# Which occupies more space?



Alice

What is the same, and what is the difference between my two wall displays?

(A)



(B)





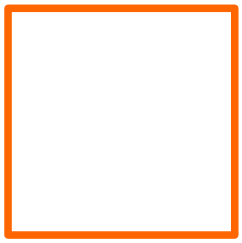
Sarah

The lengths of the ropes are the same, but...

(C)

(D)

(C)



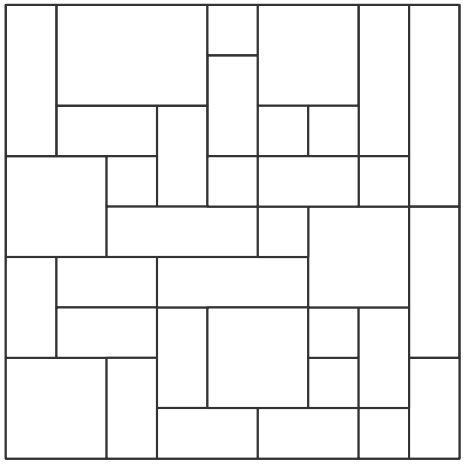
(D)



**The Territory Game**

This is a game for four players.
The aim of the game is to capture the most territory.
Each player chooses a colour.
In each round, each player throws a dice and the highest score win.
Each time someone wins, they colour in a rectangle in their chosen colour – but - you can only colour in a rectangle adjacent to one of your coloured rectangles.
The game is over when the board is completely filled in. The winner is the player who has the largest territory.

**The Territory Game Board**



# The size of space


Alice

Green

Yellow


Jon


Sarah

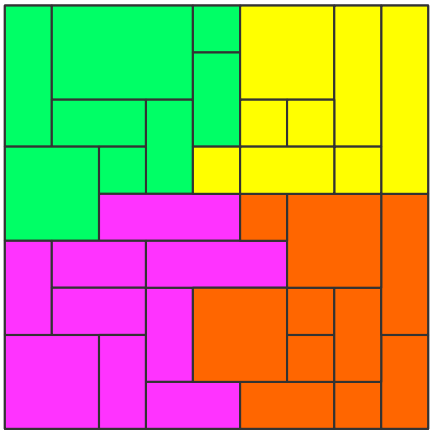
Purple

Orange


Matthew

The picture below shows a final board.

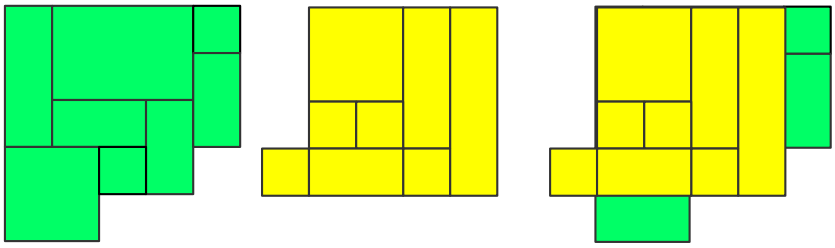
Which player has the greatest territory?



1 How did you decide who won?

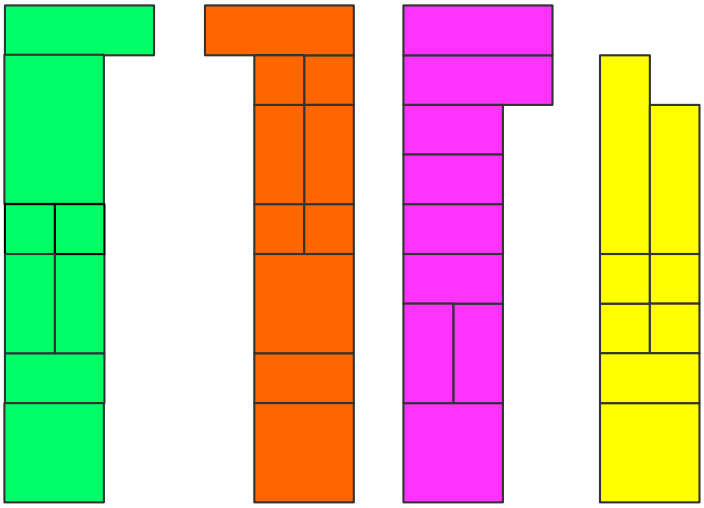

Alice

Well, I definitely beat Jon




Sarah

I think this definitely proves that I won



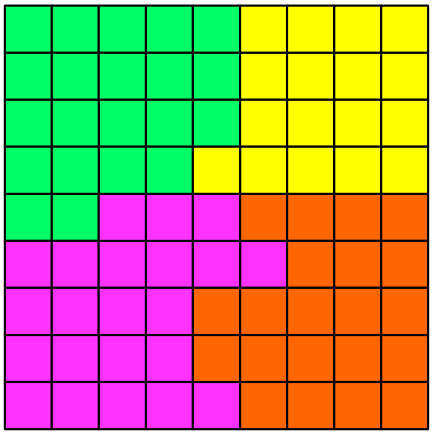
2 Can you explain the ideas of these two students?

# Measuring and Expressing Area



Matthew

I used a different method.
I divided everything into small squares.



3 How can you tell who has the most territory using Matthew's method?



Jon

That's easy. Just count the squares



Alice

I've got 21 squares

I've only got 17 squares



Jon



Sarah

I've got 22 squares

And I've got 21 squares too.



Matthew



Sarah

As I said, I am the winner.

… and I came last!



Jon



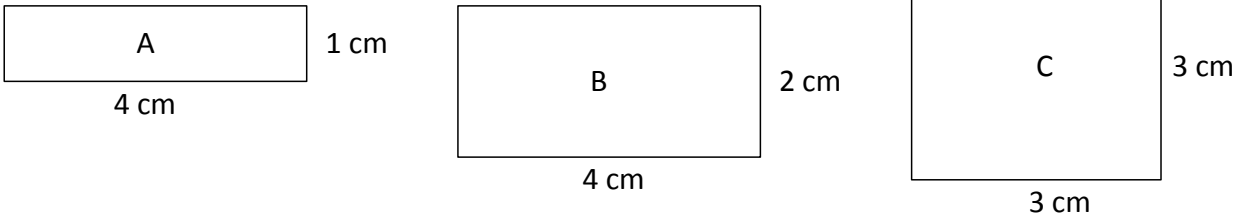
Evie

The amount of space is called the **area**. We can express the area by the number of unit squares that can fit inside.

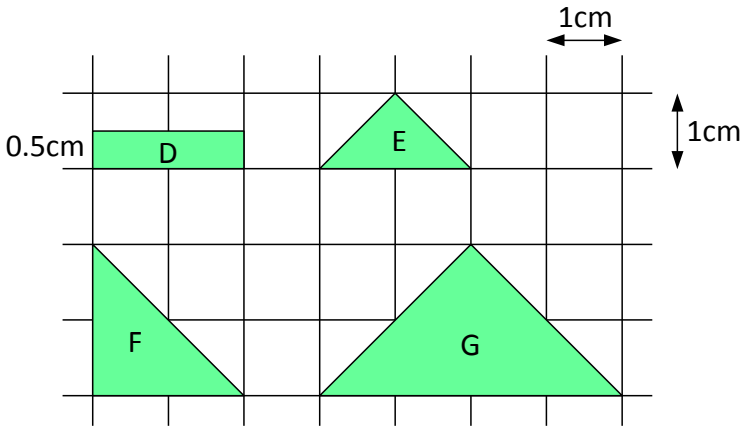
The area of a square with 1cm sides is 1 **square centimetre** and can be written as 1**cm²**.

'Square centimetre" is a unit for area.

In 1cm$^2$ , what is the area of each of these shapes?

