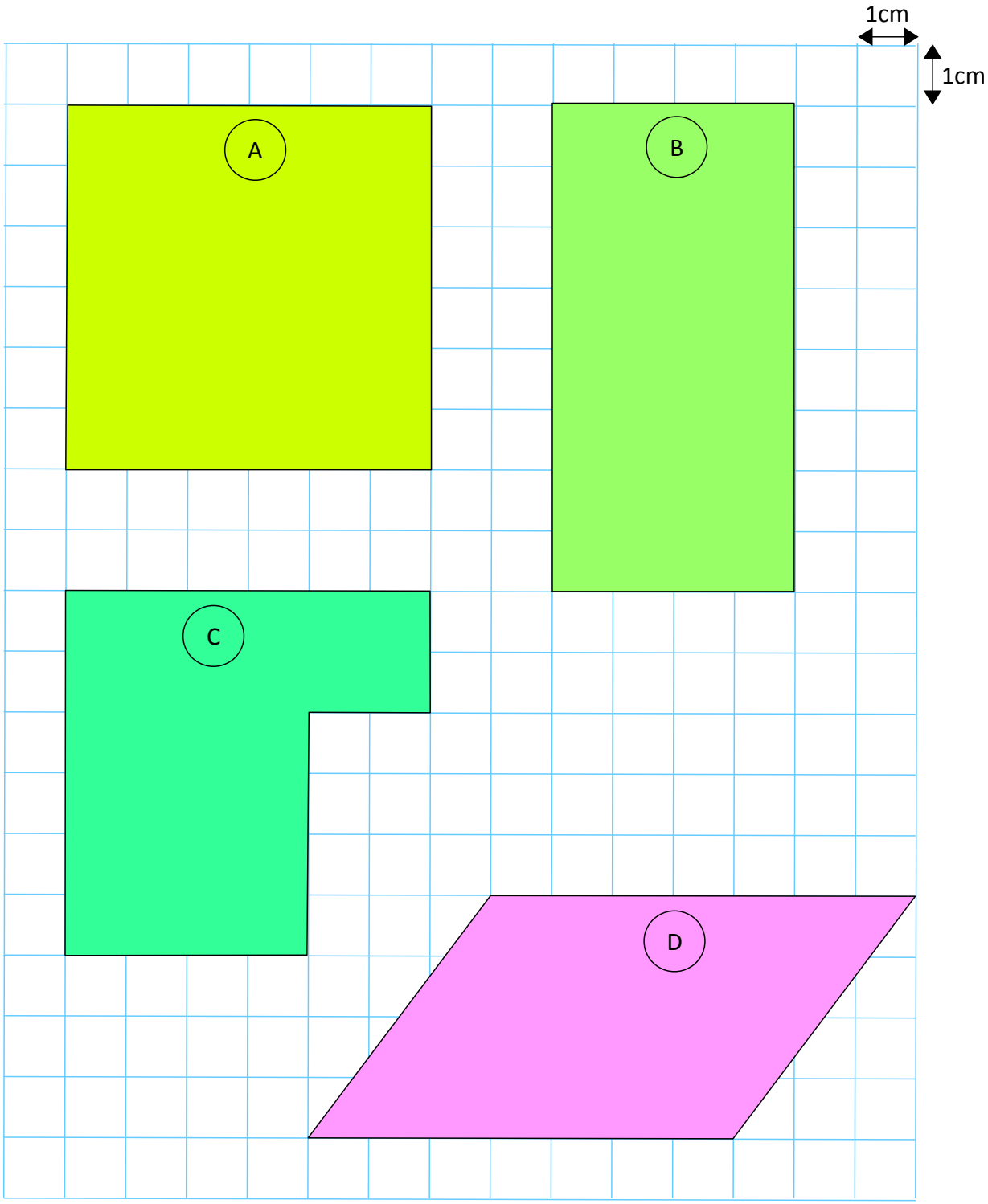
The same but different.

Look at the four shapes shown below. They have the same perimeter but not the same area.



1

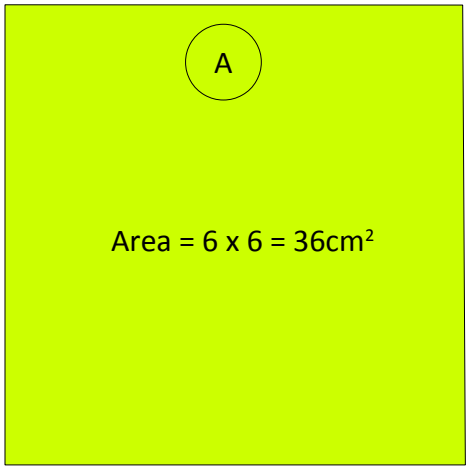
Can you put the shapes in order from the smallest to the largest area?

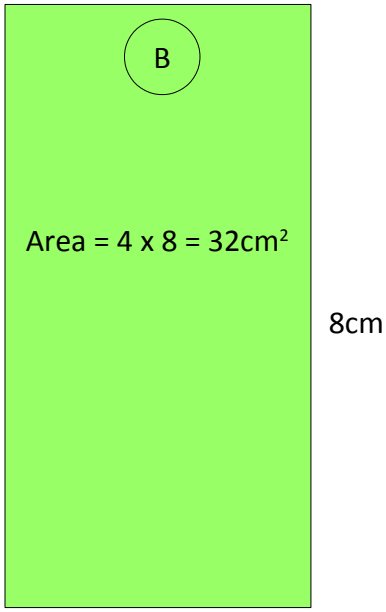
Calculating areas.



Alice

Since shape A is a square and shape B is a rectangle, we can use the formula to calculate the areas.

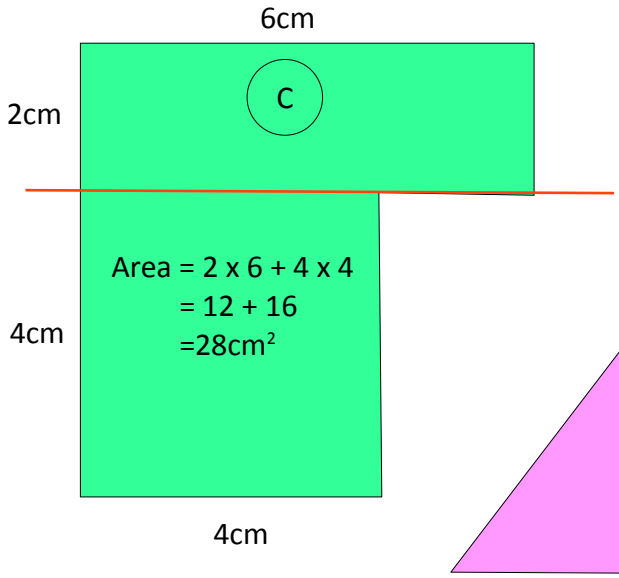


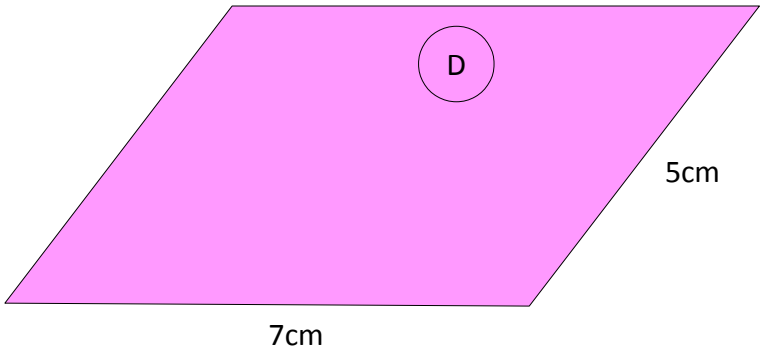




Jon

We can split C into a rectangle and a square and then use the formula.







Amber

Shape D is a parallelogram. We could try counting the squares?

I wonder if the area of shape D is 7 x 5 = 35cm²?



