Proportional Relationships



Amber

In examples (1), (2) and (4), y is proportional to x.

But not in example (3.)



Sarah

1 Can you explain what "y is proportional to x actually means?

It might help if you complete the tables before trying to answer this question.

1. The time and distance a car travelled at a speed of 60kmph (kilometres per hour):

Time x (hour)	1	2	3	4	5	6
Distance y (km)	60					

2. The side length and perimeter of a square:

Side length x (cm)	1	2	3	4	5	6
Perimeter y (cm)	4					

3. The time it takes to fill a fuel tank in the shape of a rectangular prism, and the depth of fuel:

Time x (Minutes)	1	2	3	4	5	6
Depth y (cm)	11	22				



2 In example (3), the depth of the fuel in the tank (y cm) is proportional to the time taken in filling the tank (x minutes). What is the connection between x and y.

4. The side length and area of a square:

Side length x (cm)	1	2	3	4	5	6
Area y (cm^2)	1					

Proportional Relationships



Matthew

If y is proportional to x, then when we multiply x by 2, 3, 4, … we do exactly the same to y.



Evie

It also means that y is a constant multiple of x



Jon

And so we can write a simple formula connecting x and y.

(1) The time and distance a car travelled at a speed of 60kmph (kilometres per hour):

Time x (hour)	1	2	3	4	5	6
Distance y (km)	60	120	180	240	300	360



Evie

y is always 60 times as big as x



Jon

So $y = 60\,x$

(2) The side length and perimeter of a square:

Side length x (cm)	1	2	3	4	5	6
Perimeter y (cm)	4	8	12	16	20	24



Evie

y is always 4 times as big as x



Jon

So $y = 4\,x$

(3) The time it takes to fill a fuel tank in the shape of a rectangular prism, and the depth of fuel:

Time x (mins)	1	2	3	4	5	6
Depth y (cm)	11	22	33	44	55	66



3

In example (3), the depth of the fuel in the tank (y cm) is proportional to the time taken in filling the tank (x minutes). Can you write this as a maths sentence?



Sarah

Sarah's working

Time x (mins)	1	2	3	4	5	6
Depth y (cm)	11	22	33	44	55	66
$y \div x$	11	11	11	11	11	11

y is always 11 times x, so the rule is $y = 11x$

Sarah's idea that "dividing y by x" is a good way of checking if two quantities are in proportion. It also helps you to find the connection between y and x if they are in proportion.

Using your rule

1. Use your rule to calculate the depth of fuel (y) in the tank when x =
   (i) 3.5 mins (ii) 38 mins (iii) 0 mins
   What does the value of y when x = 0 indicate about the tank?
2. How long will it take to fill the tank if the depth of the tank is 132cm?
3. Would it make any difference if the tank was not a rectangular prism?
4. The table below summarises the length and weight of the wire shown.

Length x (m)	2	3	4	5	6	7
Weight y (g)	18	27	36	45	54	63



   a) What is the quotient of the weight, y divided by the length, x?
   b) Is the weight of the wire proportional to its length? Why or why not?
   c) Write a maths sentence to express the relationship between x and y.
5. In which of the following situations, is y proportional to x?
   a) The length of daytime (y hours) and the length of night-time (x hours) in a day.
   b) The diameter of a circle (x cm) and the area of the circle (y cm^2).
   c) The quantity (x nails) and the total weight (y g) of 2g nails.
   d) Total goals scored (x) and total points (y) in the premier football league.
   e) The temperature in °C (x) and temperature in °F (y).
   f) Converting pounds (x) to dollars (y) at the bank today.

Taking a closer look at proportional relationships

4

In example (2), we looked at how the perimeter of a square changes as the side length increases. Can you find any other relations between the two sets of numbers?



Jon

In a proportional relationship, as the value of x becomes 2, 3, ... times as much, the value of y also becomes 2, 3, ... times as much.

x 5
x 2
x 3

Side length x (cm)	1	2	3	4	5	6
Perimeter y (cm)	4	8	12	16	20	24

x 3
x 2
x 5

Can you finish off Evie's idea below?



Evie

2 times
?

Side length x (cm)	1	2	3	4	5	6
Perimeter y (cm)	4	8	12	16	20	24

?
?
?

1. Matthew just had to be different. Can you complete his diagram?



Matthew

?
÷ 4
÷ 2

Side length x (cm)	1	2	3	4	5	6
Perimeter y (cm)	4	8	12	16	20	24

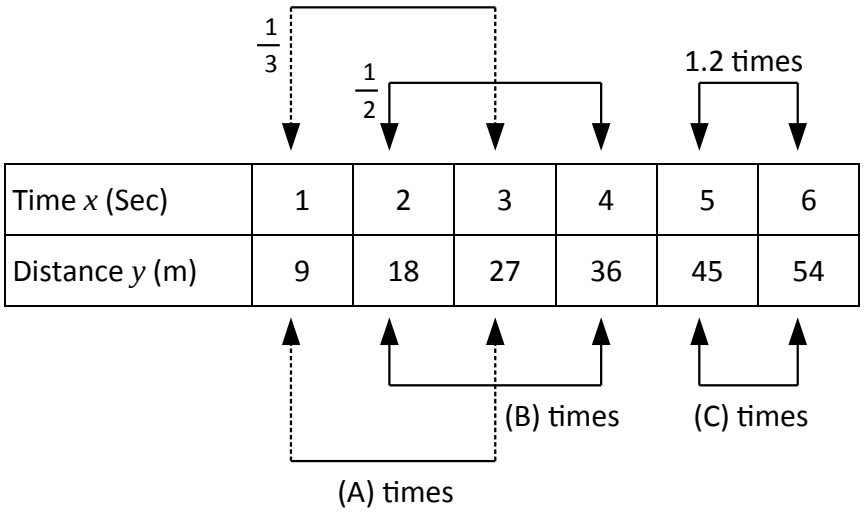
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When y is proportional to x, the following characteristics are true.

(1) When the value of x becomes 0.5 times as much, 2.5 times as much and so on, the corresponding value of y also becomes 0.5 times, 2.5 times and so on.

(2) When the value of x becomes $\frac{1}{2}$, $\frac{1}{3}$ times as much, the corresponding value of y also becomes $\frac{1}{2}$, $\frac{1}{3}$ times as much.

2. The table below shows the time and the distance a fishing boat travels at a speed of 9m per second.



Time x (Sec)	1	2	3	4	5	6
Distance y (m)	9	18	27	36	45	54



a) Is the distance proportional to the time? Why or why not?

b) Find the numbers that go into (A), (B) and (C).

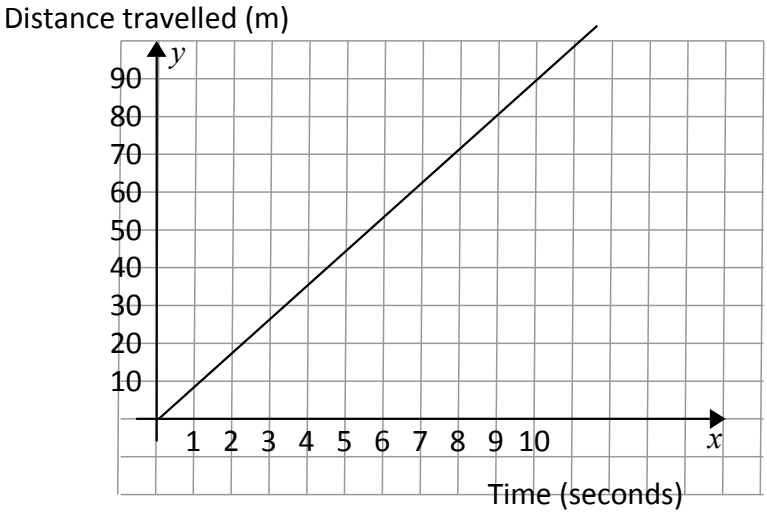
c) How far does the ship travel in 2.5 minutes?

d) What is the equation that connects y and x?

e) What do you think a graph of your results would look like?

