What do you know about algebra?

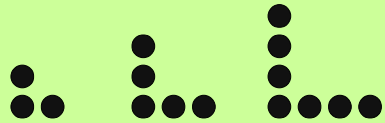
The nth term of a sequence is $3n + 4$.

Write down the first 4 terms of the sequence.

A sequence is defined by the mapping $n \rightarrow 5n - 1$.

Write down the first 4 terms of the sequence.

Look at this growing pattern:



Shape 1 Shape 2 Shape 3

Describe in words what shape 10 would look like.

Generalise your answer

If $a = 5$ and $b = 7$,

What is $2a + 3b$?

A sequence is defined by the mapping $n \rightarrow 4n - 3$

What is the 100th term?

Write these expressions concisely using the universal conventions:

$a + a + a + 2$

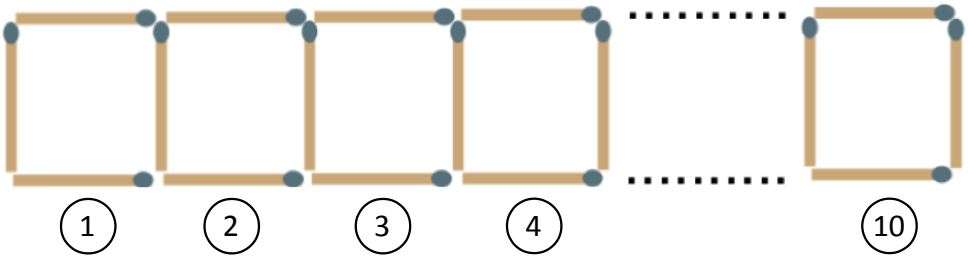
$2 \times b + 2 + b + 3$

$c \times d + d \times c + 4$

In general, how many matchsticks do we need?

Squares are made using matchstick, as shown below

1 How many matchsticks will we need for 10 squares?





Amber I could make the squares with real matchsticks and then count them.



Jon There must be an easier way!

Think about it. It might be easier to start with fewer squares.

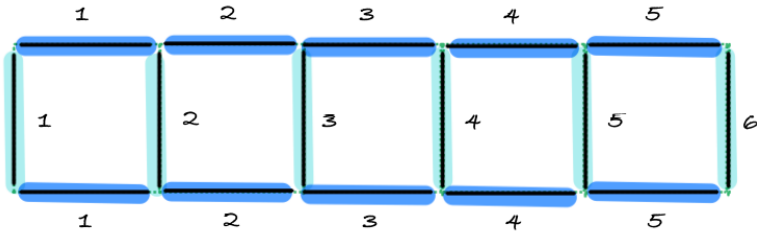
2 How many different ways can you find of counting the matchsticks needed for 5 squares?





Amber

I thought about drawing my diagram like this...



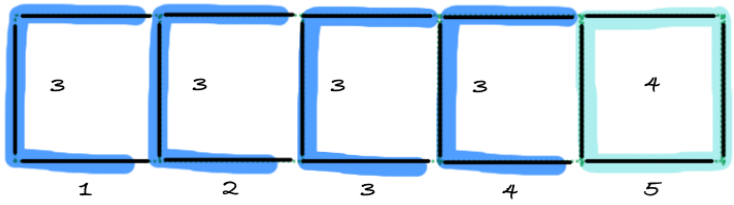
So to add then up quickly 5 + 5 + 6 = 16

I can write my maths sentence as 2 x 5 + 6 = 16



Jon

I did mine differently...



3 How did Jon count the matches for 5 squares?



Evie

Jon would have written 4 x 3 + 4 = 12 + 4 = 16.
So he got the same answer.

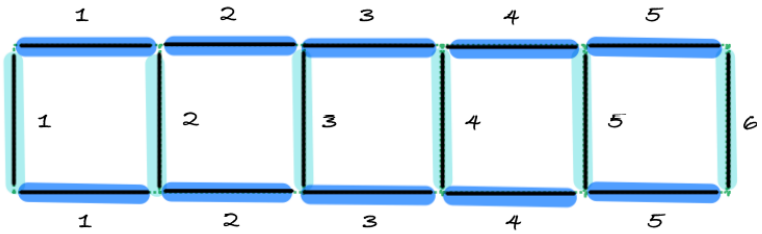


Sarah

I counted mine differently to Amber and Jon.