Matthew

2

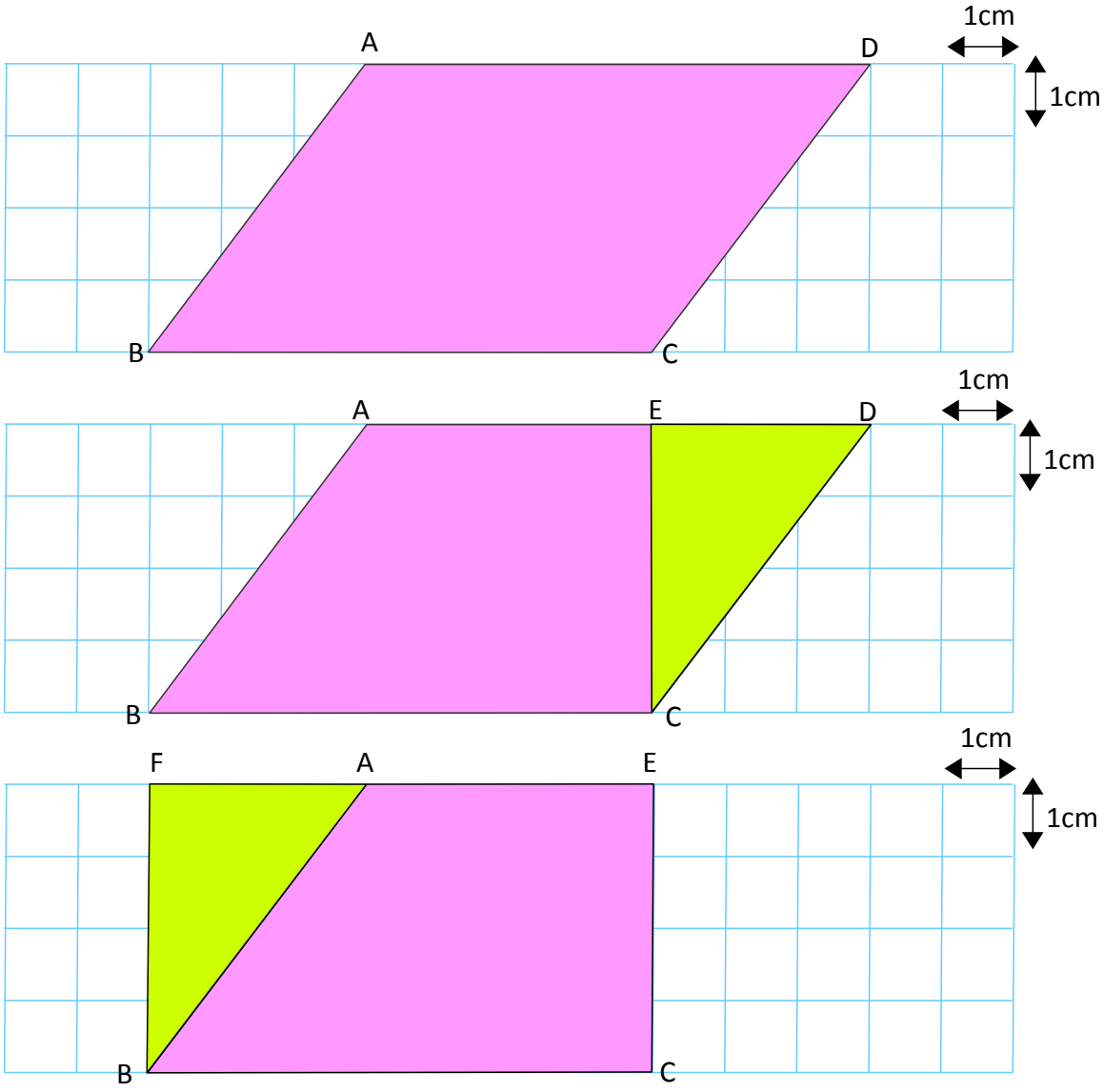
How would you find the area of the parallelogram ABCD?

How would you find the area of the parallelogram ABCD?



Amber

I transformed the parallelogram ABCD into the rectangle FBCE by moving triangle ECD into a new position FBA to work out the area.





Amber

The area of a parallelogram can be found if by changing it into a rectangle.

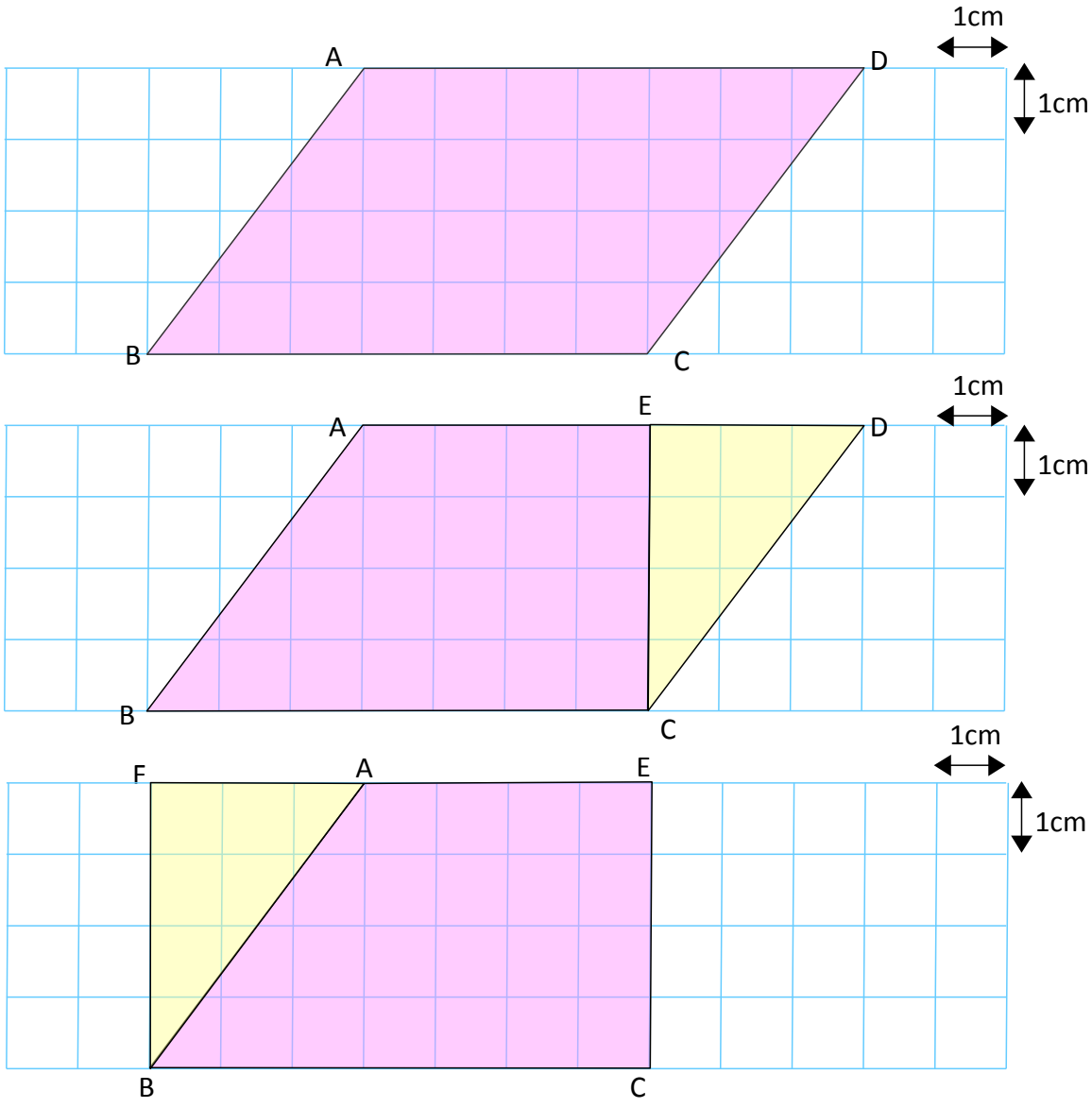
In this case the area is 7 x 4 = 28cm^2

How would you find the area of the parallelogram ABCD?



Sarah

I transformed the parallelogram ABCD into the rectangle FBCE by moving the triangle EDC into a new position FAB to work out theHJ area.