Graphs of direct proportion

The graph below represents the relationship between the amount of time, x seconds, and the distance, y metres, travelled by the fishing boat.



3. Earlier, we looked at the time it takes to fill a fuel tank in the shape of a rectangular prism and the depth of fuel. The results are shown below:

Time x (mins)	1	2	3	4	5	6
Depth y (cm)	11	22	33	44	55	66

a) Plot a graph of the depth, y cm, against the time, x minutes.

b) What do these two graphs have in common?

c) What conclusions can you draw about graphs of quantities in direct proportion?



Matthew: They are both straight lines

… and they both go through the origin. :



Evie

Three important points

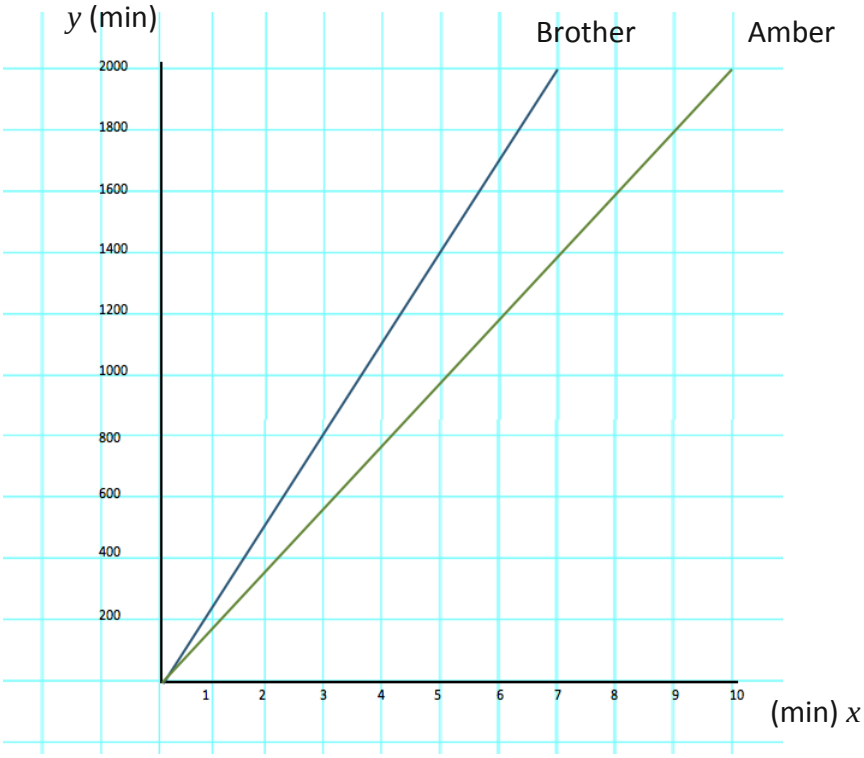
1. The graph of two quantities in a proportional relationship is a straight line that passes through the origin. This means you only need two points to plot the graph - if you are sure the quantities are in proportion.
2. In which case, the relationship between the two quantities in proportion can be written in the form $y = mx$, where m is the **constant of proportionality** and x and y are called **variables**.
3. When the value of x is chosen, there is only one corresponding value of y, and we say ***y* is a function of *x***.

Representing proportionality graphically

5

The graph below shows the time and the distance Amber and her brother ran during a race on the same running circuit.

Make different observations based on this graph.



1. Who is faster, Amber or her brother? Which part of the graph shows who is faster?

2. How far did Amber's brother run in 5 minutes?

3. How many minutes did it take for Amber to run 1400m?

4. How far apart were Amber and her brother 5 minutes after the start of the race?

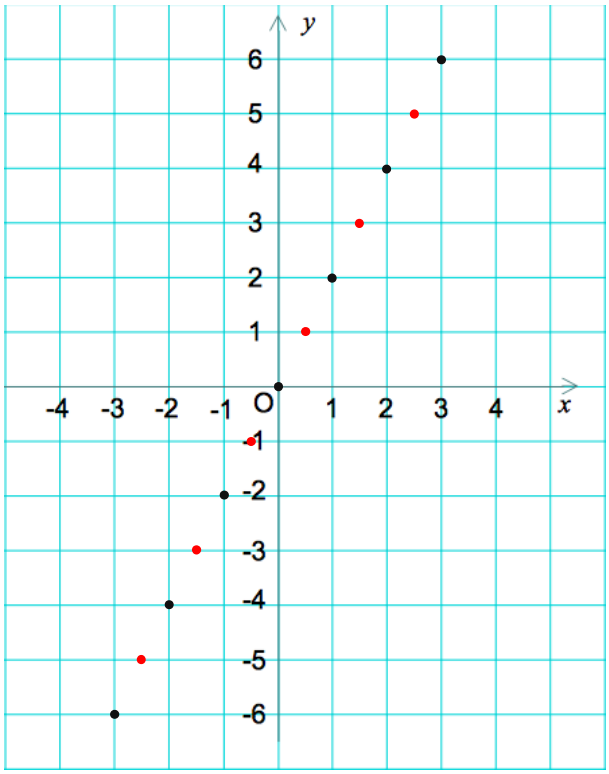
Graphs of direct proportion

Suppose we have a simple proportional relationship between, x and y, defined by $y = 2x$.
Then we can easily construct a table of values of y that correspond to the values of x.
Copy this table into your books and fill in the blanks.

x	- 3	- 2	-1	0	1	2	3
y			- 2	0		4	

We can now plot the values of x and y as coordinate pairs from the table above on a graph. You will need to consider the range of values for both x and y before deciding on your axes.

6 Plot the coordinate pairs from your table onto a suitable graph.





What happens if $x = 1.5$ or 2.5? Is that allowed?

Alice



That's a good idea. I've added some extra points in red.

Matthew



The more points you add, the closer the dots come to forming a straight line.

Sarah

Important result

Sarah is correct. If we keep on adding extra points, we end up with a straight line.
Consequently, we can draw the graph of $y = 2x$ as a straight line through the origin.

7 Plot the the graph of $y = 3x$ for $-3 \leq x \leq 3$ on a pair of suitable axes.


Amber

What does “suitable axes” mean?

As you have been given the range of x values, you just need to decide on the range of y values.


Jon


Evie

What happens if the constant of proportionality is negative?