4 What other ideas did you think of? Talk them over with your partner.
Try using diagrams and maths sentences to explain your ideas.

Look back at Amber's method.



1. How would she write down a maths sentence to count the number of matches needed for 10 squares?
2. What if she had 100 squares? … 1000 squares? … a million squares?
3. Can you generalise her results?



Amber

These are my maths sentences for the last problem...

For 10 squares, (2 x 10) + 11 = 31

For 100 squares, (2 x 100) + 101 = 301

For 1000 squares, (2 x 1000) + 1001 = 3001

For a million squares 2 x million + one million and one



Matthew

This is how I generalised Amber's result.

For any number of squares, the number of matchsticks you need is always 2 x the number of squares + the number of squares plus one.



Alice

It is far better to use algebra.

To make the results clear and concise, I wrote n for the number of squares.

In which case, the number of matchsticks needed to make n squares can be written as

(2 x n) + n + 1 or $2n + n + 1$ or even more simply $3n + 1$.

So you end up with a neat formula for the number of matches!

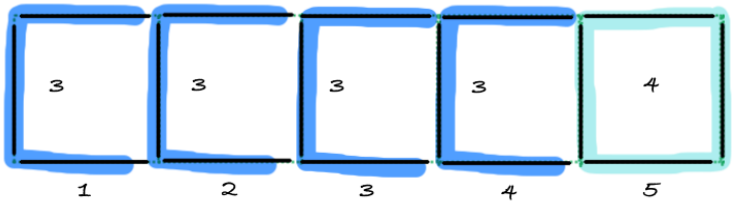
5 Using Alice's formula, write down how many matches you would need for 500 squares?



Sarah

That makes life much easier.
The number of matches needed for 500 squares is (3 x 500) + 1 = 1501

Look back at Jon's method.



1. How would he write down a maths sentence to count the number of matches needed for 10 squares?
2. What if he had 100 squares? … 1000 squares? … a million squares?
3. Can you generalise his results using algebra?



Amber

This is how Jon would calculate the number of matches...

For 10 squares, 3 x 9 + 4 = 31

For 100 squares, 3 x 99 + 4 = 301

For 1000 squares, 3 x 999 + 4 = 3001

For a million squares 3 x 999 999 + 4 = 3 000 001



Matthew

Or in general for n squares 3 x $(n - 1)$ + 4



Alice

That's interesting. Amber and Jon don't seem to have the same answer!

6

Can you explain why Jon's formula is really the same as Amber's formula?



Evie

Evie's work

Brackets in Algebra work just the same as brackets in arithmetic.

For example,

$3 \times 18 = 54$ and $3 \times (20 - 2) = 3 \times 20 - 3 \times 2 = 60 - 6 = 54$

So $3(n - 1) + 4 = 3n - 3 + 4 = 3n + 1$

That is the same as Amber's result.

The distributive law

Evie's explanation of how to **expand brackets** is an important idea in maths and is called **the distributive law**.

In general if a, b and c are any whole numbers, then

$$a(b + c) = ab + ac$$

7 Using the distributive law, expand $5(d + 2e)$.



$5(d + 2e) = 5d + 10e$
Easy!
Jon

Collecting like terms

Another useful idea in algebra is **collecting like terms**. This is really a tidying up process.

Like terms	Why are these like terms
2, -4, 0, 0.6,	All are number terms.
$2a$, $-5a$, $0.5a$,	All are a terms.
ac, ca, $3ac$. $-2ca$	All are terms containing a and c.
abc, $3bac$, $5cab$, $-2abc$	All are terms containing a, b and c.
a^2b, ba^2, $5a^2b$, $-2ba^2$, $7aab$	All all terms with a occurring twice and b once.