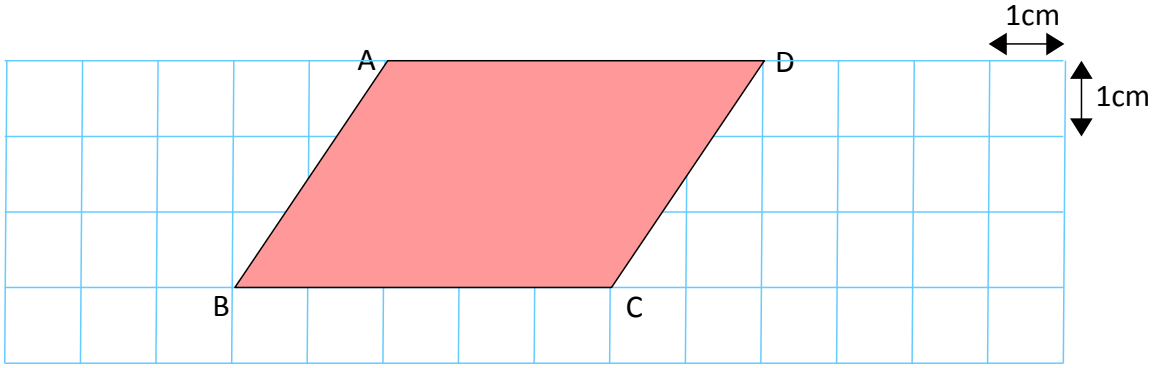
Sarah

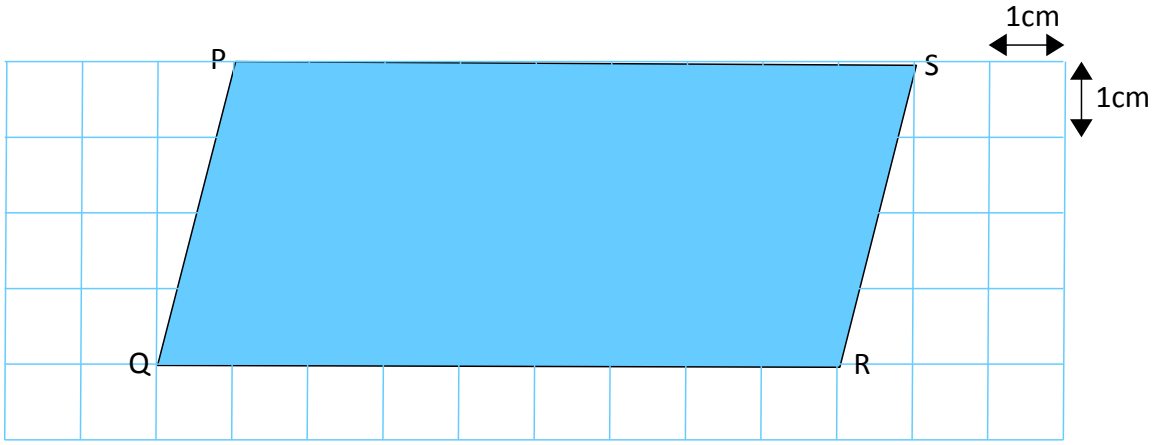
The area of a parallelogram can be found if by changing it into a rectangle.

In this case the area is $7 \times 4 = 28cm^2$

Calculating the area of a parallelogram

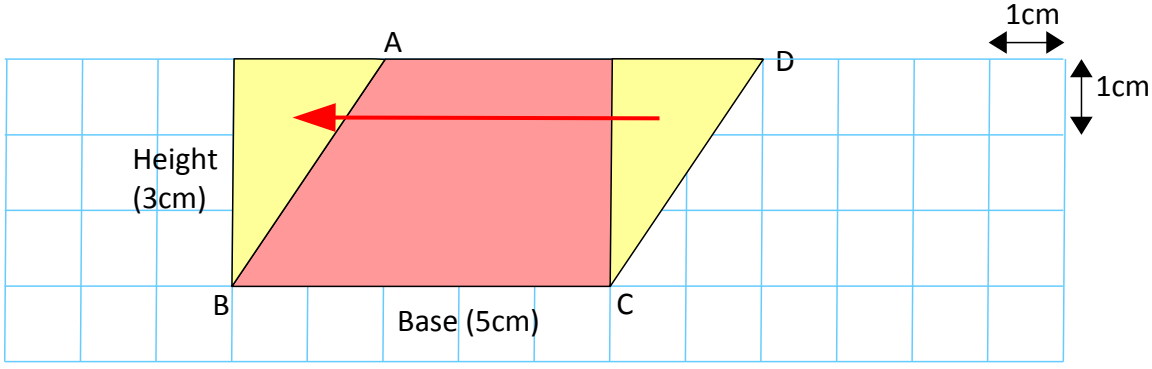
3 Based on the pupils' ideas on the previous page, calculate the area of the two parallelograms shown below by transforming them into rectangles.





4 Can you see a connection between the dimensions of the parallelogram and the dimensions of its transformed rectangle?


Jon



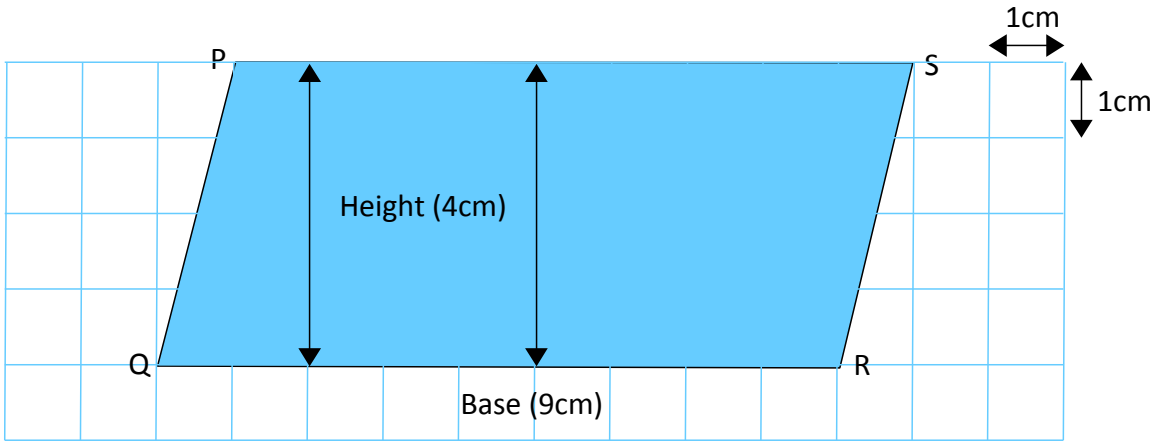
The width of the rectangle is equal to the length of the base of the parallelogram.
The height of the rectangle is equal to the distance between the base and the side parallel to it.
So in this case the area of parallelogram = 5 x 3 = 15cm^2.

Calculating the area of a parallelogram



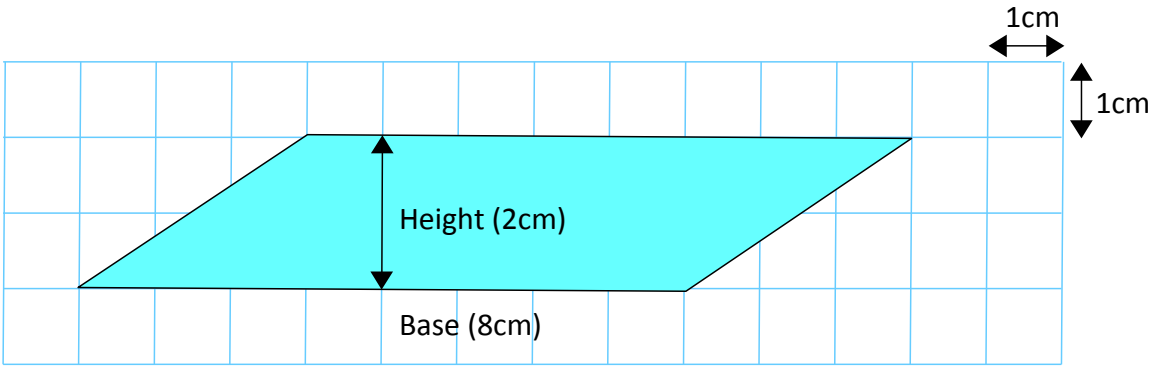
Alice

For this parallelogram the base is 9cm and the height is 4cm.
So the area of the parallelogram will be 9 x 4 = 36cm^2.



5

Look at this parallelogram. What are the dimensions of its transformed rectangle? What is the area of the parallelogram?





Sarah

The transformed rectangle has a base of 8cm and a height of 2cm.

The area of the rectangle and the parallelogram is 8 x 2 = 16cm^2.