8 To answer Evie's question, plot the the graph of $y = -2x$ for $-3 \leq x \leq 3$ on a pair of suitable axes.

It will help if you first make a suitable table. Copy the table below into your books and fill in the missing numbers.

x	- 3	- 2	-1	0	1	2	3
y			2	0		- 4	

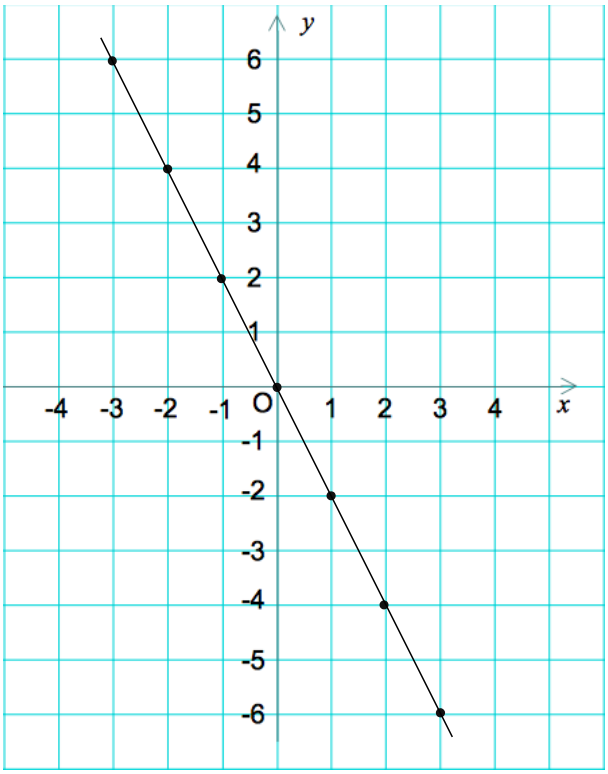
9 How do the values of y this time, compare to from $y = 2x$?


Matthew

They are the same numbers but the sign has changed.


Sarah

So the graph of $y = -2x$ is the reflection of the graph of $y = 2x$ in the y-axis.
It slopes downwards this time as x gets bigger.



Drawing graphs quickly

Earlier, we noted that if x and y are in proportion, then the relationship between x and y can be written as $y = mx$. When we plot graphs of this type of function, the graph always goes through the origin. Consequently we only need one other point to enable us to draw the graph.

Fitness check

Use this idea to find a second point on each of these graphs.

1. $y = 4x$
2. $y = 1.5\,x$
3. $y = \frac{1}{2}\,x$
4. $y = -3x$
5. $y = -0.5\,x$
6. $Y = -\frac{2}{3}\,x$

7. Sketch the graphs of the first three functions on the same grid. Label them clearly.
8. Sketch the graphs of the second three functions on a separate grid. Label them clearly.

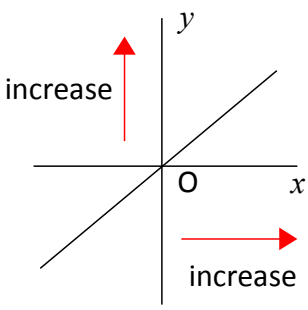
10

Compare your graphs for question 7 and question 8. What is the same and what is different about these families of graphs?

An important result

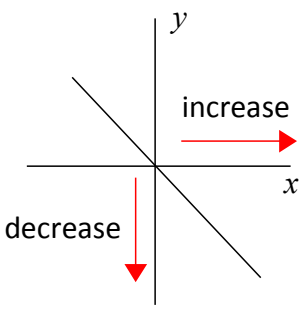
The graph of $y = mx$ is a line that passes through the origin.

When $m > 0$



The graph is going up to the right.

When $m < 0$



The graph is going down to the right.

Finding equations

Finding equations of direct proportions

11 If y is proportional to x, and when $x = 2$, $y = -8$, can you find an equation that connects y to x?



Amber

Amber's idea

Since y is proportional to x, we can write $y = mx$
where m is the constant of proportionality.
When $x = 2$, $y = -8$ so
$-8 = 2m$
$m = -4$
So the equation is $y = -4x$

Fitness check

In each of these cases y is proportional to x, find the equation connecting x and y if:

1. When $x = 2$, $y = 12$
2. When $x = -3$, $y = -18$
3. When $x = -2.5$, $y = 12.5$
4. When $x = -1.4$, $y = -7.7$
5. When $x = 2.4$, $y = 1.92$

12 In each of the questions above, can you find two more points that satisfy the rule?



Alice

Alice's working for Q1.

Since y is proportional to x, we can write $y = mx$
where m is the constant of proportionality.
When $x = 2$, $y = 12$ so
$12 = 2m$
$m = 6$
So the equation is $y = 6x$
If $x = 5$, then $y = 30$ and if $x = 8$, $y = 48$

Drawing graphs quickly

In situations (A) and (B) below, y is not proportional to x.
We know that the graph of 2 quantities in a proportional relationship will be a line that goes through 0, but what will the graphs that represent the relationship between x and y in (A) and (B) look like?

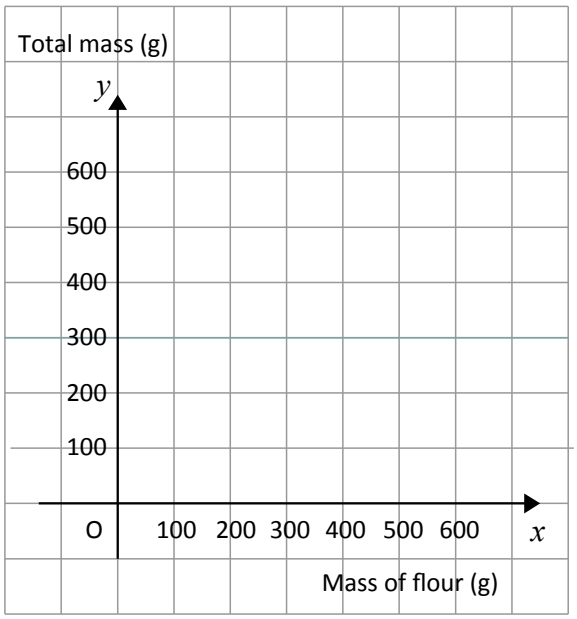
(A) The mass of flour x g in a 200g box and the total weight y g.

Mass of flour x (g)	0	100	200	300	400
Total mass y (g)	200	300	400	500	600

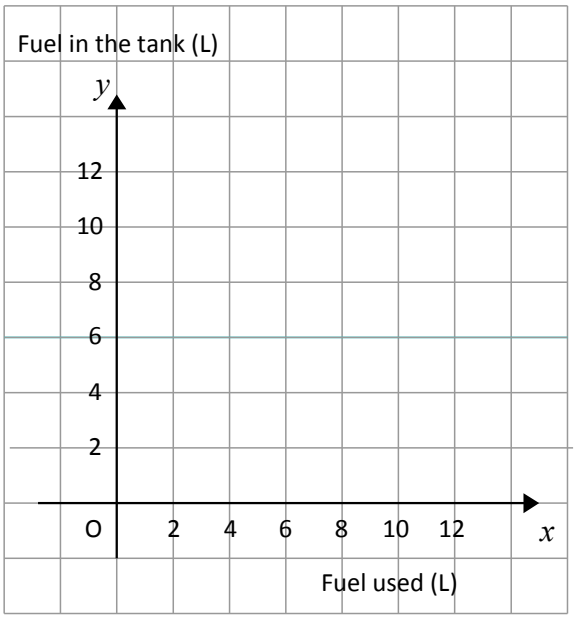
(B) The amount of fuel used (x L) from a 12L tank and the amount of fuel remaining in the tank (y L)

Fuel used x (L)	0	2	4	6	8
Fuel in the tank y (L)	12	10	8	6	4

Graph for (A)



Graph for (B)



Express the relationship between x and y with a maths sentence.