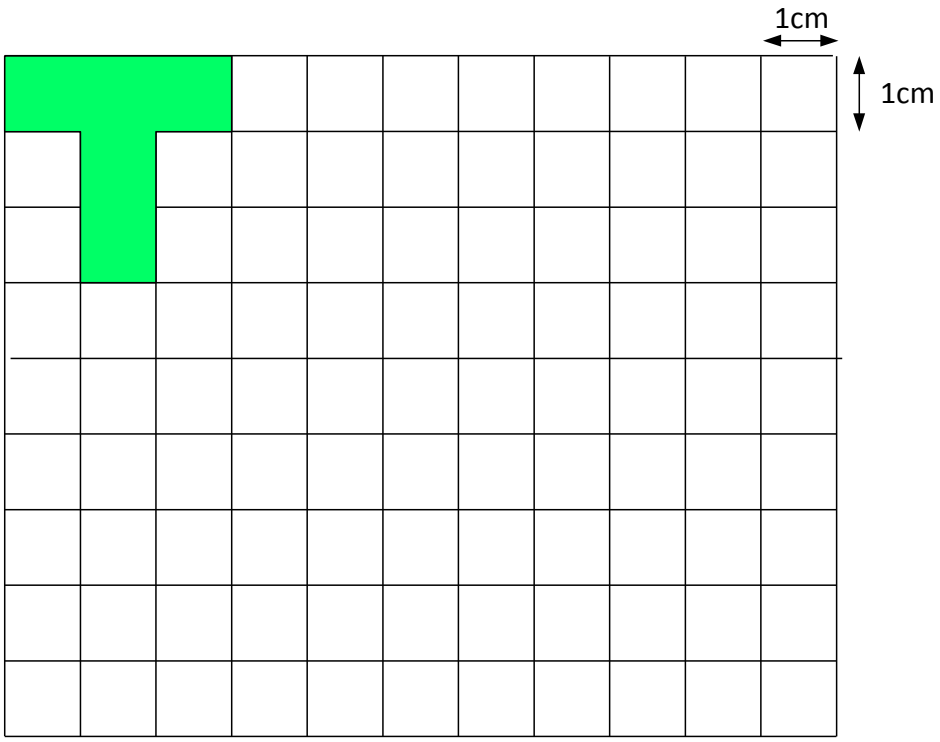


In 1cm$^2$ , what is the area of each coloured section, D, E, F and G?




4

How many different shapes can you design with an area of 5cm$^2$?



# Areas of rectangles and squares

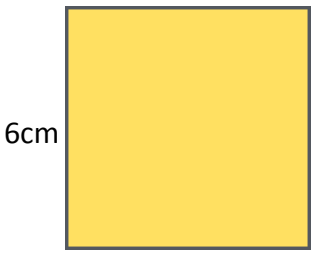
5

Think about how to find the areas of rectangles and squares below

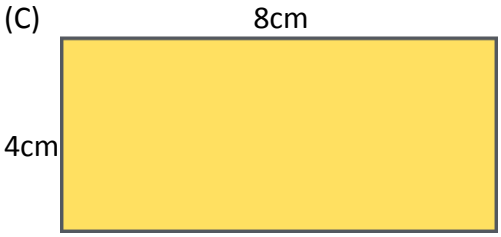
(A)




(B)



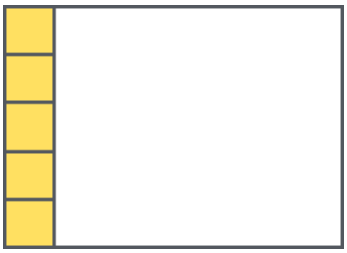

(C)



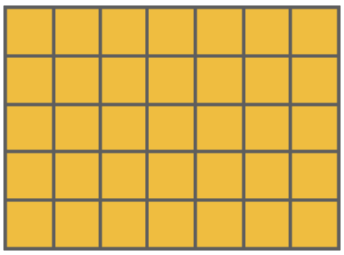

Let's think about how to calculate the area...

1) How many $1cm^2$ squares can be placed vertically in a rectangle (A)?

2) Calculate the number of $1cm^2$ squares that can be placed inside the rectangle (A). In $cm^2$, what is the area?



| Number along vertical side | | Number along horizontal side | | Total number |
|---|---|---|---|---|
| | x | | = | |
| Length of the vertical side (cm) | | Length of the horizontal side (cm) | | Area ($cm^2$) |





Jon

The number of $1cm^2$ squares that can be placed along the vertical and the horizontal sides are the same as the lengths of those sides.

Calculate the areas of square (B) and rectangle (C) shown on the previous page.

(B) ☐ x ☐ = ☐ ☐ $cm^2$

(C) ☐ x ☐ = ☐ ☐ $cm^2$

The area of rectangles and squares can be calculated as follows:



Jon

Measure the lengths of the adjacent sides.

Multiply the two numbers that express the lengths of the sides



Matthew

**Area of rectangle** = **Length** (length of vertical side) x **Width** (length of horizontal side)

**Area of a square = Side x Side**



Sarah

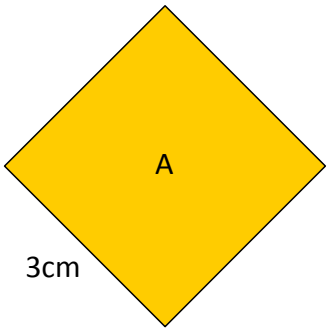
Maths sentences like the one Matthew wrote down are called formulas. Matthew has given us formulas for the areas of squares and rectangles.

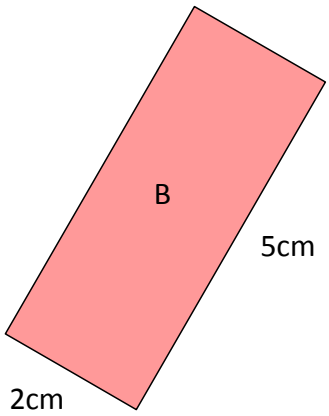
1. What are the areas of the following rectangle and square in $cm^2$

   a) A bookmark with the vertical side of 14cm and the horizontal side of 4cm?

   b) A placemat where both sides are 17cm?

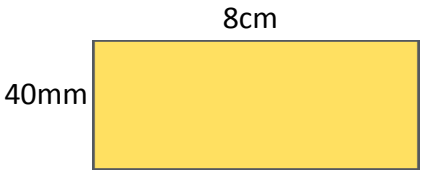
2. Calculate the areas of the shapes below.



# Measuring and Expressing Area

1. Find the area of the rectangle with a vertical side of 40mm and a horizontal side of 8cm.

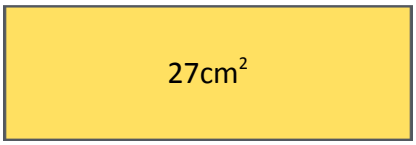




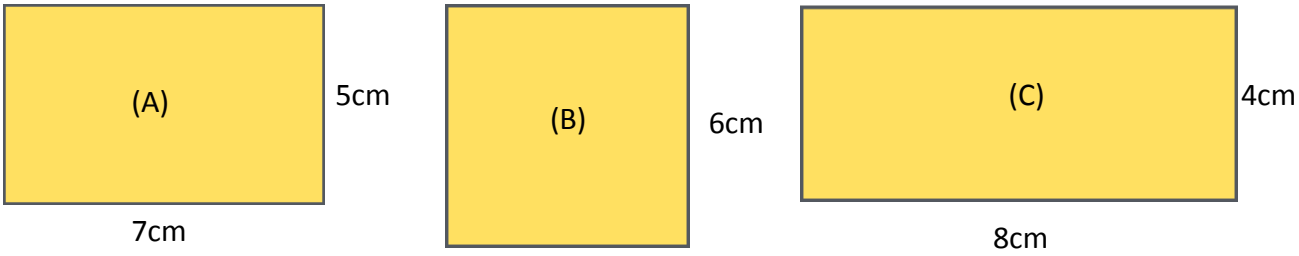

I think we need to use the same units...