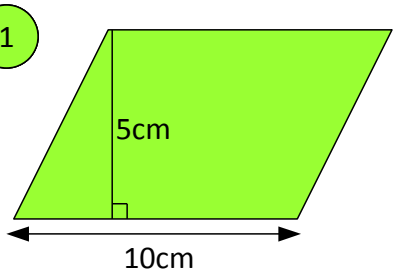
The area of parallelogram can be calculated using the formula

Area of a parallelogram = base x height

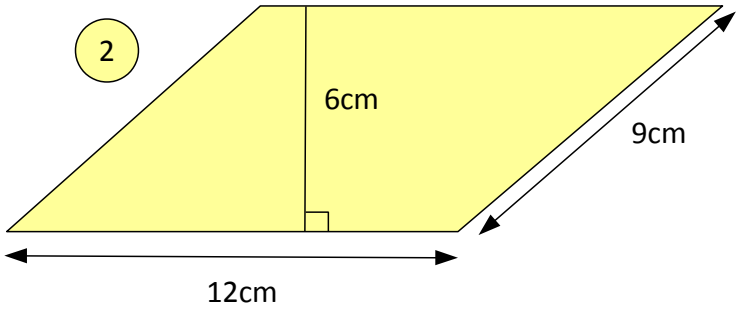
Parallelograms

Calculate the areas of each of these parallelograms:

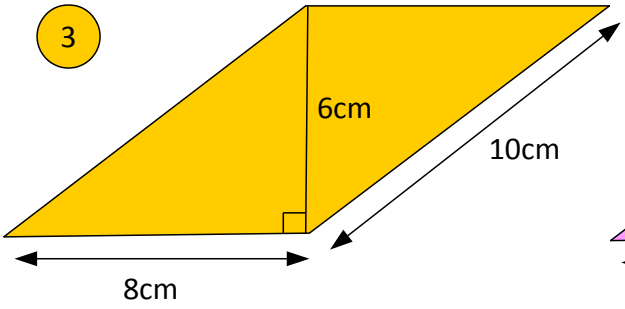
1



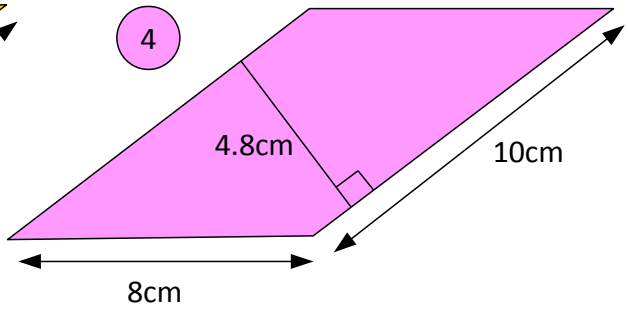
2



3



4



6

In what way is Q4 the same as Q3?
In what way is Q4 different to Q3?



Amber

Q4 and Q3 both use the same parallelogram
BUT Q4 uses a different side as the base to Q3.

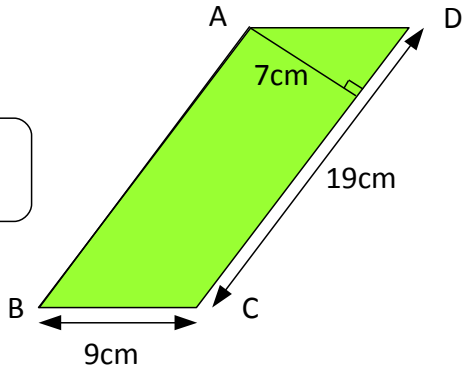


Matthew

So any side can be considered as the base!
And the height is the shortest distance to the opposite parallel side.

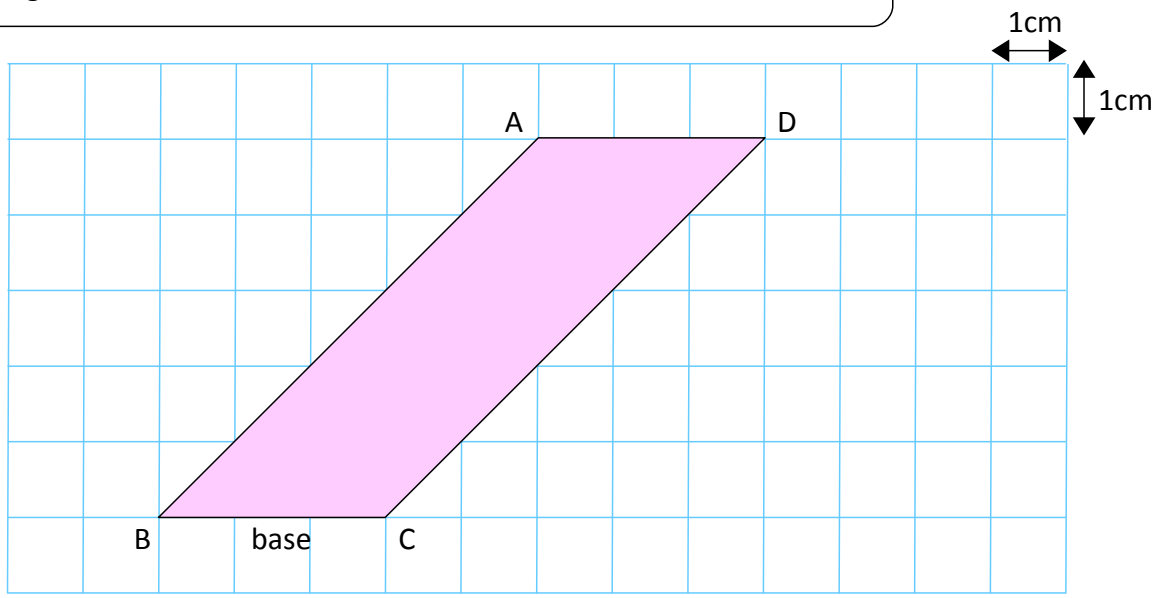
7

Look at this triangle. Which side should we take to be the base? Explain your thinking.



8

Think about how to find the area of a parallelogram ABCD on the right, when side BC is considered as the base.





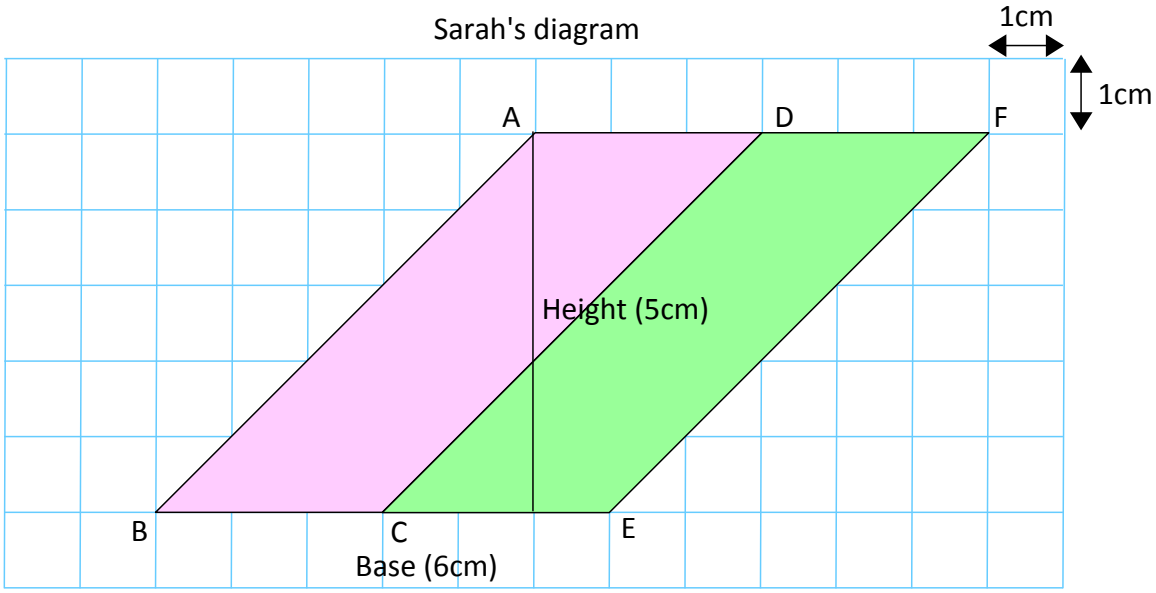
Evie

The base is 3cm but what is the height?



Sarah

Sarah's diagram



Sarah's explanation

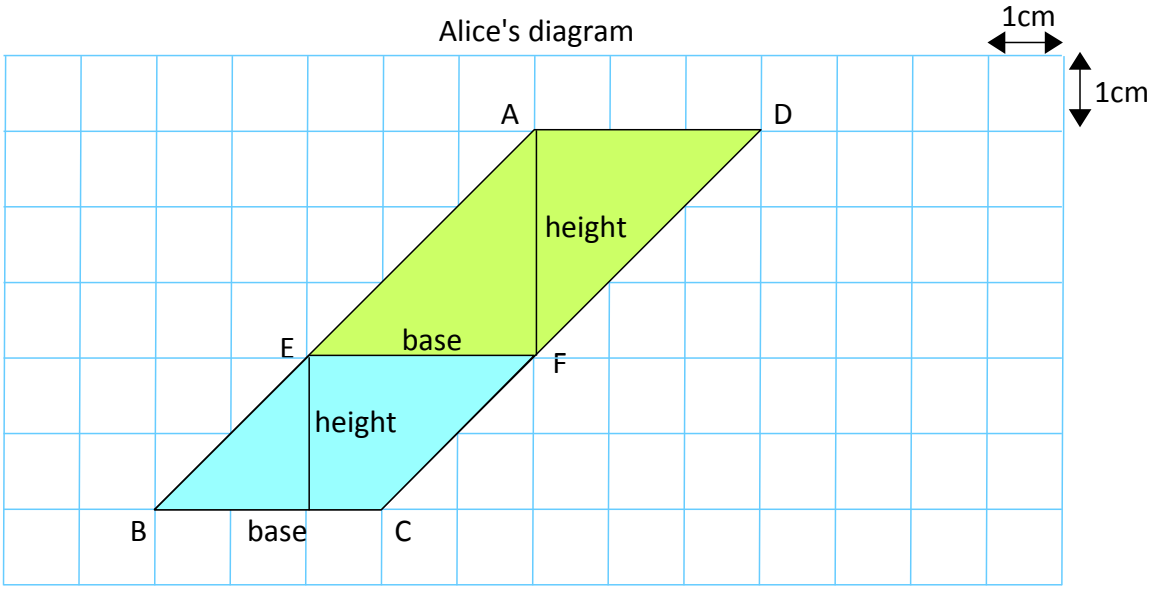
I added an identical parallelogram DCEF to the right of parallelogram ABCD to form a new parallelogram ABEF.
The area of ABEF = 6 x 5 = 30cm^2
But the area of ABEF is twice the area of ABCD
So the area of parallelogram ABCD is ½ x 30 = 15cm^2.