Matthew

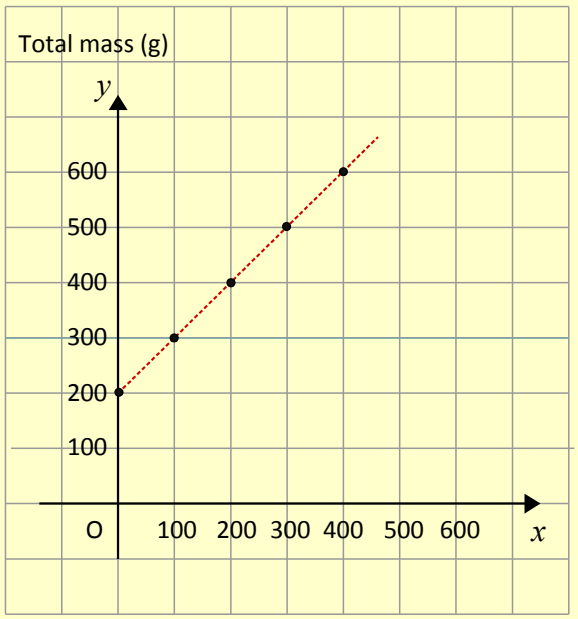
The graph of a proportional relationship goes through 0, but graphs (A) and (B)...



Evie

Evie's working

Graph for (A)



The points lie on a straight line but not through the origin.

There is a connection between x and y though because for each pair

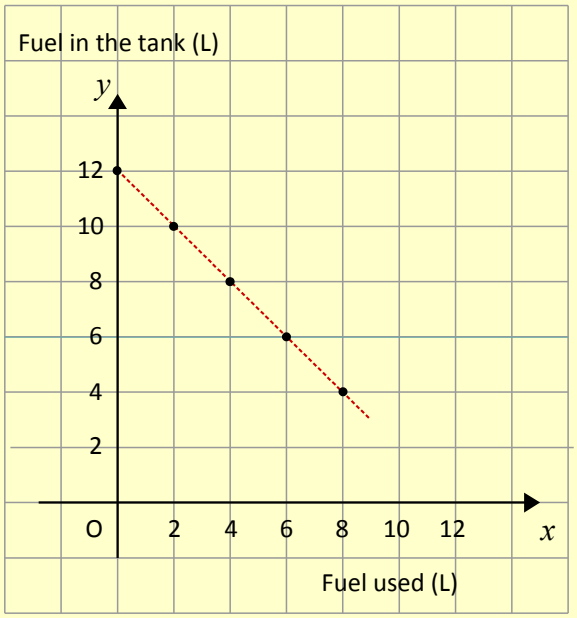
$y = x + 200$



Sarah

Sarah's working

Graph for (B)



The points lie in a straight line not through the origin and heading downwards.

There is a connection between x and y because for each pair

$y = 12 - x$

The two graphs above are example of **linear functions**. Linear functions can always be represented by a straight line graph.

Plotting graphs of linear functions

Suppose we want to plot a graph of $y = 2x + 1$, then we begin by making a table of values.

x	...	-3	-2	-1	0	1	2	3	...
y	...	-5			1		5	7	...

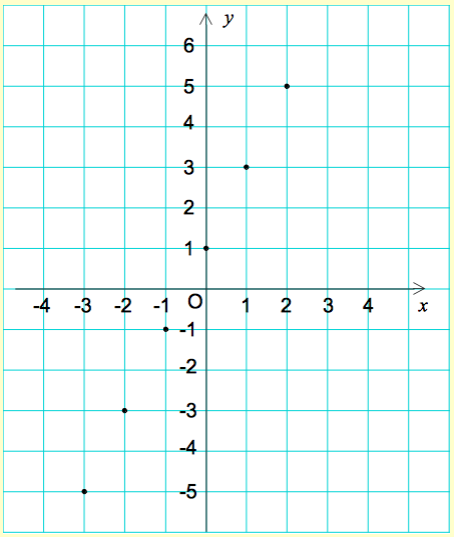
Copy the table into your books, and fill in the missing values.

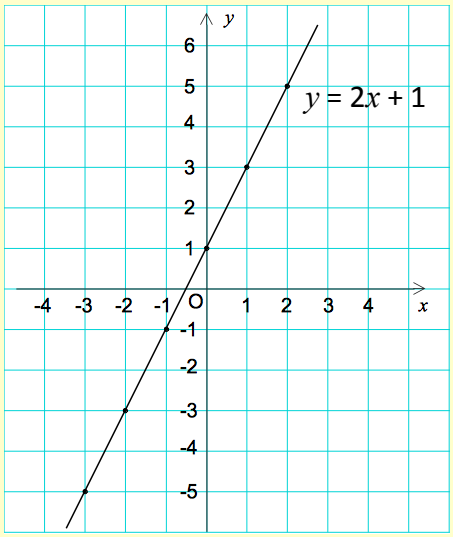
Now plot the points on a suitable graph.



Jon

Jon's working





The points lie on a straight line. The more points you plot, the closer graph becomes to a straight line.

Fitness check

1. Draw axes with values of x going from -2 to 3 and values of y going from -8 to 8.

   Make an appropriate table of values

   Draw the line $y = 3x - 2$

   What would be the value of y when $x = 7$

   Decide if the following points would lie on the extended line. Explain your thinking.

   a) (2, 4) b) (5, 13) c) (10, 27) d) (-4, -12)

1. Construct a table of values for $y = 4 - x$ for x going from -4 to 4.

   Draw a pair of suitable axes.

   Plot the points and draw the line $y = 4 - x$

   At which points does the line cross the x axis and the y axis?

   At which point are the x and y values the same?

2. Draw axes with values of x going from -3 to 3 and the values of y going from -10 to 10.

   Plot on the same axes

   (i) $y = 2x + 4$ (ii) $y = 2x - 4$

   (iii) $y = 4 - 2x$ (iv) $y = -4 - 2x$

   What can you say about the lines you have drawn?

3. Look at Fig.1. It contains 3 different graphs. You are given one of the equations.

   What are the equations of the other two lines?

   Explain your reasoning.

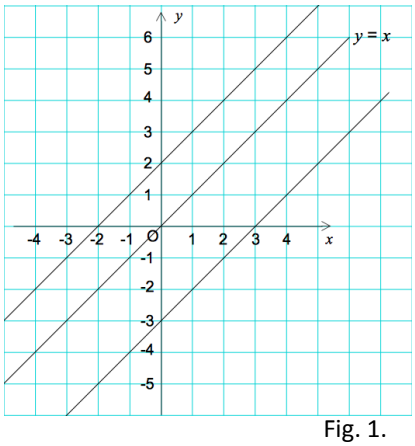


Fig. 1.

4. Look at Fig.2. It contains 3 different graphs and you are only given one of the equations.

   What are the equations of the other two lines?

   Explain your reasoning.

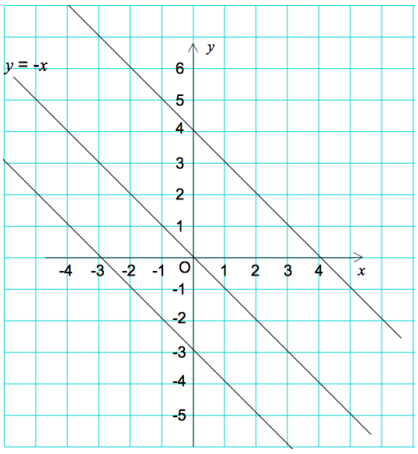


Fig. 2.