Jon

2. To draw a rectangle with an area of 27cm$^2$, and a horizontal side of 9cm, how long should the vertical side be?




Shapes (A), (B) and (C) all have perimeters of 24cm .




3. Copy the table below. Summarize the areas and lengths of the sides of the rectangles and square with the perimeters of 24cm.

| | Vertical Length | Horizontal Length | Area | Perimeter |
|---|---|---|---|---|
| (A) | | | | |
| (B) | | | | |
| (C) | | | | |
| | | | | |
| | | | | |

Even though the perimeters are the same, the areas are...



Alice

6

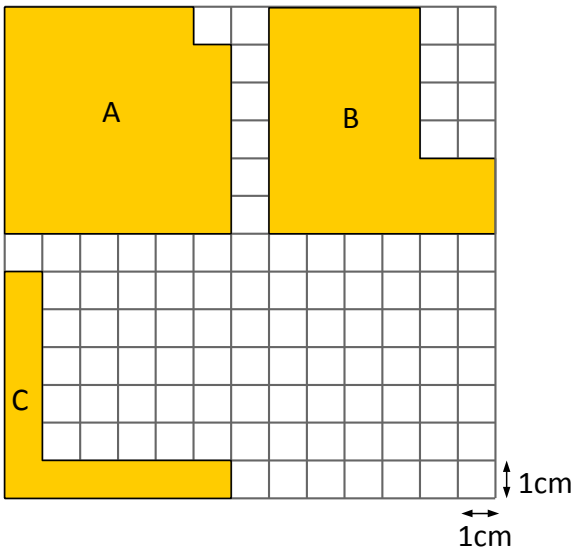
Can you add the details of two more different rectangles that also have a perimeter of 24cms?

# Calculating the area of compound shapes



Sarah

These shapes also have a perimeter of 24cm...




7 Can you work out the area of each shape?



Alice

It's easy – just count the squares.
So the area of C is 11cm$^2$.



Matthew

Perhaps we can calculate the area of shapes like this. It would be easier than counting the squares in shape A.



Jon

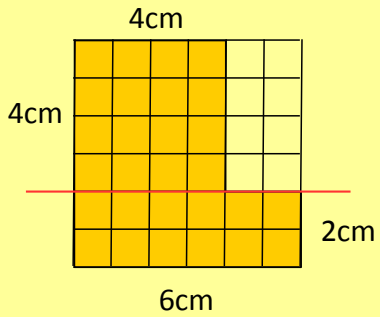
If the shape is a rectangle or a square, we have already learned how to calculate the area...

8 Can you calculate the area of shape B using Jon's ideas?
Is there more than one way you can think of doing it?



Evie

Evie's idea




My calculation is

(4 x 4) + (6 x 2) = 16 + 12 = 28

So the area of shape A is 28cm$^2$.

**Think about it by yourself...**

Is there anything you have learnt already you can use here?

**Write down all of your ideas...**

Write them down in a way that others will be able to understand!

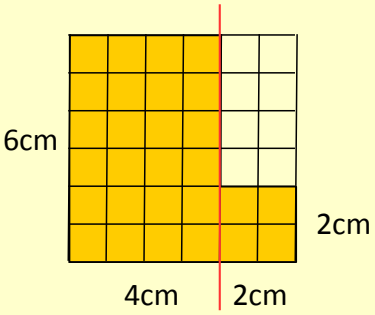
**See if you can find other ways to solve it...**

# Calculating the area of compound shapes

**Some more ideas**



Sarah



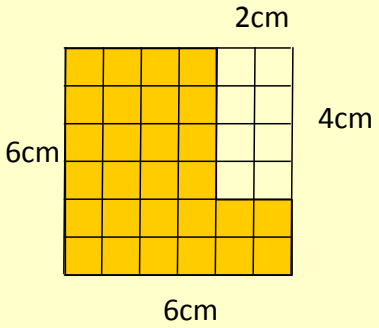

My calculation is

(6 x 4) +( 2 x 2) = 24 + 4 = 28

The area is 28cm$^2$.



Matthew




My calculation is

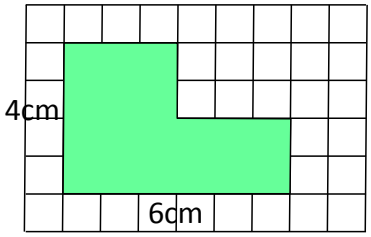
(6 x 6) - (2 x 4) = 36 - 8 = 28

The area is 28cm$^2$.

9 Can you explain Sarah's calculation and how it relates to the diagram? What is different about Matthew's idea?

**Fitness check**

Look at this shape. How many different ways can you find to calculate the shaded area?




10 Look at the calculations below. Can you explain how each of these calculations fits the diagram?



Jon

4 x 3 + 2 x 3 = 12 + 6 = 18

The area is 18cm$^2$.



Alice

3 x 2 + 6 x 2 = 6 + 12 = 18

The area is 18cm$^2$.



Evie

4 x 6 - 2 x 3 = 12 + 6 = 18

The area is 18cm$^2$.



Matthew

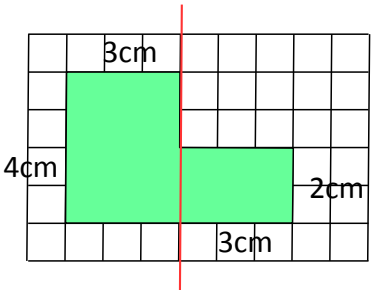
(4 x 9) ÷ 2 = 36 ÷ 2 = 18

The area is 18cm$^2$.

# Calculating the area of compound shapes



Jon

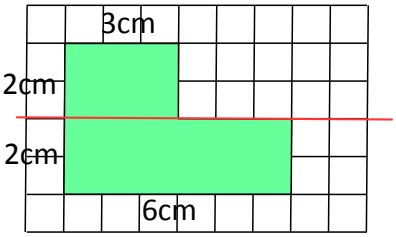



4 x 3 + 2 x 3 = 12 + 6 = 18

The area is 18cm$^2$.



Alice

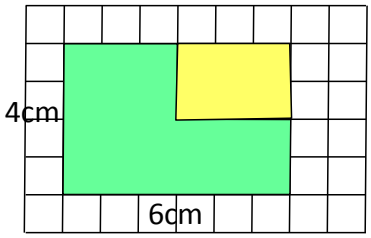



3 x 2 + 6 x 2 = 6 + 12 = 18

The area is 18cm$^2$.



Evie

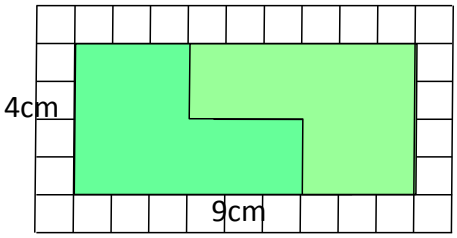



4 x 6 - 2 x 3 = 12 + 6 = 18

The area is 18cm$^2$.



Matthew




(4 x 9) ÷ 2 = 36 ÷ 2 = 18

The area is 18cm$^2$.

11 What idea is common to all these solutions?

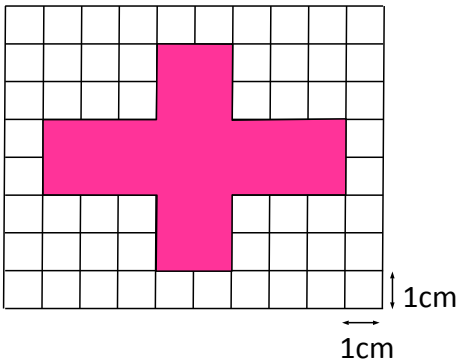


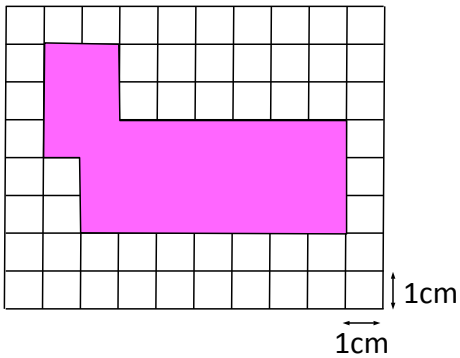
Amber

All of the students changed the shape into familiar rectangles and then...