Alice

I did it a different way.

Alice's diagram



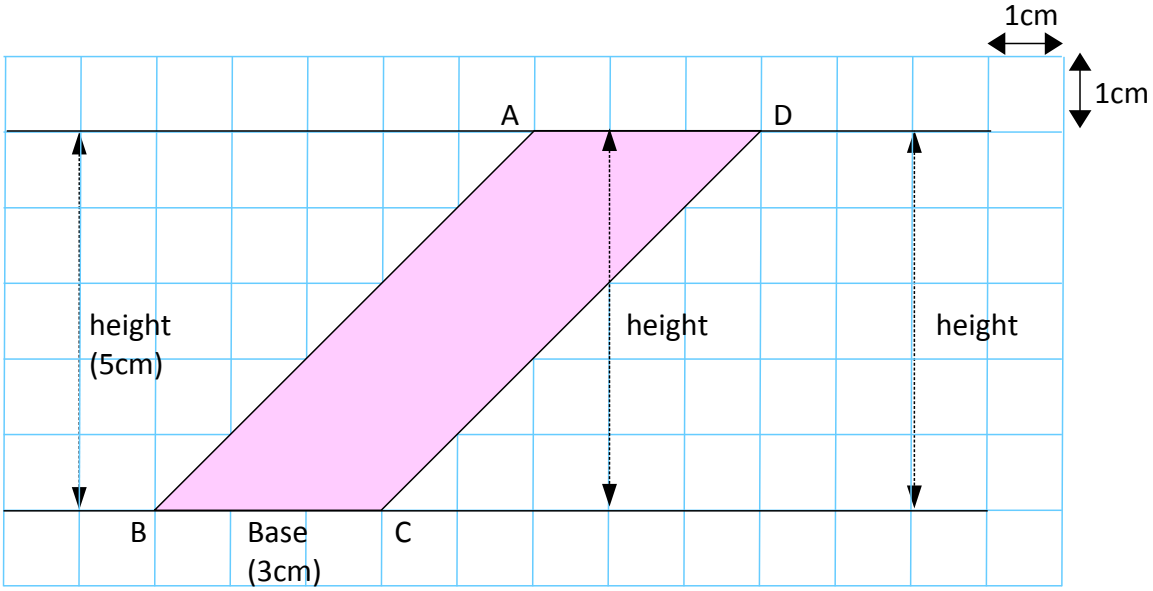
9

Look at Alice's diagram. What calculations would you expect her to use to calculate the area of parallelogram ABCD?.



Matthew

I got an even better way of deciding on the height of this type of parallelogram.



Matthew's explanation

I have extended the base BC and the opposite parallel side AD. As shown in my diagram above, the height of a parallelogram is the distance between the two parallel lines

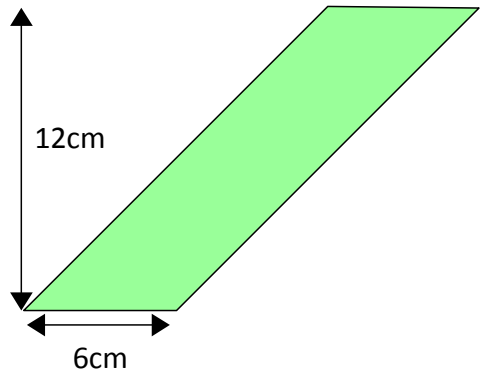
Parallelograms

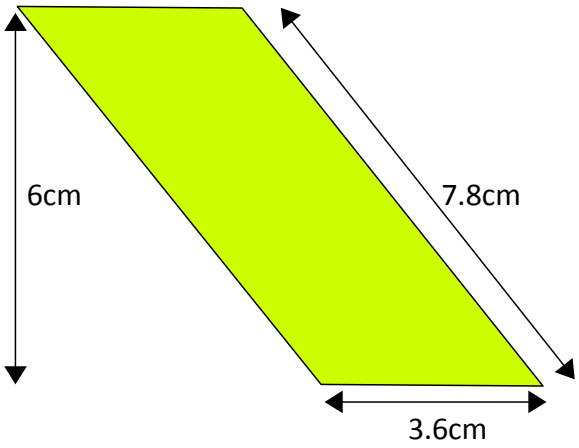
The area of parallelogram can be calculated using the formula

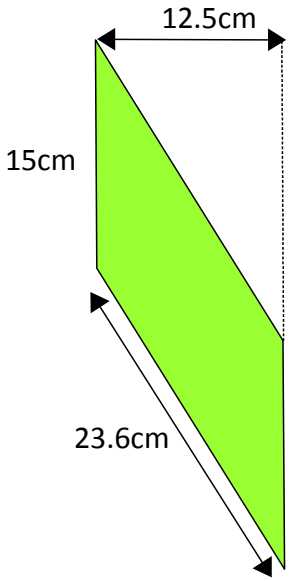
Area of a parallelogram = base x height

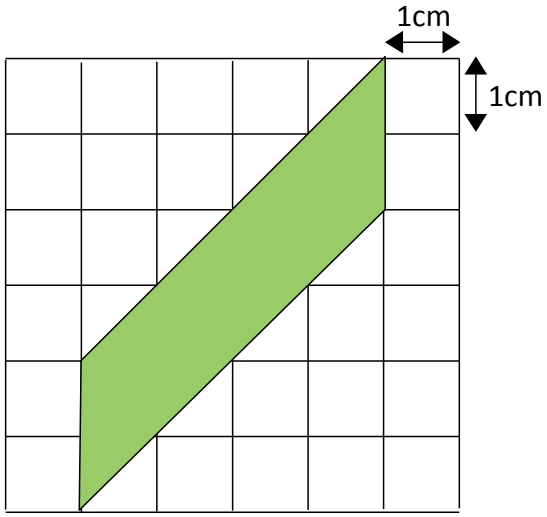
Fitness check

In each of these questions find the area of the coloured shape. Make sure you write down the calculation you are using.



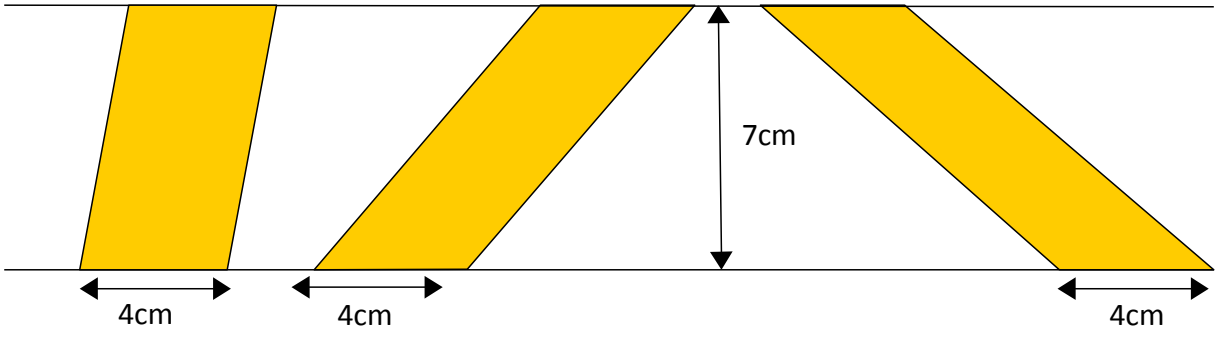






10

Can you explain why each of the parallelograms below have the same area?.



What is the area of the shape?



Alice

No matter what its shape, the area of a parallelogram can be calculated if we know the base and the height.