5. There is £20 annual membership fee for joining the local athletics club.

   There is then a charge of £2 per visit to use its facilities.

   Non-members pay £5 per visit.

   Alice is a member, Matthew is not a member.

   Complete this table to show their costs for visiting the club over the next few months.

Number of visits	0	5	10	15	20	25
Total cost (Alice)(£)	20	30	40	50	60	70
Total cost (Matthew)(£)	0	25	50	75	100	125

   Using the same axes, plot both of these graphs.

   After how many visits have they paid exactly the same amount?

6. The cost £y of printing x brochures is given in the table below.

Number of brochures (x)	25	50	75	100	150
Cost in £ (y)	28	36	44	52	60

   a) Plot a graph of this information

   b) Use you graph to estimate the cost of 60 brochures.

   c) How many brochures could be printed for £50

7. A salesman has a basic monthly salary of £1000 plus 8% commission on sales. Copy and complete this table showing his potential monthly income.

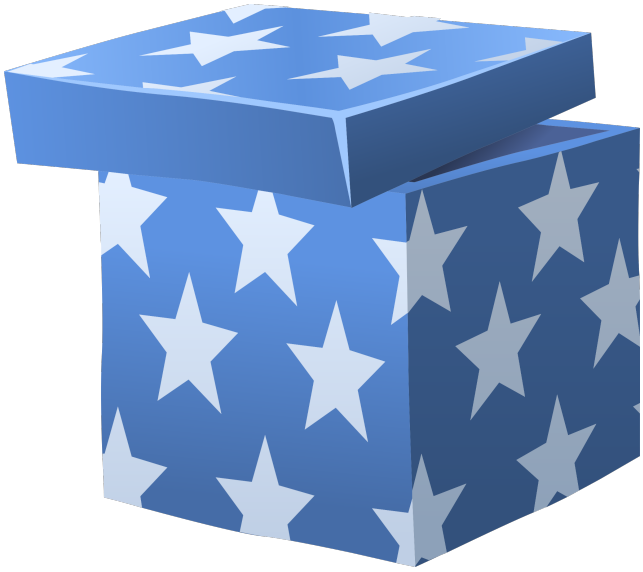
Sales (thousands) (x)	0	2	5	10	15
Monthly salary in £ (y)	1000	1160	1400	1800	2200

a) Plot these results on a suitable graph.

b) How much would he earn if his sales were £7000 in July?

c) What were his sales if his salary was £2300 in September?

How many sweets are in the box?



A class are given 'clue cards' to work out how many sweets are in the boxes. Each of the 5 pupils have a card with a different clue on it.



Amber

My cards says there are 3 boxes of sweets



My card says we do not know how many are in the first box of sweets



Evie



Jon

Mine says the second box has twice as many sweets as the first box

The third box has 3 more than the second box...



Matthew



Sarah

Altogether there are 28 sweets

Forming equations

To understand these clue cards, we need to write the information down in a way that makes sense.

There are 3 boxes of sweets



We do not know how many are in the first box of sweets



n

The second box has twice as many sweets as the first box



n $\quad$ $2n$

The third box has 3 more sweets than the second box...



n $\quad$ $2n$ $\quad$ $2n + 3$

Altogether there are 28 sweets



$n + 2n + 2n + 3 = 28$

Collecting up like terms gives

$$5n + 3 = 28$$

13

Can you now solve this equation? How many sweets are in each box? How can you check to see if you are right?

Forming equations



Jon

I have written my own clue sharing question...



I have four boxes of sweets
I do not know how many are in the second box.
There is one less in the first box.
There is the same number in the third box as the first box.
There are four times more sweets in the fourth box than in the second box.
There are 54 sweets in total

How many sweets are in each box?

14 Form an equation and try to solve it. Make sure you check your answer.



Sarah

Here is my clue sharing question...



I have four boxes of sweets
I do not know how many are in the first box.
There are two more in the second box than the first box of sweets.
There are three less in the third box than in the first box
In the fourth box, there is one less than twice as many as in the first box.
There are 73 sweets in total

How many sweets are in each box?

15 Form an equation and try to solve it. Make sure you check your answer.

Question Creation



Question creation - Have fun creating problems for your friends to solve. Make sure you can answer your own question and that the answers make sense.



Amber

My question is about singing hamsters.

Clue Cards



I have three groups of singing hamsters, I do not know how many are in the first group

Clue Cards



In the second group, there are five more than in the first group.

Clue Cards



In the third group, there are three times as many as in the second group.

Clue Cards



Clue Cards



Altogether there are 55 singing hamsters!

Clue Cards



How many are in each group?

16

Form an equation and try to solve it. Make sure you check your answer.

Solving Equations

Finding the solution to an equation is referred to as solving equations. In order to solve an equation, we need to change the original equation into the form, $n = \square$. Generally we do this one step at a time.

Look back at our first equation in this section $5n + 3 = 28$

The "equals" sign means that both sides have the same value.

In order to maintain this equality, whatever we do to one side we must also do the same to the other side.

Let's see how this works out in practice.

$5n + 3 = 28$

$- 3 \quad -3$ (Subtract 3 from both sides)

$5n = 25$

$\div 5 \quad \div 5$ (divide both sides by 5)

$n = 5$

This is called a two step equation because we had to carry out two operations before arriving at our solution.

Properties of equalities

1. If we add the same number or expression to both sides of an equality, then it remains an equality.
2. If we subtract the same number or expression from both sides of an equality, then it remains an equality.
3. If we multiply both sides of an equality by the same number, then it remains an equality.
4. If we divide both sides of an equality by the same number, then it remains an equality.
5. If both sides of an equality are switched, it remains an equality.

One Step Equations

1. Solve $n + 6 = 13$

   $n + 6 = 13$ (Isolate the unknown)

   $n + 6 - 6 = 13 - 6$ (take away 6 from both sides)

   $n = 7$

2. Solve $3n = -15$

   $\frac{3n}{3} = \frac{-15}{3}$ (Divide both sides by 3)

   $n = -5$

Solving Equations

Solve the following one step equations

1. $12 + n = 8$
2. $x - 7 = -2$
3. $5y = -45$
4. $\frac{a}{8} = 7$
5. $b + 11 = 8$
6. $c - 15 = 3$
7. $7d = 84$
8. $\frac{e}{9} = 14$

Two Step Equations

When solving two step equations, the idea is to first reduce a two step equation to a one step equation. For example

1. Solve $3n - 7 = 35$

   $\mathbf{3n} - 7 = 35$ (Isolate the unknown term)

   $3n = 42$ (Add 7 to both sides to form a one step equation)

   $n = 14$ (divide both sides by 3)

2. Solve $\frac{x - 4}{3} = 8$

   $\frac{\mathbf{x - 4}}{3} = 8$ (Isolate the unknown term)

   $x - 4 = 24$ (Multiply both sides by 3 to form a one step equation)

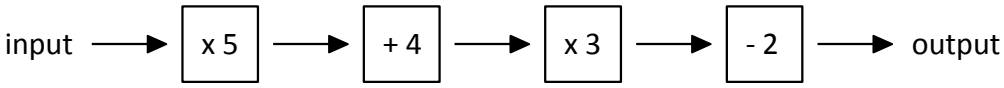
   $x = 28$ (Add 4 to both sides)

Solve the following two step equations

1. $5 + 2n = 29$
2. $4x - 17 = 27$
3. $5y + 15 = -45$
4. $\frac{a}{4} - 3 = 15$
5. $13 - 3b = 34$
6. $6c + 5 = -7$
7. $-4d + 6 = 54$
8. $\frac{e+2}{5} - 1 = 9$

Function machines and equations

Look at this function machine



1. What is the output if the input is 3?


Alice

Alice's working

$3 \times 5 = 15$
$15 + 4 = 19$
$19 \times 3 = 57$
$57 - 2 = 55$

2. What if the input is 6?


Sarah

Sarah's working

$6 \times 5 = 30$
$30 + 4 = 34$
$34 \times 3 = 102$
$102 - 2 = 100$

17 Try different input numbers. Tell your partner your output number and challenge them to find your input number.


Matthew

My output number is 220.
What is my input number?