**Fitness check**

Calculate the area of the two shapes below in as many ways as possible.
In each case, show how your calculation matches the diagram.

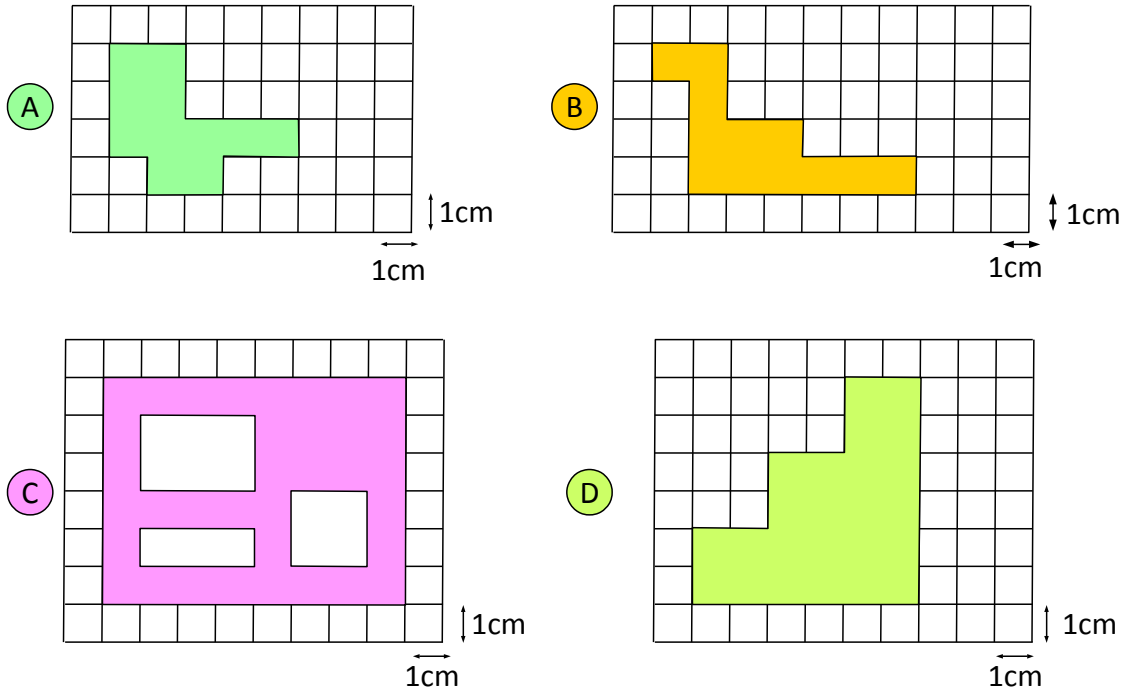


# Calculating the area of compound shapes

Fitness check

1. Write down three different calculations that could be used to calculate the areas of these coloured shapes.




2. Three possible calculations for Amber's shape are:

   6 x 6 – 4 x 3; 3 x 6 + 2 x 3; 4 x 3 + 2 x 6

   Can you draw Amber's shape?

3. Three possible calculations for Evie's shape are:

   3 x 2 + 2 x 2 + 1 x 1; 3 x 4 + 2 x 2 – 1 x 1; 4 x 4 – 2 x 2 – 1 x 1

   Can you draw Evie's shape?

12 Compare the areas of these two wall displays.



80cm

120cm



1m

1m

1. What is the area of the first picture in $cm^2$?

[ ] x [ ] = [ ] $cm^2$

2. What is the area of the second picture in $cm^2$?

[ ] x [ ] = [ ] $cm^2$

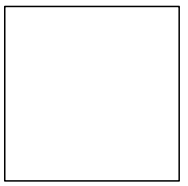


Jon

To express the area of a large space like a school hall, we use a square with 1m sides as the unit.

The area of a square with 1m sides is called **1 square metre**.

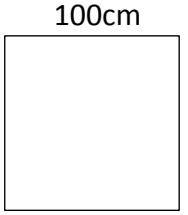
It is written as $1m^2$



1m

1m

13 How many $cm^2$ is $1m^2$?



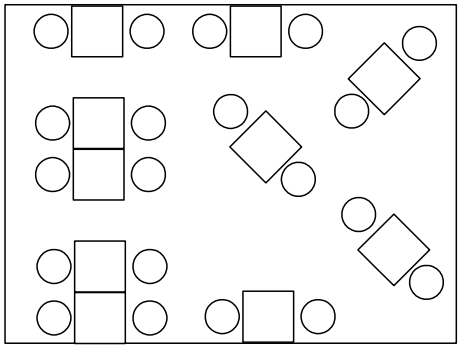
100cm

100cm

$1m^2$ = [ ] $cm^2$

14

The coffee shop floor is rectangular, as shown. What is the area of the coffee shop floor in $m^2$?




1. How many $1m^2$ tiles would be needed to carpet the floor of the shop?
2. Calculate the area of the coffee shop by using the formula for the area of a rectangle.

Even when the lengths are expressed in m, you can still use the formula to find the area...



Jon



3. What is the area of a square carpet with 2m sides in $m^2$?
4. What is the area of a square carpet with sides of 4m long?
5. What is the area of the same carpet in $cm^2$?

15

How many people can stand within a 1m square?

Make a square with 1m sides using a large sheet of paper and try it out!

16 Calculate the area of a rectangular field that measures approximately 200m by 150m





Evie

That's easy. 200 x 150 = 30000
So the area of the field is 30000m$^2$.

**A new unit of measure**

For large areas, we often measure the area in **hectares** where

**1 hectare** is written as **1ha** and represents an area of 10 000m$^2$.

17 What is the area of the field in hectares?



Matthew

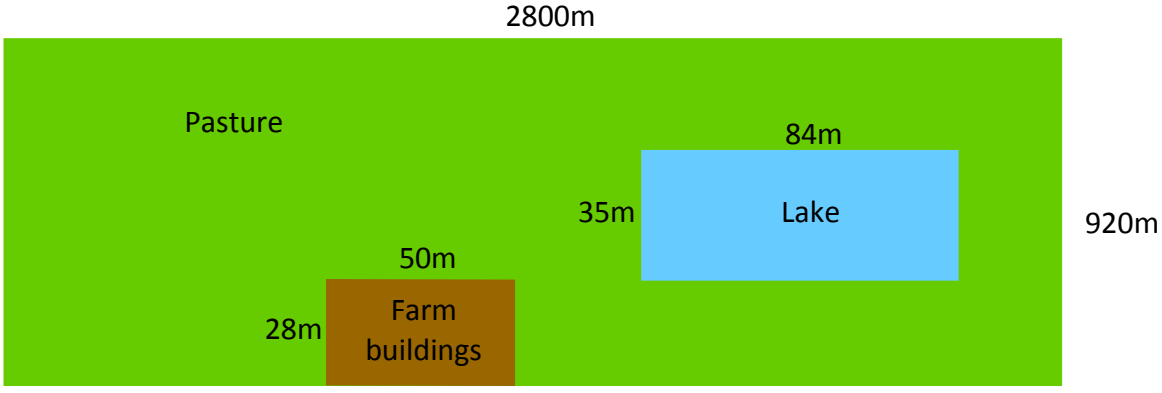
Matthew's working

*The area of the field is 30 000m$^2$.*

*That is 3 x 10 000m$^2$.*

*So the area is 3ha.*

1. What is the area of a square field with 300m sides in m$^2$?
2. What is the area of the same field in hectares?
3. If the area of a rectangular shaped farm is 24ha, what might be the dimensions of the farm?
4. Look at the picture below of a sheep farm in Wales. What is the area of pasture in m$^2$? In hectares?