Fitness check

Simply:

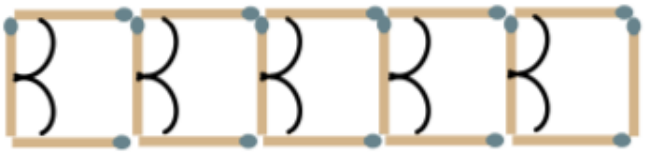
1. $5(4r + 2)$
2. $3(8f + 3)$
3. $9(2p - 4)$
4. $6(3s - 1)$
5. $4(2k - 6)$
6. $10(6q - 3)$
7. $2(4 – 3a)$
8. $7(2 – b)$
9. $12(5 – 4n)$
10. $5(4r + 2) + 4(2r – 6)$
11. $9(2p - 4) + 10(6q - 3)$
12. $3(2c + d) + 4(d - 3c)$

Generalising

Let's look at some other ideas of how to count the number of matchsticks needed to make 5 squares



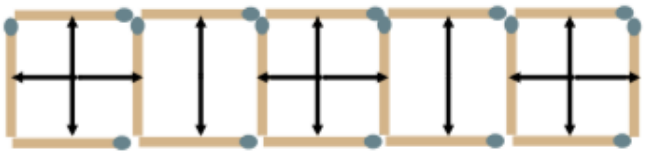
Evie



3 + 3 + 3 + 3 + 3 + 1 = 16



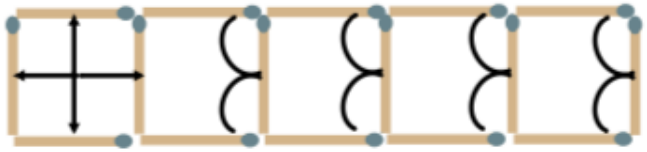
Matthew



4 + 2 + 4 + 2 + 4 = 16



Alice



4 + 3 + 3 + 3 + 3 = 16



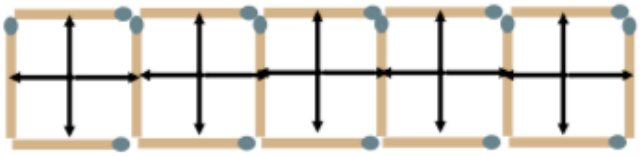
Sarah



2 + 2 + 2 + 2 + 2 + 5 + 1 = 16



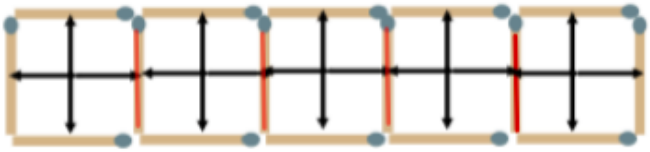
Amber



4 + 4 + 4 + 4 + 4 = 20



Jon



4 + 4 + 4 + 4 + 4 – 4 = 16

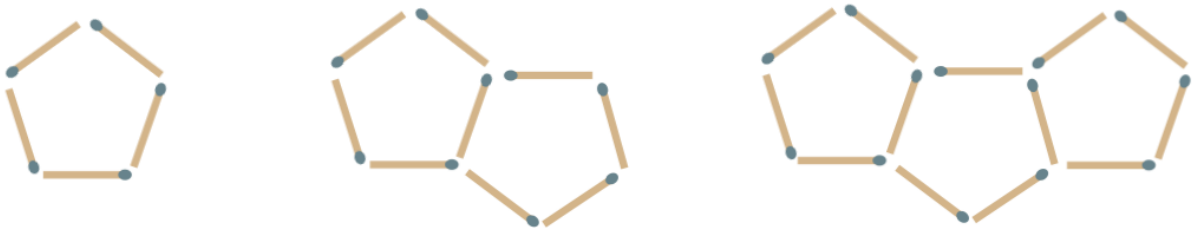
Look at each method. Can you explain why Amber's idea is incorrect?
For the others, write down a concise maths sentence to count the number of matches needed for 10 squares..

1. What if there were 100 squares? … 1000 squares? … a million squares?

2. Can you generalise each result?

3. Can you write a concise algebraic expression for the number of matchsticks required using each method?

How many matches do you need?



Shape 1 Shape 2 Shape 3



Alice

Sometimes it helps me to put my results in a table and see if I can spot any patterns.

Shape number	1	2	3	4	5
Number of matches	5	9	13	17	21

+ 4 + 4 + 4 + 4



Alice

So the Shape 10 will have 5 + 9 x 4 = 5 + 36 = 41 matches.
And Shape 100 will have 5 + 99 x 4 = 5 + 396 = 401 matches.