My output number is 295.
What is my input number?


Evie


Jon

My output number is 1495.
What is my input number?

18 Can you think of a way of working backwards from the output to find the input?

Backtracking

Look at our function machine. Is there a way of working backwards?
If we know the output, is there any easy way to find the input?

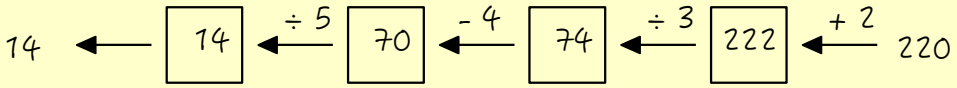


3. Let's think about Matthew's question. What is the input if the output is 220?

Amber's idea


Amber

You can just work backwards. Start with the output, work right to left and reverse each operation.
So



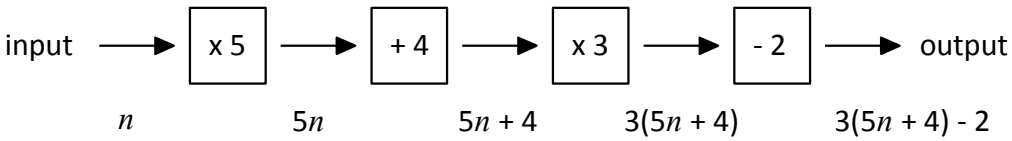
Matthew's input was 14

This process of working from the output to the input is called **backtracking**.

4. Use backtracking to find the input if the output is:
   (a) 295 (b) 1495 (c) 3010 (d) 14 965

Linking backtracking to solving equations

Earlier we used n to stand for a number we don't yet know.
We can use this idea with our function machine by calling our input n.



So the function machine can be represented algebraically by $3(5n + 4) – 2$.
And we can use backtracking to solve any equation in the form $3(5n + 4) – 2$ = something.

5. Solve the equation $3(5n + 4) – 2 = 520$ using backtracking.

Solving equations and backtracking



Alice

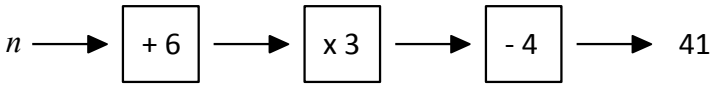
Alice's working

$3(5n + 4) - 2 = 520$
$3(5n + 4) = 522$ (Add 2 to both sides)
$5n + 4 = 174$ (Divide both sides by 3)
$5n = 170$ (Take away 4 from both sides)
$n = 34$ (Divide both sides by 5)

Representing function machines as algebraic functions

We can now represent any function machine in algebraic notation. We can then use backtracking to solve any related equations.

For example, look at this function machine.

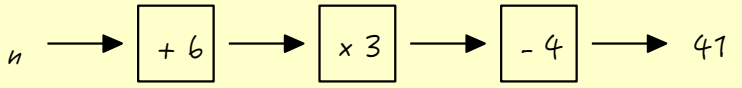


1. Represent this information as an algebraic equation.
2. Solve your equation using backtracking.



Sarah

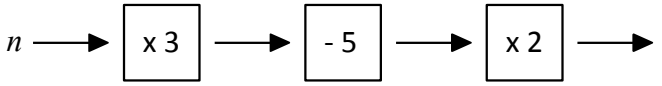
Sarah's working

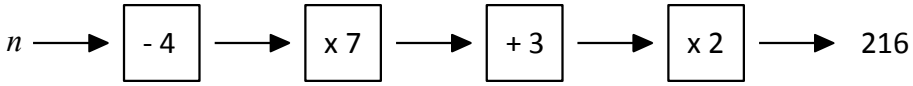


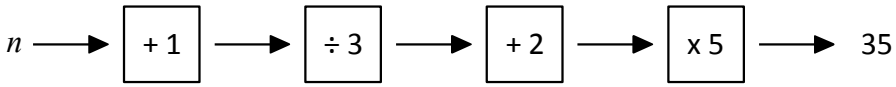
$n + 6$ $\quad$ $3(n + 6)$ $\quad$ $3(n + 6) - 4 = 41$

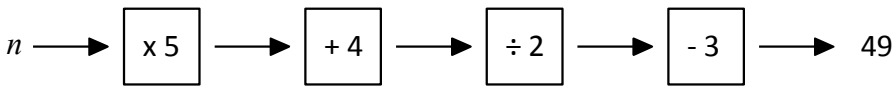
$3(n + 6) - 4 = 41$
$3(n + 6) = 45$ (Add 4 to both sides)
$n + 6 = 15$ (Divide both sides by 3)
$n = 9$ (Take away 6 from both sides)

For each of these function machines, write down an algebraic equation and then solve it using backtracking.

1. 

2. 

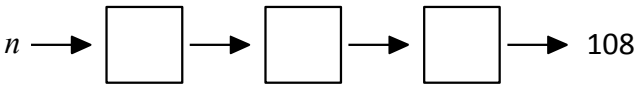
3. 

4. 

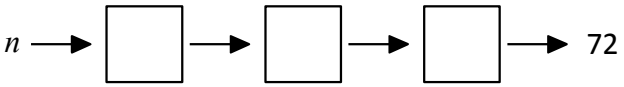
Backtracking

Complete the function machine diagram to represent these equations. Use backtracking to solve each equation.

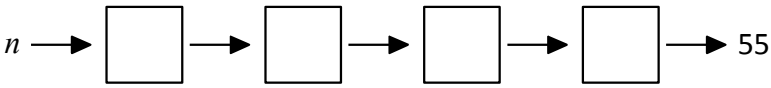
5. $(3n - 9) \times 6 = 108$



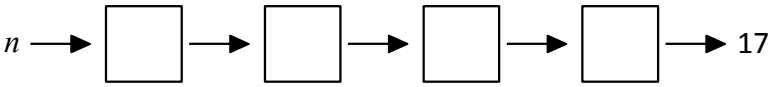
6. $4(n - 5) - 8 = 72$



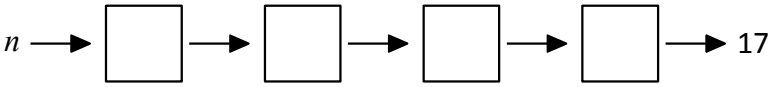
7. $\frac{5n - 4}{3} + 8 = 55$



8. $\frac{2(n + 1)}{5} + 9 = 17$



9. $\frac{5(4n - 2)}{7} = 30$



Solve these equations by backtracking

10. $2(n - 5) = 18$

11. $\frac{3n - 4}{5} = 13$

12. $\frac{4n - 6}{7} + 9 = 39$

13. $\frac{3(n + 2)}{4} - 7 = 11$

14. $\frac{2(5n - 3)}{3} + 14 = 52$

Changing the subject

Earlier in this chapter, we looked at functions written in the form $y = 3x + 2$.
Given a value for x, we could then easily find the corresponding value of y.
For example if $x = 2$, then $y = 3x2 + 2 = 6 + 2 = 8$
But what if we knew the value of y and we want to find the corresponding value of x?
For example, what is x if $y = 11$?