Evie

I wonder if we can find the area of other geometric figures we have studied?

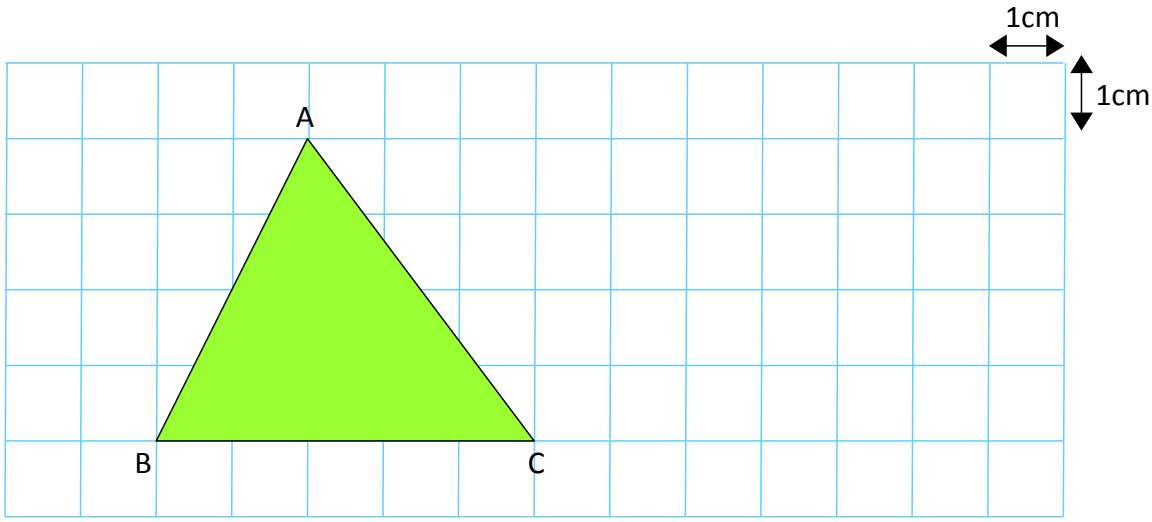


Jon

I want to try and find the area of triangles...

11

Think about how to find the area of the triangle ABC.





Alice

I wonder if we can change the shape into something that we know how to calculate the area of already?

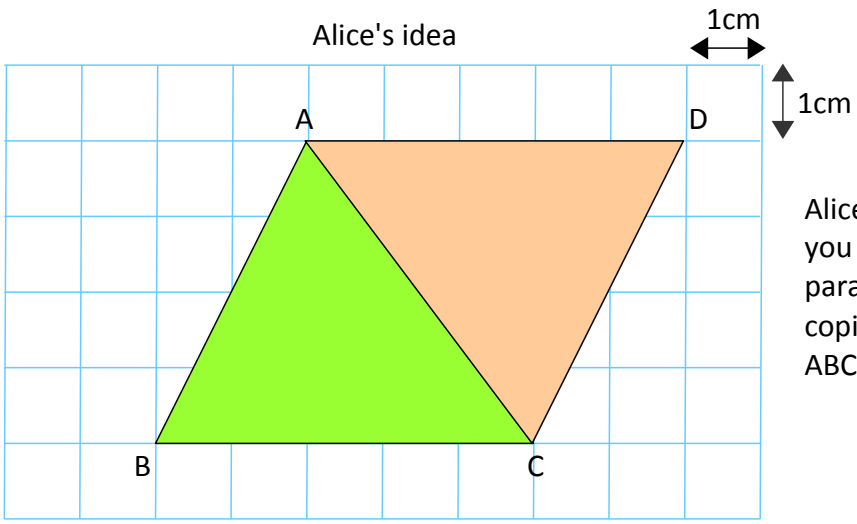
Finding the area of a triangle

Explain the three students' ideas below using diagrams and math sentences



Alice

Alice's idea

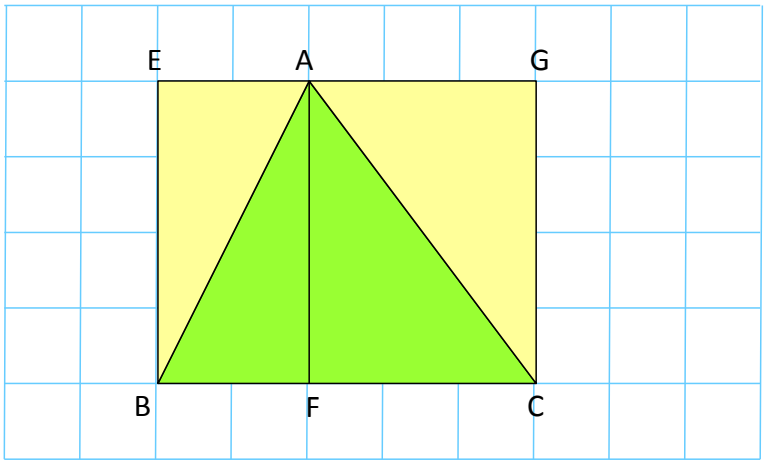


Alice used the fact that you can make a parallelogram using two copies of the triangle ABC, and...



Evie

Evie's idea

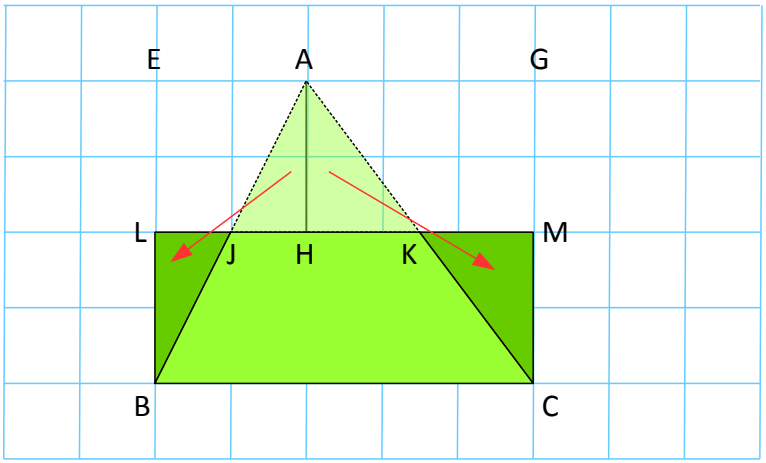


Evie made rectangle EBCG using 2 copies each of triangle ABF and triangle AFC, and...



Sarah

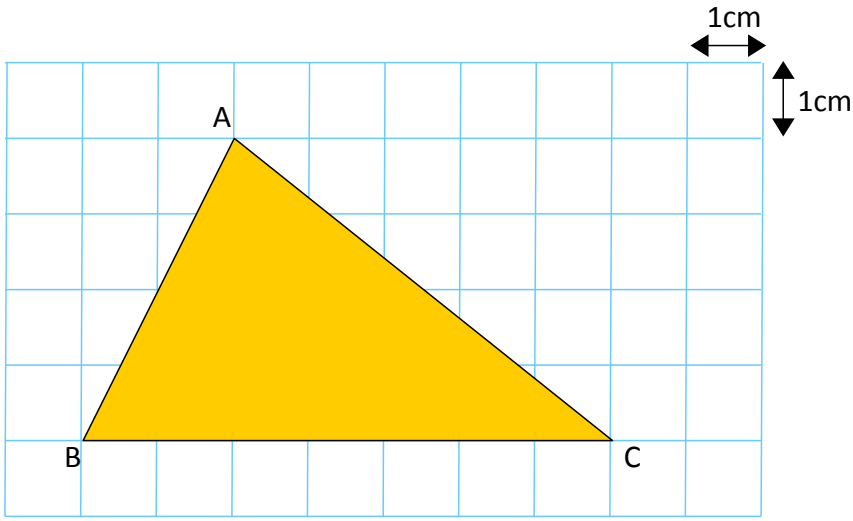
Sarah's idea



Sarah made rectangle LBCM by moving triangle AHJ and triangle AJK and...

Finding the area of a triangle

12 Based on Alice's idea on the previous page, calculate the area of the triangle ABC on the right



Let's come up with a formula for calculating the area of a triangle.

The area of the triangle above is a half the area of a parallelogram.