Jon

I've got an easier way of working it out. Use your general result!
We know we have $5 + 4(n - 1)$ matches for shape n.
So using the distributive law that is $5 + 4n - 4 = 1 + 4n$
For shape 10 we have 1 + 4 x 10 = 1 + 41 = 41 matches.



Matthew

So you can work out the rule from the table!

8 What do you think the rule might be for this number sequence 3, 8, 13, 18, 23, …?

Some important terminology

A **number sequence**, in mathematics, is a string of numbers that follow a particular pattern. The individual elements in a sequence are called terms. Some of the simplest sequences can be found in multiplication tables.

Quite often, we want to generate number sequences without drawing pictures. You need to be familiar with two different ways this might arise.

Sometimes you will be given the rule as a **mapping**. For example, the rule above could be written as

$$n \to 1 + 3n \text{ or } n \to 3n + 1$$

But sometimes you will be given a **general rule** for the n^{th} **term**. For example, the rule above could be written as

$$n^{th} \text{ term} = 1 + 3n \text{ or } n^{th} \text{ term} = 3n + 1$$

Finding the general rule in a number sequence.

Earlier, you were asked to find a rule for the n^{th} term of the sequence 3, 8, 13, 18, 23, ...



Amber

Amber's work

n	1	2	3	4	5	6
value	3	8	13	18	23	28

+ 5 + 5 + 5 + 5 + 5

The nth term is $3 + 5 \times (n - 1) = 3 + 5n - 5 = 5n - 2$

9

Using Amber's idea, find the rule for the n^{th} term for this sequence 1, 8, 15, 22, 29, ...



Matthew

Matthew's work

n	1	2	3	4	5	6
value	1	8	15	22	29	35

+ 7 + 7 + 7 + 7 + 7

The nth term is $1 + 7 \times (n - 1) = 1 + 7n - 7 = 7n - 6$

Fitness check

In each of these questions, use Amber's idea to find the general rule for the n^{th} term.

1. 1, 5, 9, 13, 17, 21, ...
2. 2, 6, 10, 14, 18, ...
3. 3, 6, 9, 12, 15, ...
4. 2, 5, 8, 11, 14, ...
5. 5, 11, 17, 23, 29, ...
6. 7, 13, 19, 25, 31, ...



Jon

I think I have noticed something...

10 What do you think Jon noticed?



Alice

Alice's explanation

If you look at 3, 6, 9, 12, 15, ⋯
The rule for the n^{th} term is 3n
That's the three times table!
If you look at 2, 5, 8, 11, 14, ⋯
The rule for the nth term is 3n − 1
Because it is the three times table take away one each time.
So as 5, 11, 17, 23, 29, goes up in 6's, it is the six times table take away one.
The rule for the nth term is 6n - 1



Sarah

So all we have to do is decide which times table the sequence has come from and check the first term. That is much easier.

11 Use Evie's idea to write down the rule for the nth term of this sequence
7, 11, 15, 19, 23, ...



Jon

Jon's work

7, 11, 15, 19, 23, ⋯
Going up in 4's, so that's the 4 times table.
So part of the rule will be 4n, but that would start with 4, so I need 3 more.
So the rule for the nth term is 4n + 3

Fitness check



In each of these questions, write down the rule for the n^{th} term.

1. 9, 14, 19, 24, 29, ...
2. 11, 21, 31, 41, 51, ...
3. 2, 10, 18, 26, 34, ...
4. -2, 5, 12, 19, 26, ...

12

Can you work out the rule for their n^{th} term in this sequence?
23, 20, 17, 14, 11, ...



Amber

The terms are going down! It's a decreasing sequence.



Matthew

Matthew's work

23, 20, 17, 14, 11, ⋯
The terms are going down by 3 each term.
So it's just like taking away the three times table from 26.
So I am thinking (26 − 3), (26 − 6), (26 − 9), ...
n^{th} term is 26 − 3n
Check 4^{th} term 26 − 3 × 4 = 26 − 12 = 14
It works!



Question creation - Have fun creating problems for your friends to solve. Make up your own decreasing number sequences and challenge your friends to find your rule.

Fitness check



In each of these questions, write down the rule for the nth term.
In each case, work out the 20th term.

