2. In each of these cases, y is proportional to x. Find the missing values.

a)

x	4	7	?
y	12	?	36

b)

x	2	8	?
y	3	?	21

c)

x	2	5	?
y	5	?	35

d)

x	3	12	?
y	4	?	36

3. These tables below show the cost of a certain brand of cheese at two local supermarkets. Which supermarket offers the best value for the cheese?

S1

Mass (g)	300g	500g
Cost (£)	£3.60	£6.00

S2

Mass (g)	400g	600g
Cost (£)	£5.00	£7.50

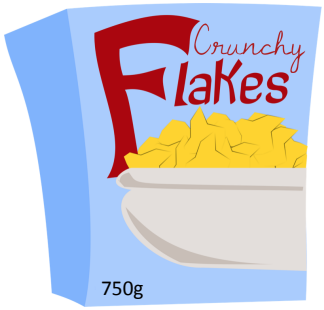
4. Which of these "Special offers" gives the best value for money?



Unit pricing

Sometimes foods packed in the "giant" or "family" size may seem like the best buy. You may think that buying one large container will not cost as much as two or three smaller packages. But larger containers do not always end up costing you less than smaller ones. It is important to look at the cost per unit and compare this cost.

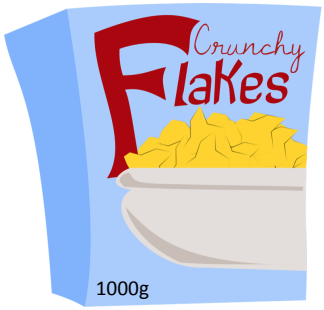
The "unit price" tells you the cost per 100g or some other unit of weight or volume of a food package. It is usually posted on the shelf below the food. The shelf tag shows the total price (item price) and price per unit (unit price) for the food item. The lower the “unit” price, the better the value. In this case, the larger box is better value.



Crunchy Flakes Cornflakes 750G

£3.49

(£0.47/100g)



Crunchy Flakes Cornflakes 1K

£3.99

(£0.40/100g)

Now look back at the ice cream question. These goods should have included the price per 100ml to help the shopper make an informed decision.



500ml

£4.30

(0.86/100ml)

SPECIAL OFFER



600ml

£5.20

(£0.87/100ml)

SPECIAL OFFER



800ml

£6.90

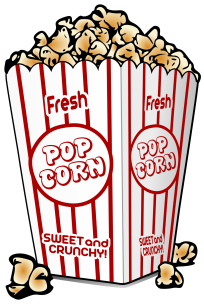
(£0.86/100ml)

SPECIAL OFFER

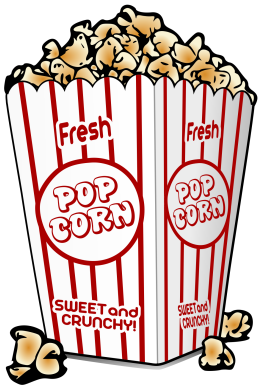
Fitness check

In each of these cases, work out the unit price to decide which is the better buy.

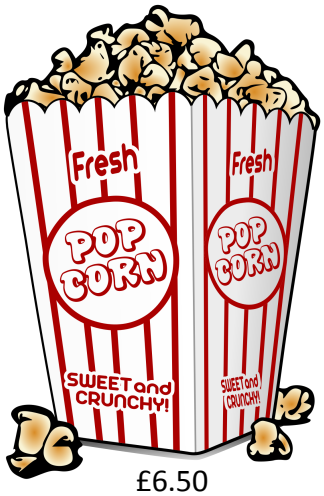
1. Cinema Popcorn



£3.00
Regular
(200g)



£5.00
Large
(350g)



£6.50
Jumbo
(800g)