1. What is the base and the height of the parallelogram?
2. Which parts of the triangle correspond to the base and the height of the triangle?
3. Calculate the area of the triangle above using BC as the base



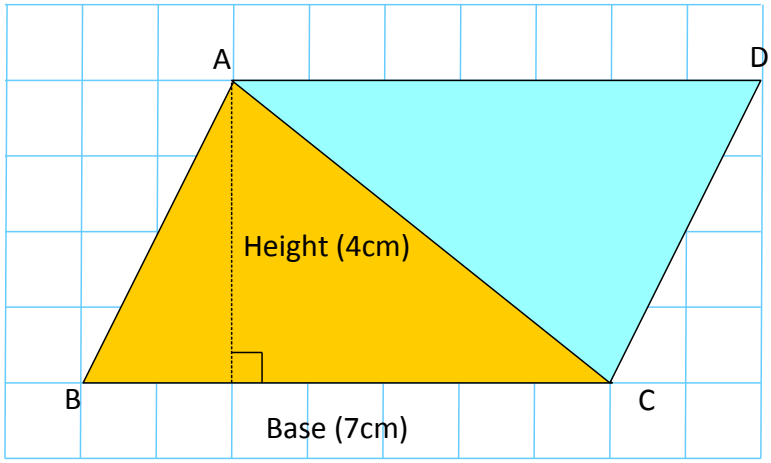
Alice

I used the fact that you can make a parallelogram using two copies of the triangle ABC.

The area of the parallelogram is 7 x 4 = 28cm^2.

The area of triangle ABC will be ½ x 28 = 14cm^2.

Using Alice's method



The area of a triangle can be calculated using the following formula:

Area of triangle = Base x Height ÷ 2



Alice

This means that the Area of a triangle = Area of a parallelogram ÷ 2, doesn't it?



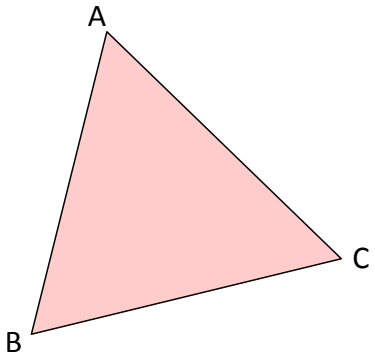
Jon

I wonder if we can make this formula using Evie or Sarah's ideas too?



Amber

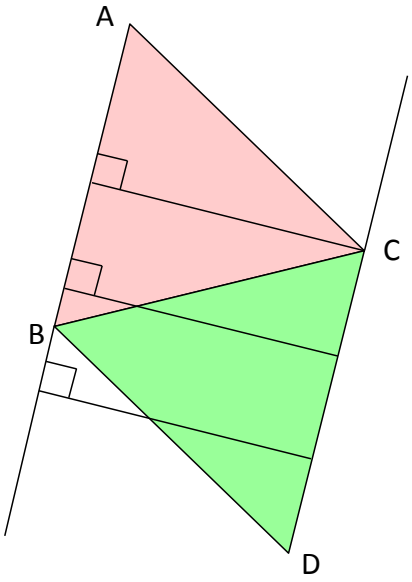
Can the base be any side of the triangle? For example, can AB be treated as the base in this triangle?





Matthew

If we think back to the area of a parallelogram formed of two identical triangles, then the height would be the distance between the parallel lines AB and CD





Evie

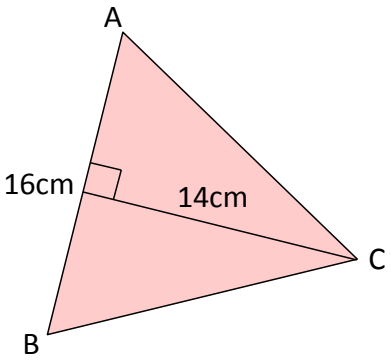
I think the most sensible one to choose would be the one that goes through C.

Evie is absolutely right. If we want to use AB as the base, then the corresponding "height" is the direct distance from C to the side AB.



Alice

So to calculate the area of this triangle, I can use the formula (Base x height) ÷ 2





Jon

So the area of triangle ABC is

$(16 \times 14) \div 2 = 224 \div 2 = 112\text{cm}^2$

Altitudes

The direct line joining a vertex of a triangle to the opposite side is called an **altitude** of the triangle.
The altitude will always be at right angles to the base.

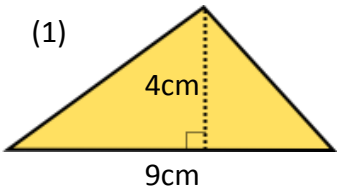
13

How many different altitudes will a triangle have?
Will they always be different lengths? Explain your thoughts.

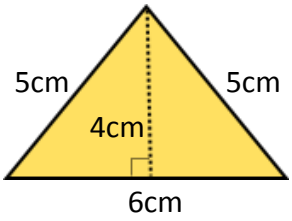
Triangles

Calculate the areas of each of the following triangles.

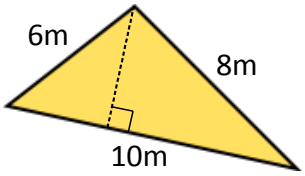
(1)



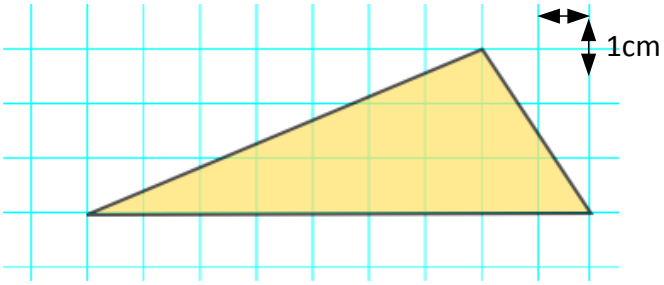
(2)



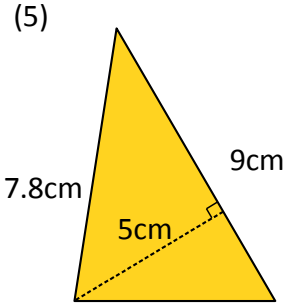
(3)



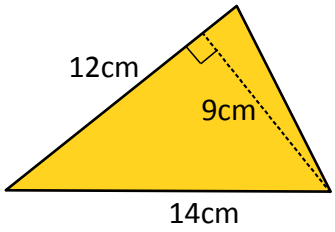
(4)



(5)



(6)



14 Think about how to find the area of triangle ABC on the right when we consider BC as the base.



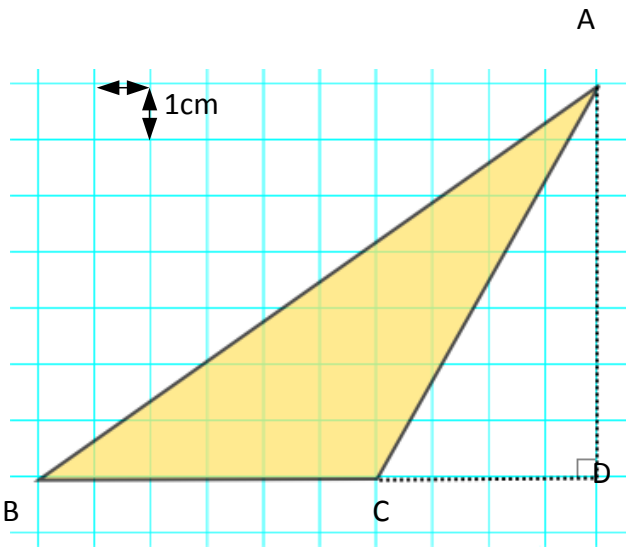
Sarah

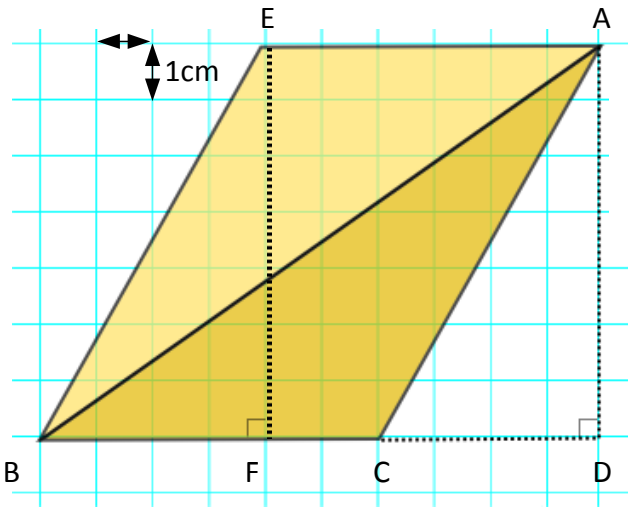
The base is ...



Jon

The height is...