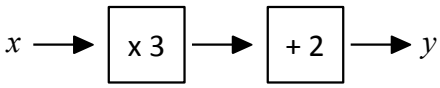

Sarah

Sarah's working

It's easy.
You just solve the equation $11 = 3x + 2$
It's just a two step equation.
So $11 = 3x + 2$ is the same as $3x + 2 = 11$ (Switch sides)
So $3x = 9$ (Take away 2 from both sides)
$x = 3$ (Divide both sides by 3)

But sometimes we are given lots of values of y and asked to find the corresponding values of x.
In this case, it would be easier if we had a function that turned y numbers into x numbers.
This process is called **changing the subject**.

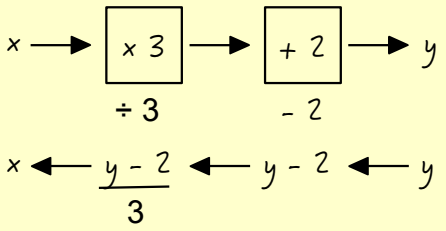
We can do this quite easily if we use backtracking
To begin with it is useful to think of an appropriate function machine.
For example, we could express the function $y = 3x + 2$ as follows:



19 Can you use your backtracking skills to write x in terms of y?


Alice

Alice's working



So I can write $x = \frac{y - 2}{3}$

Changing the subject

Fitness check

For each of these functions, make x the subject.

1. $y = 5x - 3$
2. $y = 6x + 7$
3. $y = x - 5$
4. $y = 2x + 8$
5. $y = -4x + 3$
6. $y = -2x - 5$
7. $y = 7 - x$
8. $y = 15 - 4x$

Flat out … and no brakes

Speedway racing can be pretty exciting.

Four riders race around a dirt oval track on single-speed motorcycles with no brakes. They slide sideways around the corners slowing down by angling the front wheel and by dragging their left foot in the dirt.

They then accelerate flat out down the straight until the next bend.

Generally races last for 4 laps.



The speed v m/s of the bikes down the straights can be calculated using the formula $v = u + at$ where u is the speed coming out of the corner, a is the acceleration in m/s^2 and t is the time in seconds.

This is a well known formula that you will meet again later.

1. Calculate the speed (v m/s) of the bike after t seconds if:
   a) u = 16m/s, a = 5.3m/s^2, t = 1.4s
   b) u = 17.5m/s, a = 5.7m/s^2, t = 2.6s
   c) u = 18.2m/s, a = 5.8m/s^2, t = 3.1s
   d) u = 19.2m/s, a = 5.9m/s^2, t = 3.4s

2. Calculate the acceleration (a m/s^2) if:
   a) v = 22.4m/s, u = 15.4m/s, t = 1.2s
   b) v = 31.2m/s, u = 16.1m/s, t = 2.5s
   c) v = 35.8m/s, u = 17.2m/s, t = 3.1s
   d) v = 19.3m/s, u = 15.4m/s, t = 0.9s

Changing the subject

In the previous two questions, we used the formula $v = u + at$, where v was the final speed, u was the initial speed, a was the acceleration and t was the time taken.

In question 2, it would have been helpful to make a the subject of the formula.

This is another example of changing the formula.

20 Can your backtracking skills to make a the subject of the formula?



Evie

Evie's work

$v = u + at$

$u + at = v$ (Switch sides)

$at = v - u$ (Take away u from both sides)

$a = \frac{v - u}{t}$ (Divide both sides by t)

3. Use Evie's new formula to calculate the acceleration of a high speed train if:
   a) v = 40m/s, u = 0m/s, t = 180s.
   b) v = 60m/s, u = 20m/s, t = 130s.
   c) v = 80m/s, u = 30m/s, t = 108s.
   d) v = 83m/s, u = 0m/s, t = 180s.

4. Change the subject of these formulae to the letter indicated in brackets.
   a) $v = u + at$ (u)
   b) $v = u + at$ (t)
   c) $e = 3c + 7d$ (c)
   d) $e = 3c + 7d$ (d)
   e) $s = 3pq + 4r$ (r)
   f) $s = 3pq + 4r$ (p)
   g) $f = g + 3hj$ (g)
   h) $f = g + 3hj$ (h)
   i) $c = \frac{(a + b)}{2}$ (a)
   j) $f = \frac{(2g + 3h)}{5}$ (g)
   k) $f = \frac{(2g + 3h)}{5}$ (h)
   l) $s = \frac{(u + v)}{2} \times t$ (t)

Expanding brackets

1. Look at this rectangle.
   Write down an formula for the perimeter, P.





Jon

Easy.
P = $2x + 2y$

Or you could write
P = $2(x + y)$



Alice

2. Now look at this compound shape.
   Write down a formula for the perimeter in two different ways.





Sarah

P = $2x + 4y$

P = $2(x + 2y)$



Matthew

Expanding brackets

As you can see in the examples above, brackets can be useful in tidying up algebraic expressions.

Sometimes you will be given an expression involving brackets, such as $2(x + 2y)$ and asked to **expand** or **multiply out the bracket**.

3. Expand $2(2x + 5y)$



Amber

Amber's work

$2(2x + 5y)$ = 2 lots of $2x + 5y$
And $2x + 5y + 2x + 5y = 4x + 10y$
So $2(2x + 5y) = 4x + 10y$

I can see an easier way of doing this.



Evie

21 What do you think Evie might have spotted?

The distributive law


Evie

Evie's explanation

To work out 2(2x + 5y)
You just multiply each term in the bracket by 2.

2(2x + 5y) = 2 x 2x + 2 x 5y = 4x + 10y

The distributive law

Evie is actually using a very important idea in Algebra called the **distributive law**.

That is that $a(b + c) = ab + ac$

Fitness check

Expand the brackets in these expressions

1. $3(3a + 2b)$
2. $5(c + 3d)$
3. $2(4e - f)$
4. $6(2g - 5h)$
5. $-2(5a + 3b)$
6. $-4(3c - 2d)$
7. $3(2e - 3f + 4g)$
8. $5(3h - 2j + 6k)$

Factorising

When you rewrite an expression using brackets, you are said to be **factorising** an expression.

For example, $3a + 12b$ can be written as $3(a + 4b)$ because 3 is a factor of both terms.