# Changing units

18

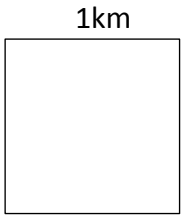
Alice's town is roughly shaped like a rectangle on her map with 4km and 6km sides. Think about how to express the area of the town.





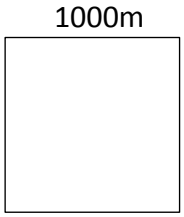
Jon

To express a large area like the area of a town or county, we use a square with 1km sides as the unit




The area of a square with 1km sides is 1 square kilometre and is expressed as $1km^2$

1. What is the approximate area of Alice's town in $km^2$?
2. How many $m^2$ is $1km^2$?
3. How many hectares (ha) is $1km^2$?






Alice

Alice's working

1. Area of the rectangle = $4 \times 6 = 24km^2$
2. $1km = 1000m$

   So $1km^2 = 1000 \times 1000 = 1000000m^2$

   So $1km^2$ is the same as $1000000m^2$.
3. $1\ km^2 = 100 \times 10000m^2 = 100ha$

**Changing units**

Convert

1. $5000cm^2$ into $m^2$
2. $5km^2$ into $m^2$
3. $5km^2$ into hectares
4. 10 hectares into $m^2$
5. 10 hectares into $km^2$
6. $500000m^2$ into hectares

# Measuring and Expressing Area

## Measuring small areas

A First Class stamp measures 24mm by 20mm.

What is the area covered by a stamp?

What do you think are the most appropriate units to measure this area?




24mm

20mm



Jon

To express a small area like the area of a stamp, we need a smaller unit. Generally, use a square with 1mm sides as the unit.

The area of a square with 1mm sides is 1 square millimetre and is expressed as $1mm^2$

1. What is the area of the stamp in $mm^2$?
2. What is the area of the stamp in $cm^2$?
3. How many $mm^2$ are in $1cm^2$?



Amber

Amber's working

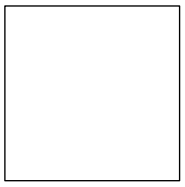
1. Area of the rectangle = $24 \times 20 = 480mm^2$

2. Working in cm, area = $2.4 \times 2.0 = 4.8cm^2$

3. So $1cm^2 = 10 \times 10 = 100mm^2$

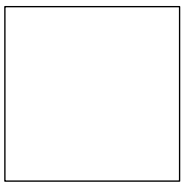
So $1cm^2$ is the same as $100mm^2$.

1cm



1cm

10mm



10mm

## Changing units

Convert

1. $5000mm^2$ into $cm^2$
2. $40cm^2$ into $mm^2$
3. $1m^2$ into $mm^2$
4. $20000000mm^2$ into $cm^2$
5. $20000000mm^2$ into $m^2$
6. $200cm^2$ into $mm^2$

# Fitness check

1. Find the areas of the rectangles and squares below:

   a) A rectangle with a vertical side of 10cm and a horizontal side of 23 cm.

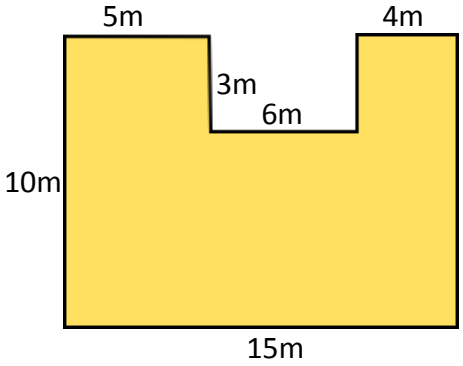
   b) A rectangle with a vertical side of 50mm and a horizontal side of 3cm.

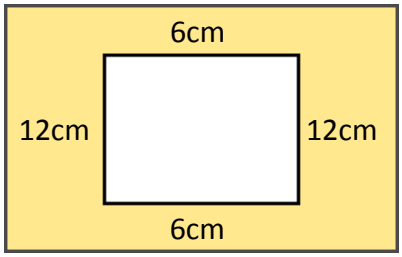
   c) A square with a perimeter of 16m.

2. We are making a rectangle with a vertical side of 3cm by bending a 20cm wire. What is the area of the rectangle in $cm^2$?




3. Find the areas of the shaded figures below:




4. Investigate the areas of various shapes.

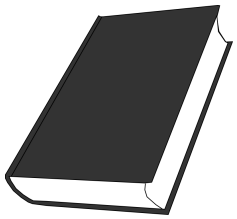
   You could look at:

   - Doors
   - Tables
   - Books
   - Rooms

   Try estimating first.







# Appropriate units

1. Fill in each of the boxes with a unit of area or a word.

   a) The area of a square with 1m sides is 1 ☐

   b) The area of a square with 1km sides is 1 ☐

   c) The area of a square with 1cm sides is 1 ☐

   d) Area of a rectangle = ☐ x ☐

   e) Area of a square = ☐ x ☐

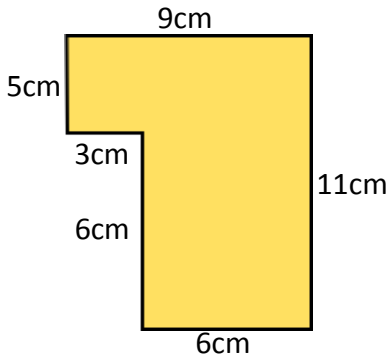
2. Write an appropriate unit of area in each [ ].

   a) The area of a small basketball court is 420 [ ]

   b) The area of a farmer's field is 1376 [ ]

   c) The area of a desktop in a classroom is 2400 [ ]

3. Think about how to calculate the area of a compound shape, such as shown below.






Evie

Evie's working

My calculation is

$(5 \times 9) + (6 \times 6) = 45 + 36 = 81$

So the area is $81cm^2$

   a) Can you explain Evie's way of calculating the area, showing where the segment line should be?

   b) Can you find the area of the shape using a different method?

Do you understand the units and the formulas for area?

Do you understand the units for area?

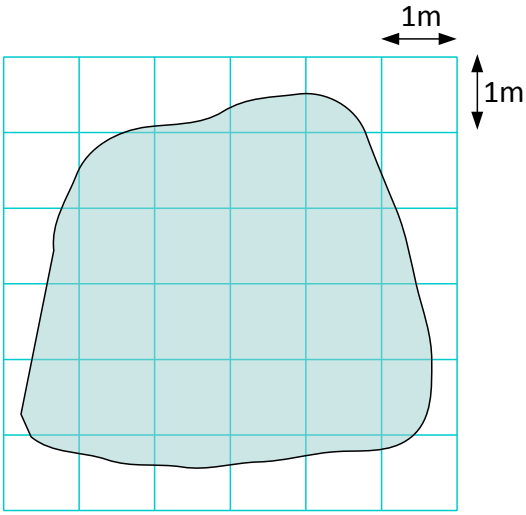
Do you know how to calculate the area of shapes like these?

# How big is the fish pond?



Matthew

It all depends on what you are measuring. You could mean …




19

Can you finish off Matthew's sentence?



Alice

Perimeter?



Sarah

Or the area covered by the pond?

Or even the volume of water in the pond!



Jon

20

Can you think of a way of estimating the perimeter of the pond?



Amber

You could use a piece of string placed carefully around the drawing.

21

Can you think of a way of estimating the area of the pond?



Evie

You could count the squares.

22

Using these ideas, estimate the perimeter and the area of the pond.

# How big is the fish pond?



Evie

The area must be less than $34m^2$.