1. 19, 14, 9, 4, -1, …
2. 8, 6, 4, 2, 0, …
3. 100, 90, 80, 70, 60, …
4. 0, -7, -14, -21, -28, ...

12 Can you match these sequences with the rule for their n^{th} term?

12, 18, 24, 30, 36, …	$18 - 6n$
12, 7, 2, -3, -8, …	$7 + 5n$
12, 0, -12, -24, -36, ...	$12n$
12, 6, 0, -6, -12, …	$24 - 12n$
12, 24, 36, 48, 60, …	$17 - 5n$
12, 17, 22, 27, 32, …	$6 + 6n$



Alice

Do number sequences always have to go up or down by the same amount each time?
What about 1, 3, 6, 10, 15, … ?

13 Can you carry on with Alice's number sequence? Can you explain in words what is happening?

Linear sequences

Number sequences that go up or down by the same amount each time are called **linear sequences**. The difference between successive terms is called the **common difference**. A linear sequence is completely defined if you know the first term and the common difference.

14 The first term of a linear sequence is 4, and the common difference is 11. Write down the first four terms of the sequence and the rule for the n^{th} term.

In the next section, we will be looking at some non-linear sequences.

Triangle numbers



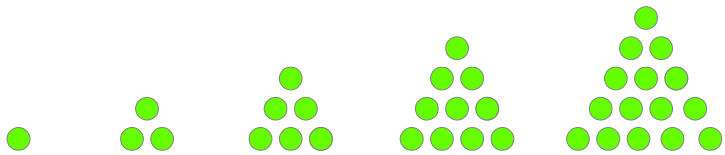
Alice

On the previous page, Alice asked about the sequence

1, 3, 6, 10, 15, ...

As you probably noticed, this is not a linear sequence, but it is easy to extend as the difference between the terms grows by one each time.

In fact, this sequence of numbers can easily be modelled by a series of triangles, as can be seen below.



These numbers are known as the **triangle numbers**.



Sarah

I've thought of another interesting sequence of numbers

1, 4, 9, 16, 25, ...

15

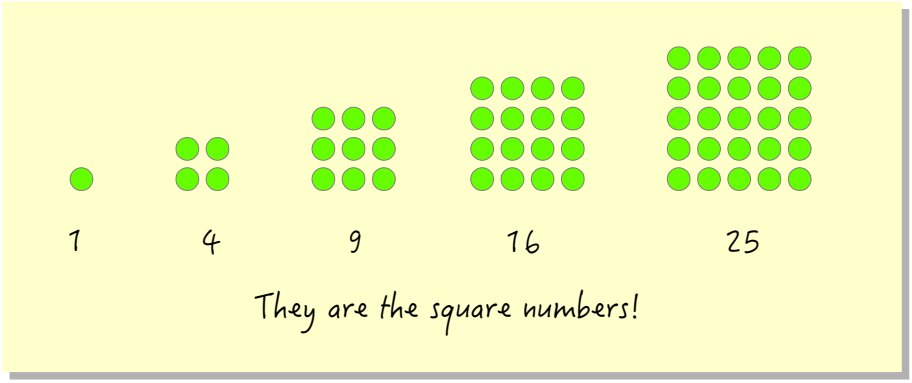
Can you write down the next 3 numbers in Sarah's sequence?

Think about how we might write the n^{th} term for this sequence.



Jon

Jon's solution



Other sequences

In each of these number sequences, write down the next three terms and try to generalise your answer by writing an expression for the nth term.

1. 1, 4, 9, 16, 25, …
2. 2, 5, 10, 17, 26, …
3. 0, 3, 8, 15, 24, …
4. 1, 8, 27, 64, 125, …
5. 1, 2, 4, 8, 16, …
6. 3, 6, 12, 24, 48, …
7. 2, 6, 18, 54, 162, …
8. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$
9. $\frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \frac{9}{11}, \ldots$
10. $\frac{1}{4}, \frac{4}{9}, \frac{9}{16}, \frac{16}{25}, \frac{25}{36}, \ldots$



Question creation - Have fun creating problems for your friends to solve. Make up your own challenging sequences and challenge your friends to find your rule.