2. Fizzy Drinks



500ml
£1.20

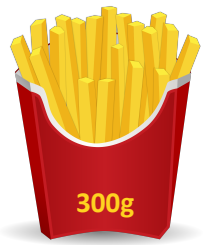


3 x 500ml
£2.40

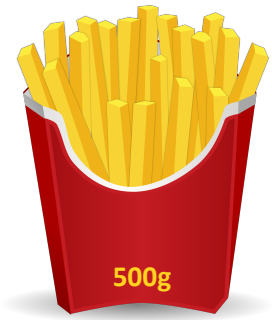


1.6L
£2.50

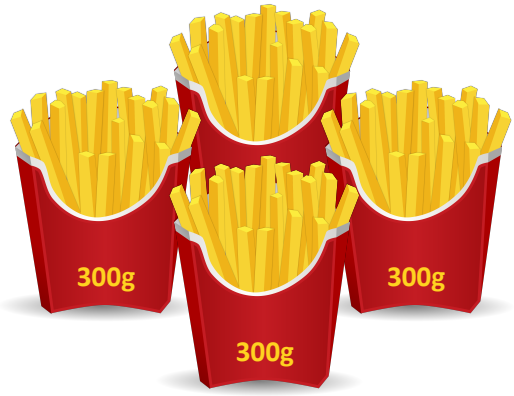
3. Fries



Saver Deal
£1.00



Big Deal
£1.80

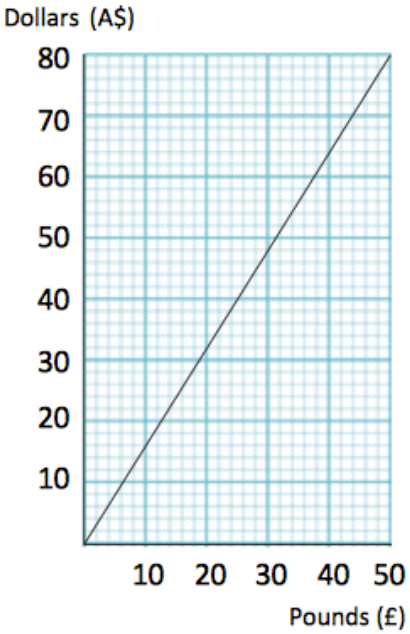


Family Pack
£3.85

Algebra 2b: Mastery questions

1. This graph can be used to change between pounds (£) and Australian dollars (A$).

   Convert £40 into A$.



A	B	C	D
A$25	A$60	A$64	A$70

2. Using the same conversion graph, convert $40 into £.

A	B	C	D
£20	£25	£64	£70

3. What is the output from this function machine?

Input 5 → [+ 4] → [x 3] → ? output

A	B	C	D
19	23	27	40

4. What is the output for this function machine?

Input 8 → [x 6] → [- 4] → [÷ 2] → ? output

A	B	C	D
46	20	25	22

5. What is the input for this function machine?

Input ? → [- 3] → [÷ 4] → 12 output

A	B	C	D
0	6	45	51

6. This function machine calculates the cost, £C, of a rail journey of d kilometres.

   Input d → [x 0.6] → [+ 10] → ? output

   What is the cost of a journey of 80km?

A	B	C	D
£14.80	£50.80	£58	£480

7. Using the same function machine as in Q6, the correct formula for working out the cost, £C, of a journey of d kilometres is

A	B	C	D
$d = 10 + 0.6C$	$C = 10 \times 0.6 + d$	C = 10 d + 0.6	C = 0.6 d + 10

8. This formula relates the number of chirps per minute (x) made by a snowy tree cricket to the ambient temperature in degrees Fahrenheit (y).

   $y = 0.25x + 36$

   What is the temperature in °F if the number of chirps per minute is 60?

A	B	C	D
15	24	51	52

9. The solution of the equation $8x = 36$ is

A	B	C	D
28	4	4.5	288

10. The solution of the equation $x - 7 = 21$ is

A	B	C	D
14	28	3	147

11. The solution of the equation $\frac{x}{3} = 18$ is

A	B	C	D
6	15	21	54

12. Which of these tables shows that y is directly proportional to x?

A

x	6	9	12
y	3	6	9

B

x	6	9	12
y	9	12	15

C

x	6	9	12
y	2	3	4

D

x	6	9	12
y	12	15	18

13. If y is directly proportional to x, what is the missing value in this table?

x	4	10	12
y	10	25	?

A	B	C	D
20	30	40	50

14. If y is directly proportional to x, what is the missing value in this table?

x	3	7	?
y	18	42	54

A	B	C	D
8	9	11	12

15. A recipe for 10 flapjacks needs 20g of butter, 30g of sugar and 50g of oats. How many flapjacks can I make if I have 160g of butter, 180g of sugar and 220g of oats?

A	B	C	D
80	60	40	4

16. A craft project for 5 pupils requires 10m of string, 15 cotton reels and 60 beads. How much of each commodity do I need for 12 pupils?

A	25m of string 30 cotton reels 120 beads	B	24m of string 36 cotton reels 144 beads
C	24m of string 30 cotton reels 144 beads	D	25m of string 36 cotton reels 120 beads

17. If 450g of cheese cost £2.70, what is the price per 100g?

A	B	C	D
30p	50p	60p	70p

18. A local supermarket offers 4 different multi-packs of identical packets of crisps. Which of these offers the best value for money?

A	B	C	D
3 for £1.05	5 for £1.70	8 for £2.15	10 for £3.50

19. The local hardware shop sells screws in bags containing multiples of 100. Which of these represents the best value for money?

A	B	C	D
200 for £1.98	300 for £2.94	500 for £4.90	800 for £7.80