Jon

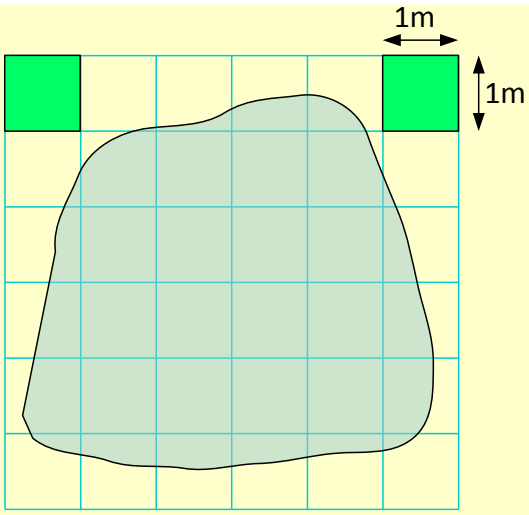
And it must be more than $14m^2$.

23 Can you explain Evie's idea? Can you explain Jon's thinking?

Evie's working



Evie




My calculation is

$6 \times 6 = 36$

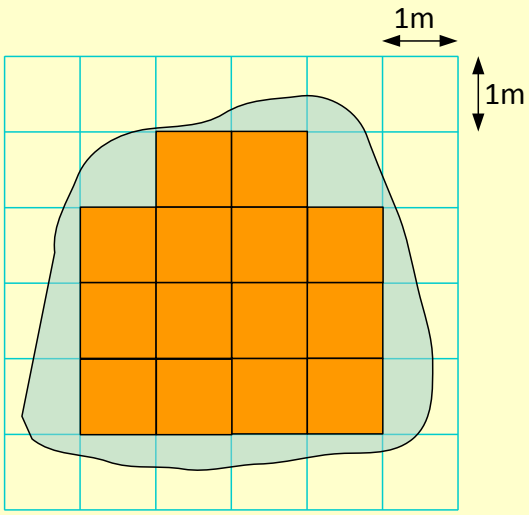
$36 - 2 = 34$

So the area is less than $34cm^2$

Jon's working



Jon




I can fit 14 whole squares completely within the pond.

So the area is more than $14cm^2$

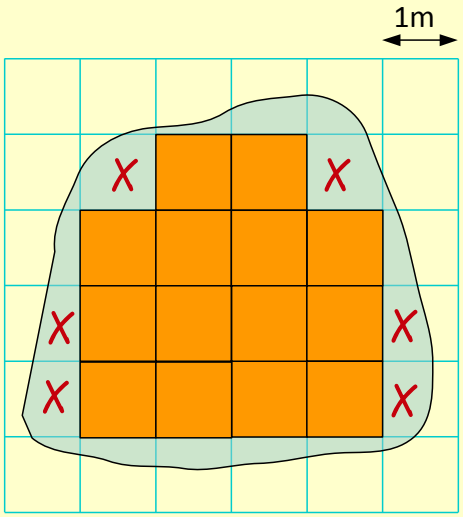
# How big is the fish pond?



Sarah

I filled in more of the pool using "half" squares. I think the area is actually bigger than 17m$^2$.

Sarah's working




I can fit 14 whole squares, and there are another 6 squares that are more than half covered.

So my calculation is

$14 + 6 \times 0.5 = 14 + 3 = 17$

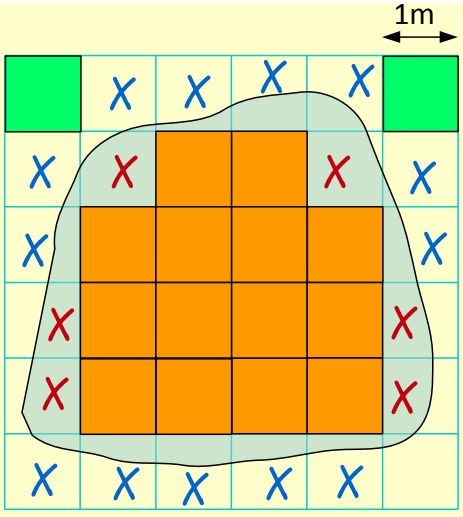
So the area is more than 17m$^2$.



Matthew

If you use the same idea for the squares outside the pool, then the area is less than 27.5m$^2$.

Matthew's working




I can fit 2 whole squares outside of the pool, and there are another 13 squares that are less than half covered.

So my calculation is

$36 - 2 - 13 \times 0.5$

$= 34 - 6.5 = 27.5$

So the area is less than 27.5m$^2$.



Amber

I think I can do better by mixing and matching up parts of squares!

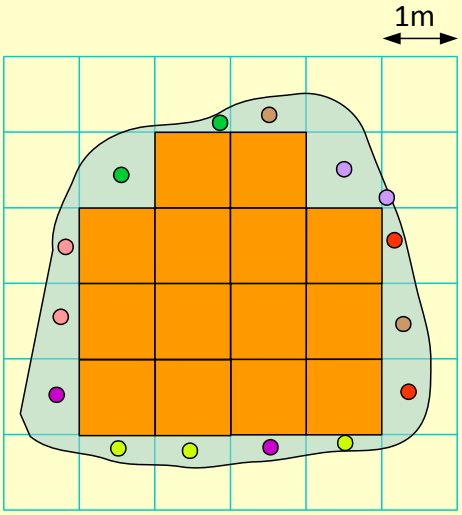
# How big is the fish pond?



Amber

First of all I am going to count squares that are completely within the pool and those which are almost inside.

Amber's working




I can fit 14 whole squares and then I have created 7 more whole squares, by mixing and matching. I then have three small parts left which would make about half a square.