We simply write $3a + 12b = 3(a + 4b)$

Fitness check

Factorise these expressions

1. $9a + 15b$
2. $5c + 10d$
3. $24e - 6f$
4. $21g - 9h$
5. $-25a + 10b$
6. $-36c - 24d$
7. $6e - 9f + 12g$
8. $32h - 16j + 8k$

Algebra 3b: Mastery questions

1. Which of these tables shows that y is directly proportional to x?

A

x	2	3	4	5
y	4	9	16	25

B

x	2	3	4	5
y	8	9	10	11

C

x	2	3	4	5
y	8	12	16	20

D

x	2	3	4	5
y	5	7	9	11

2. In the following table, y is directly proportional to x. What is the constant of proportionality?

x	2.5	5	10	20
y	7.5	15	30	60

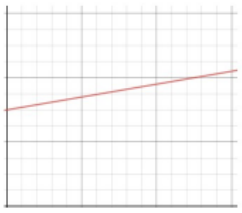
A	B	C	D
2	3	5	10

3. If y is directly proportional to x, which of these statements could not be true?

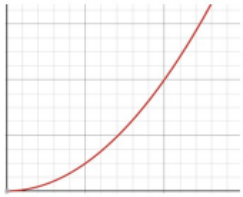
A	$\frac{y}{x} = 5$	B	$\frac{x}{y} = 7$
C	$\frac{y}{6} = x$	D)	$\frac{4}{y} = x$

4. Which of these graphs show that y is directly proportional to x ?

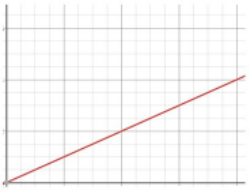
A



B



C



D



5. Which of these points lies on the graph of $y = 2.5x$

A	B	C	D
(2.5, 1)	(5,2)	(3,7.5)	(7.5,3)

6. If you were asked to plot the the graph of $y = 4x$ for $-3 \le x \le 3$ on a pair of suitable axes. Which of these would be the correct range of values for y?

A	$-3 \le y \le 3$	B	$6 \le y \le 6$
C	$-9 \le x \le 9$	D)	$-12 \le x \le 12$

7. If you were asked to plot the line $y = 5x - 3$ which of these points would not lie on the line?

A	B	C	D
(2, 7)	(3, 12)	(6,32)	(8, 37)

8. Look at Fig.1. It contains 2 different graphs. You are given one of the equations. What is the equation of the other graph?

A	$y = x - 3$	B	$y = 2x - 3$
C	$y = -2x + 3$	D)	$y = -2x - 3$

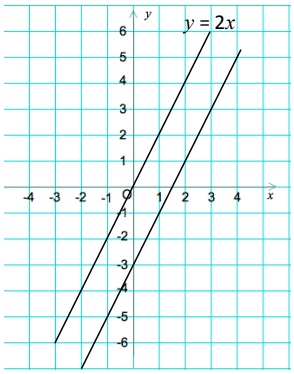


Fig. 1.

9. Mrs Pearson pays one rate for 100 units of electricity she uses and a different rate for any additional units used as shown in Fig.2.

   How much must she pay for using 900 units of electricity?

A	B	C	D
£180	£100	£126	£116

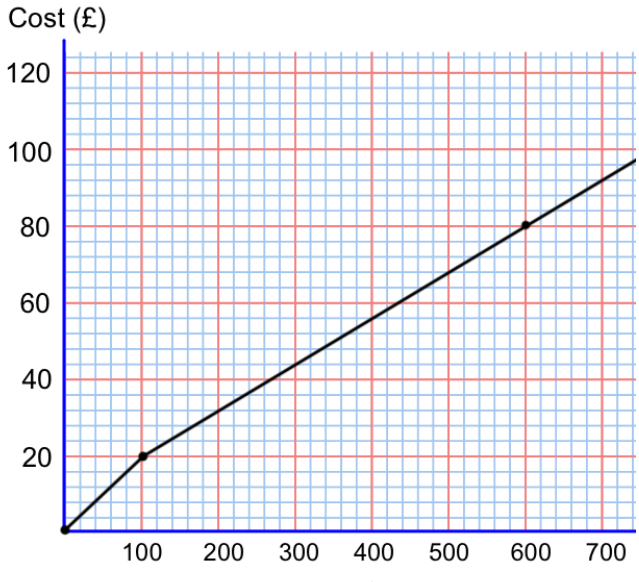
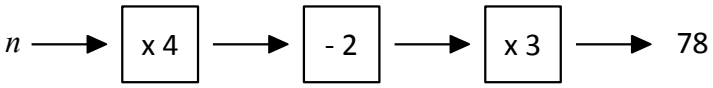


Fig. 2.

10. Will is n years old. Layla is 3 years older than Will. Luke is twice as old as Layla.
The total of their ages is 61. Work out the age of each person.
Which of these equations can be used to solve this question?

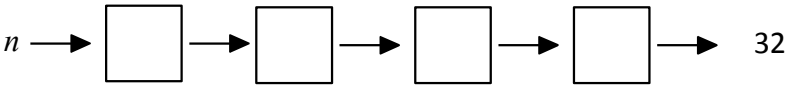
A	B	C	D
$3n + 3 = 61$	$4n + 3 = 61$	$4n + 6 = 61$	$4n + 9 = 61$

11. Look at this function machines. Which of these equations does it represents?



A	$3(4n - 2) = 78$	B	$4(3n - 2) = 78$
C	$4n - 6 = 78$	D	$4n - 2 \times 3 = 78$

12. This function machine represents the equation $\frac{7n - 2}{4} + 8 = 32$



Which operation belongs in the third box?

A	B	C	D
x 7	÷ 4	+ 8	- 2

13. Which option gives you the correct method for solving this equation?

$$5n - 6 = 74$$

A	Add 6, divide by 5
B	Divide by 5, add 6
C	Add 6, multiply by 5
D	Subtract 6, divide by 5

14. Which option gives you the correct method for solving this equation?

$$\frac{a}{4} - 3 = 15$$

A	Subtract 3, divide by 4
B	Add 3, divide by 4
C	Multiply by 4, add 3
D	Add 3, multiply by 4

15. Solve the equation $5(x - 2) + 6 = 31$

A	B	C	D
x = 4	x = 5	x = 6	x = 7

16. Solve the equation $\frac{x + 3}{3} - 1 = 3$

A	B	C	D
$x = 3$	$x = 6$	$x = 9$	$x = 12$

17. Which option is correct in making x the subject of the formula $y = 7x + 4$?

A	Subtract 4, divide by 7
B	Divide by 7, subtract 4
C	Multiply by 7, subtract 4
D	Subtract 4, multiply by 7

18. Which option is correct in making a the subject of the formula $v = u + \boldsymbol{at}$?

A	Divide by t, subtract u
B	Divide by u, subtract t
C	Subtract u, divide by t
D	Subtract u, divide by a

19. Expand

$3(2x - 5)$

A	B	C	D
$5x - 8$	$6x - 8$	$6x - 15$	$6x - 5$

20. Expand and simplify

$4y - 2(y - 3)$

A	B	C	D
$2y - 6$	$2y + 6$	$3y - 6$	$3y + 6$

21. Expand and simplify

$3(5a - 2) + (a - 4)$

A	B	C	D
$9a - 10$	$16a - 6$	$18a - 18$	$16a - 10$

22. Factorise

$5x + 20$

A	B	C	D
$5(x + 4)$	$5(x + 20)$	$25x$	$5 + x + 20$

23. Factorise fully

$24y - 72$

A	B	C	D
$8(3y - 9)$	$3(8y - 24)$	$12(2y - 6)$	$24(y - 3)$