Pascal's Triangle

The array of numbers shown below is known as Pascal's triangle. It is very useful in various branches of maths and biology as you will find out later.

Pascal's Triangle

			1				row 0
		1		1			row 1
	1		2		1		row 2
1		3		3		1	row 3
	1		4		6		4		1		row 4
1		5		10		10		5		1	row 5
	1		6		–		–		–		–		1		row 6
1		–		–		–		–		–		–		–	row 7

16

Find as many rules or patterns as you can amongst the numbers and use your ideas to generate the next three rows of Pascal's triangle.

Pascal's Triangle

Sometimes Pascal's triangle is written in a different form.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

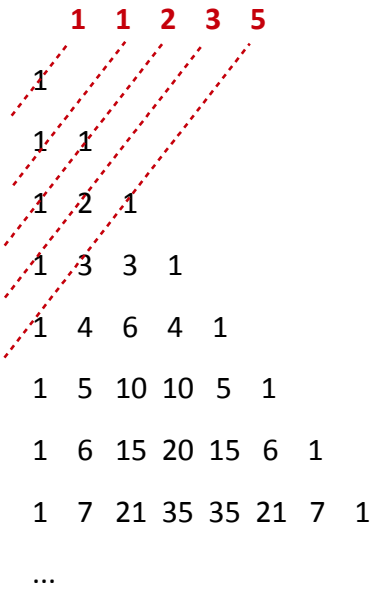
1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

...

By adding the numbers diagonally as shown below, generate a new sequence



17 This new sequence is known as **Fibonacci's sequence**. Can you generate the first 10 numbers in this sequence without using Pascal's triangle?

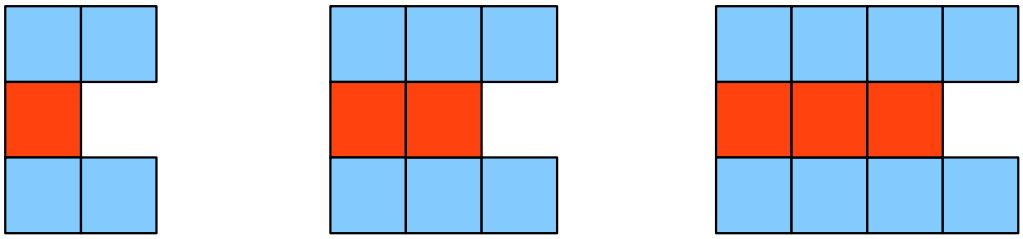
Did you spot the triangle numbers within Pascal's triangle?

Algebra 3a: Mastery questions

1. The nth term of a sequence is $5n - 4$. What is the 20th term of this sequence?

A	104	B	96	C	80	D	20

2. Here is a pattern made of red and blue tiles.



Shape 1 Shape 2 Shape 3

A shape has 42 blue tiles. How many red tiles does it have?

A	41	B	83	C	20	D	22

3. In the previous question, the general rule for the number of blue tiles in shape n is

A	$n + 2$	B	$4n$	C	$4n + 2$	D	$2n + 2$

4. In question 2, the general rule for the total number of tiles in shape n is

A	$n + 3$	B	$3n + 2$	C	$3n + 1$	D	$4n + 2$

5. The first three terms of a linear sequence are

83, 78, 73, …

What is the rule for the nth term of this sequence?

A	$-5n + 88$	B	$-5n + 78$	C	$5n + 78$	D	$-5n$

6. What name best describes this sequence?

1, 3, 6, 10, 15, ...

A	The factors of 30	B	Odd, odd, even, even, odd
C	Prime numbers	D	Triangle numbers

7. Find the nth term for this sequence

$$9, 15, 21, 27, \ldots$$

A	$6n + 3$	B	$6n$	C	Add 6	D	$9n + 6$

8. What name best describes this sequence?

$$1, 1, 2, 3, 6$$

A	Fibonacci	B	Odd, odd, even, odd, even
C	Prime numbers	D	Triangle numbers

9. The first three terms of a linear sequence are

$$\frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \ldots$$

What is the rule for the nth term of this sequence?

A. $\frac{3n}{3n+2}$ B. $\frac{n+2}{n+4}$ C. $\frac{n+2}{2n+3}$ D. $\frac{2n+1}{2n+3}$