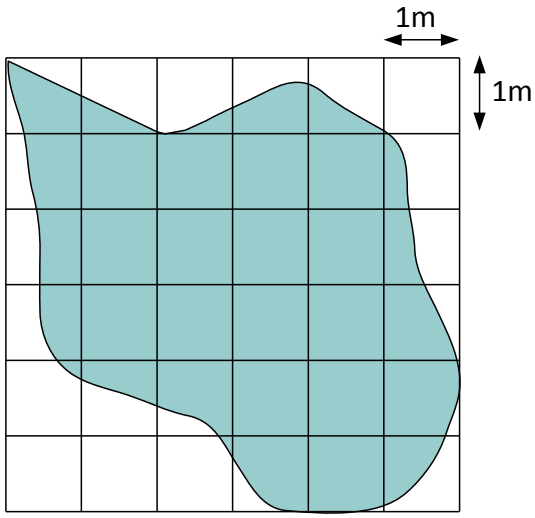
So my calculation is

$14 + 7 + 0.5 = 21.5$

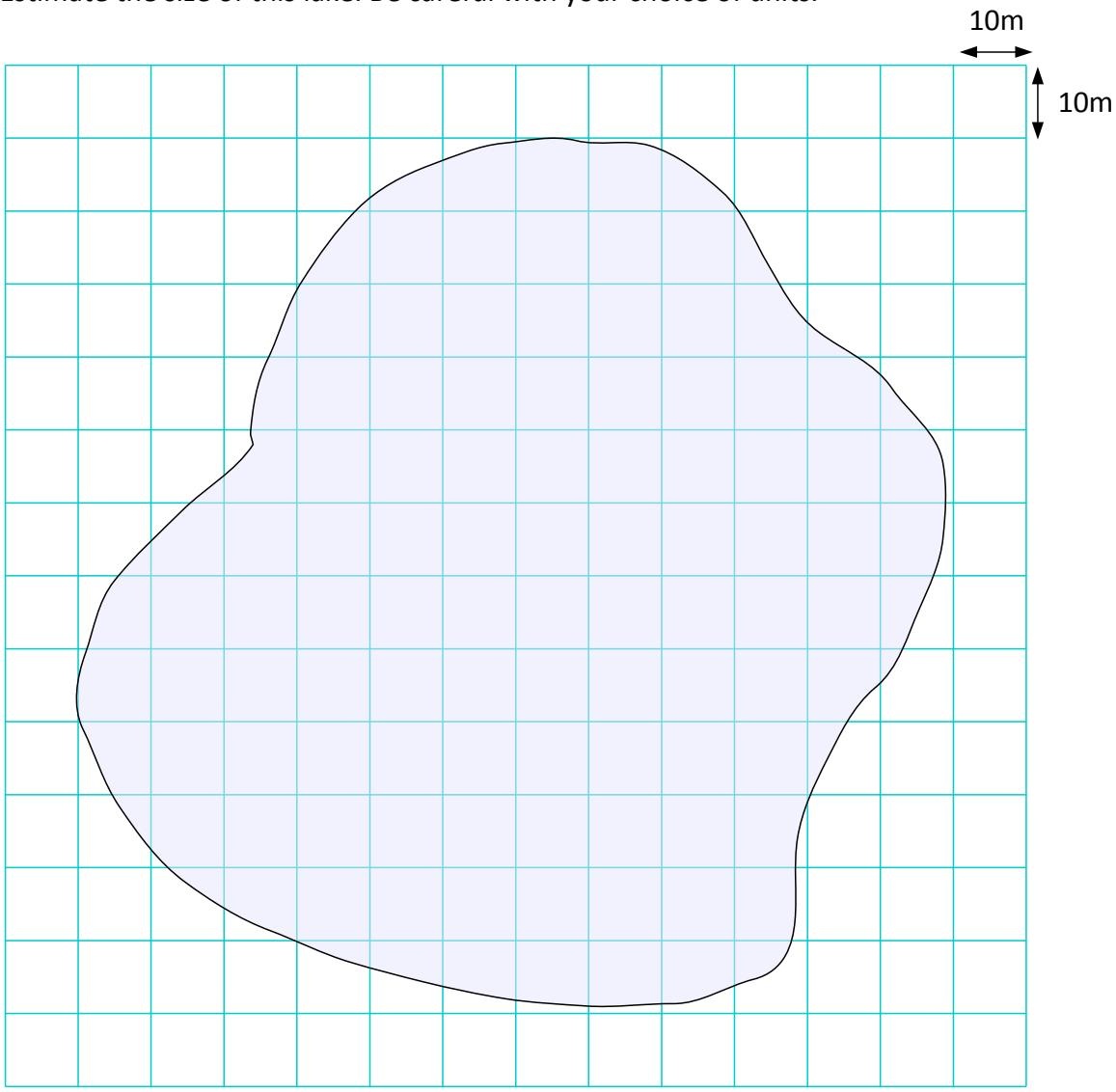
So I estimate the area is approximately $21.5m^2$.

24 Can you explain Amber's idea to your partner?

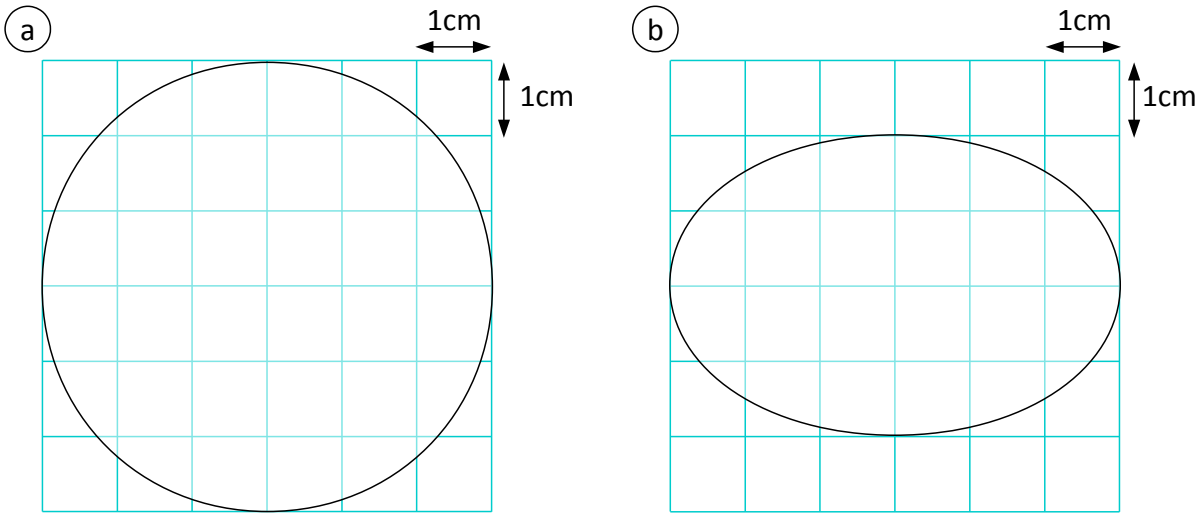
1. Now, using Amber's method, estimate the area of this pond.



2. Estimate the size of this lake. Be careful with your choice of units.



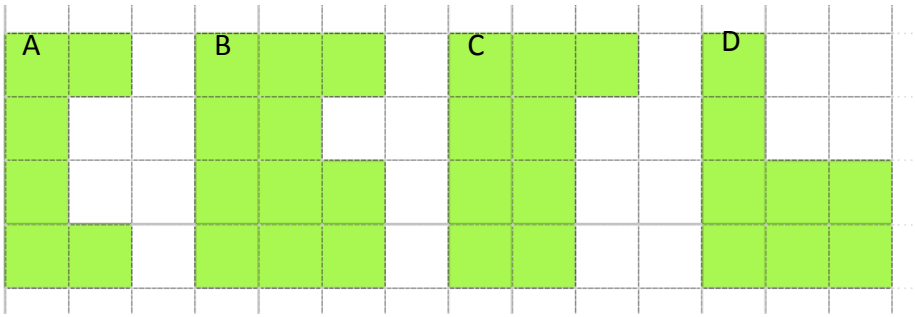

3. Estimate the area of: a) this circle and b) this oval



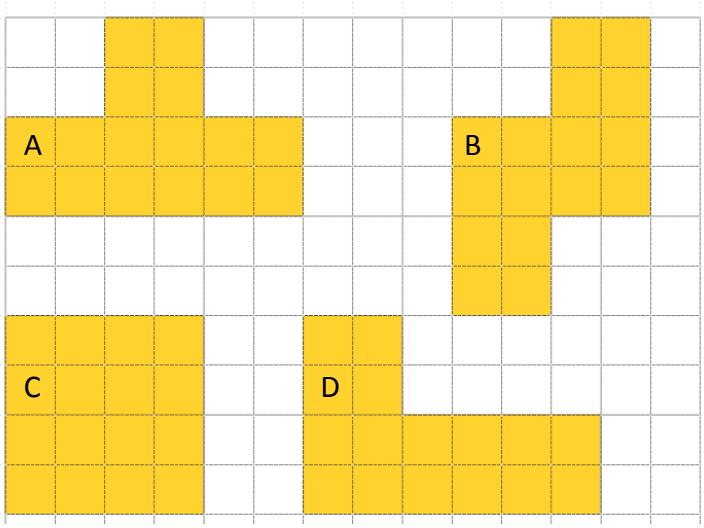
# Geometry 1c: Mastery questions

In questions 1 – 12, four possible answers are given. Only one of them is correct. Record the letter which you think indicates the correct answer. You must be able to give reasons for your choice.

1. Here are some shapes drawn on a 1cm square grid. Three of them have a perimeter of 14cm. Which one does not have a perimeter of 14cm?



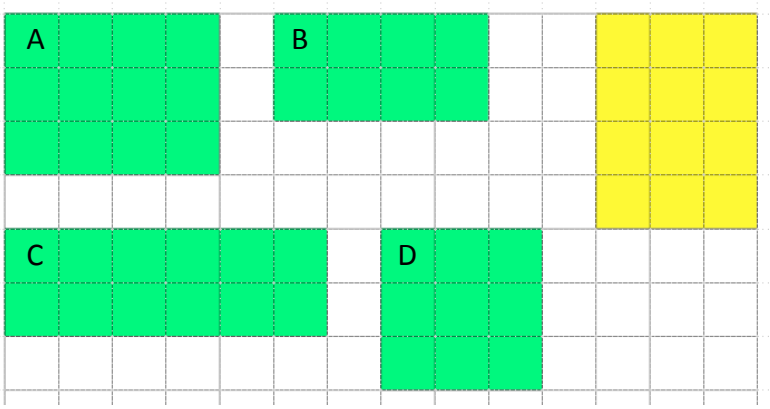

2. All these shapes have an area of 16cm$^2$, but which one has a different perimeter to the others?




3. One side of a square is 6cm. What is the perimeter of the square?

| A | 12cm | B | 24cm | C | 36cm | D | 6cm |
|---|---|---|---|---|---|---|---|

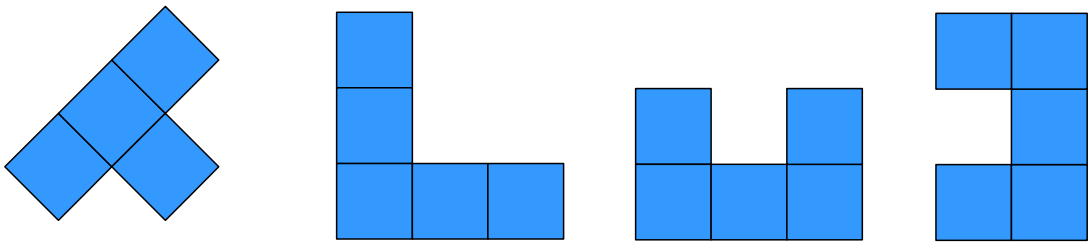
4. Which of these rectangles has the same area as the yellow shape?



5. Amber has drawn two rectangles on centimetre squared paper. The first one has length 6 squares and width 3 squares. The second has length 5 squares and a width of 4 squares. What is the combined area of the rectangles?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A | 36cm | B | 36cm$^2$ | C | 38cm$^2$ | D | 18cm$^2$ |

6. What is the total area of these 4 shapes, each made from 1cm square tiles?



| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A | 18cm$^2$ | B | 19cm$^2$ | C | 20cm$^2$ | D | 21cm$^2$ |

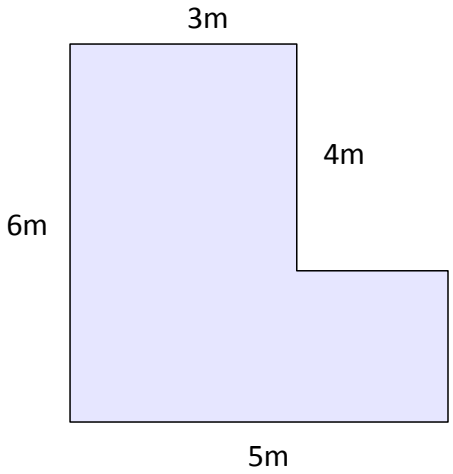
7. A rectangle measures 7 cm long and 3 cm wide. What is the area of the rectangle?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A | 10cm | B | 10cm$^2$ | C | 20cm | D | 21cm$^2$ |

8. Alice has drawn a square using 1cm square paper. What could the area of the square be?

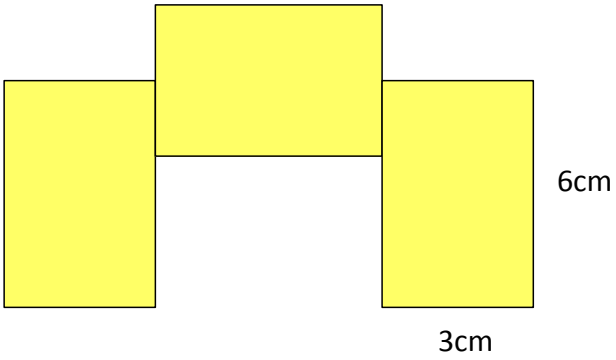
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A | 20cm$^2$ | B | 24cm$^2$ | C | 30cm$^2$ | D | 36cm$^2$ |

9. Amber, Jon and Taro were asked to calculate the area of the floor of the PE store. They each did it a different way. Who is correct?



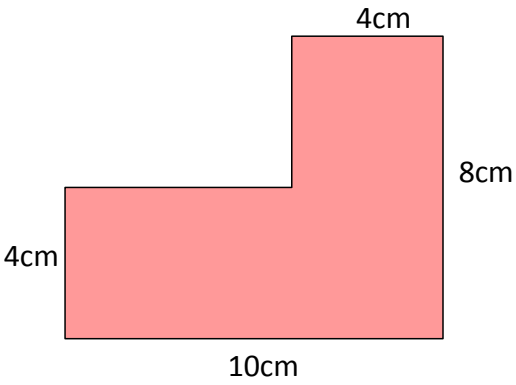

| | |
|---|---|
| A | Amber: 6 x 3 + 2 x 2 |
| B | Jon: 5 x 2 + 3 x 4 |
| C | Taro: 6 x 5 – 2 x 4 |
| D | They are all correct |

10. This shape is made up of 3 identical rectangles. What is the area of the shape?



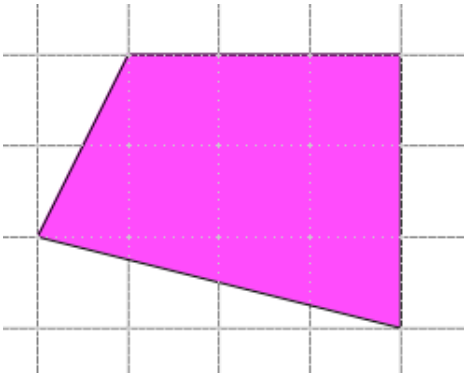

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A | $18cm^2$ | B | $36cm^2$ | C | $54cm^2$ | D | $63cm^2$ |

11. Calculate the area of this shape.




| | |
|---|---|
| A | $26cm^2$ |
| B | $56cm^2$ |
| C | $72cm^2$ |
| D | $1280cm^2$ |

12. Taro drew this shape on 1cm square paper. What is the area of this shape?

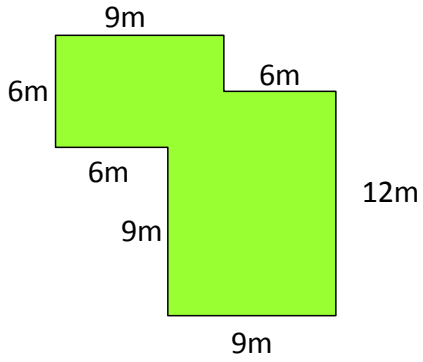


| | |
|---|---|
| A | $6cm^2$ |
| B | $9cm^2$ |
| C | $12cm^2$ |
| D | You can't work it out |

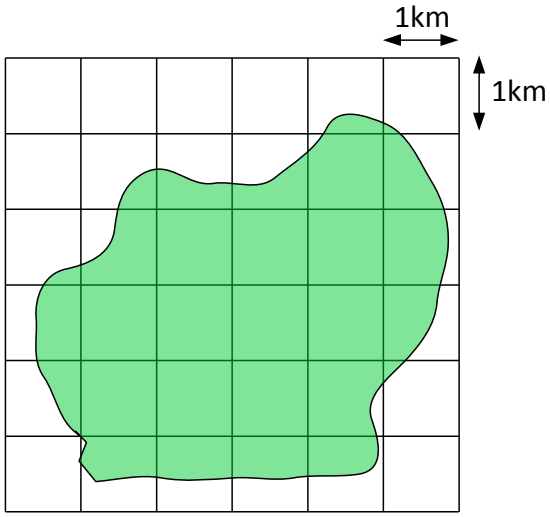
# Area 1 – Mastery questions

In questions 13 – 15, you must show all your working.

13. Calculate the area of this shape.




14. Estimate the area covered by this forest.




15. Matthew says his rectangle has a perimeter of 30cm and an area of 50cm$^2$. Sarah says her rectangle also has a perimeter of 30cm but has an area of 56cm$^2$. Can they both be correct? Use drawings to support your thinking.