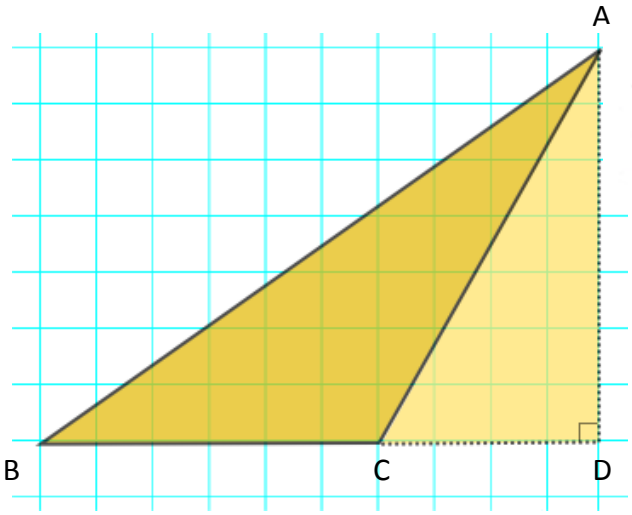






Amber

Amber used the fact that two triangles ABC can be put together to make a parallelogram EBCA, then...





Sarah

Sarah subtracted the area of the triangle ACD from the area of the triangle ABD to...

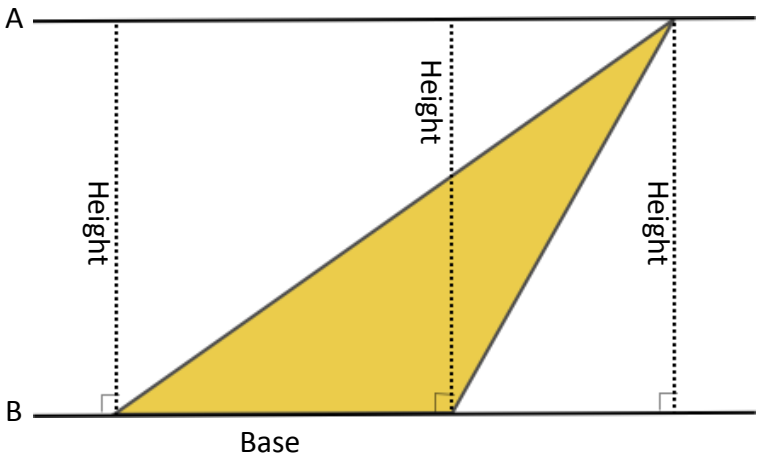
What did Sarah think about using triangles ABD and ACD?



Jon

(2) Based on these two students' ideas, find the area of the triangle ABC

As shown on the right, the height of a triangle can be found outside of the triangle.



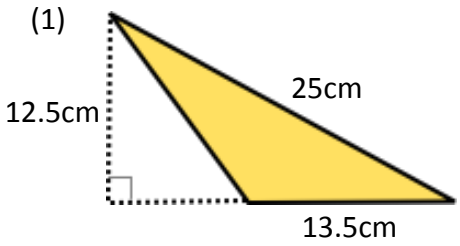
3) What is the height of the triangle if we consider BC as the base?

4) Check to see if we can use the formula for the area of a triangle that we found earlier by substituting the length of BC as the base and the height found above.

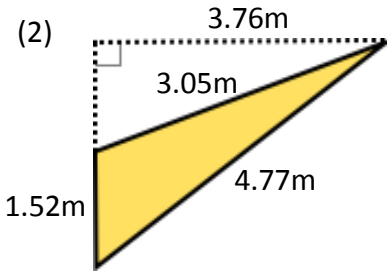
Triangles

Calculate the area of each of the following triangles

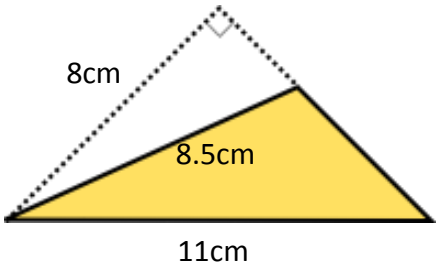
(1)



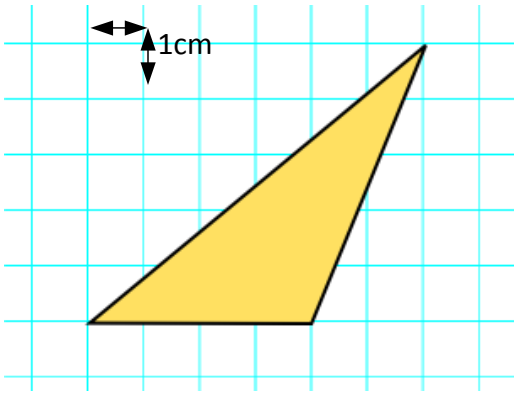
(2)



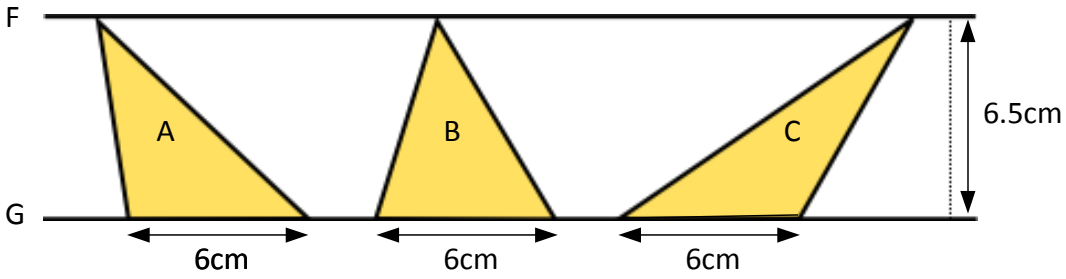
(3)



(4)



15 Find the areas of triangles A, B, C below.



Lines F and G are parallel

Explain why the areas of triangles A, B and C are equal

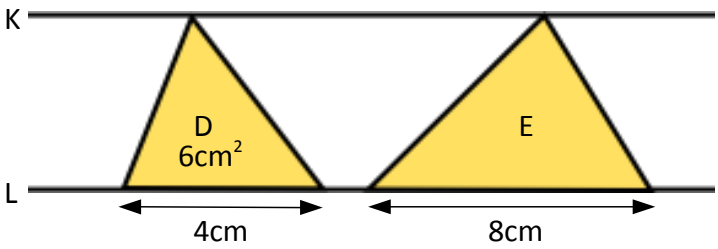
Even if triangles are shaped differently, if the lengths of their bases and heights are equal, their areas will also be equal

4) What is the area of the triangle E on the right?



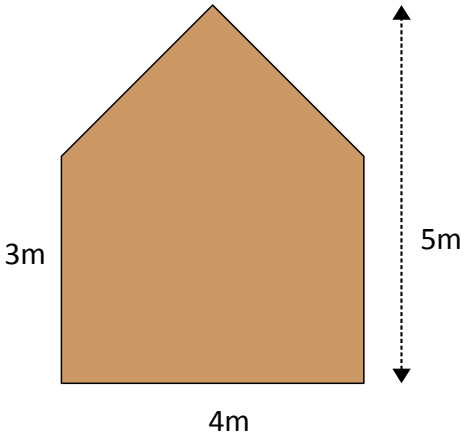
Jon

The area of triangle D is $6cm^2$
Lines K and L are parallel



Compound shapes

16 How could you calculate the area of this shape?

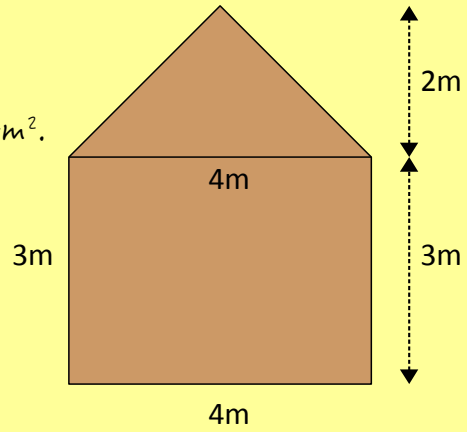


Evie's working



Evie

I split the shape into a rectangle and a triangle.
The area of the rectangle is $3 \times 4 = 12m^2$.
The area of the triangle is $4 \times 2 \div 2 = 8 \div 2 = 4m^2$.
So the total area is $12 + 4 = 16m^2$.

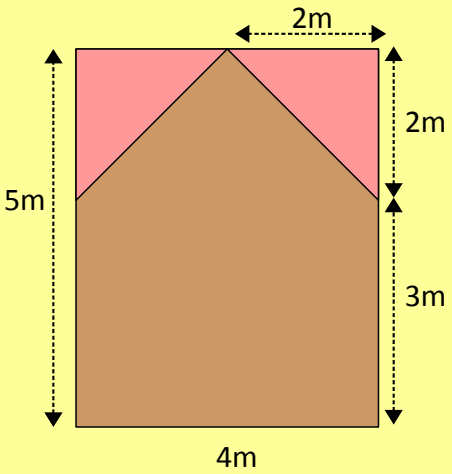


Jon's working



Jon

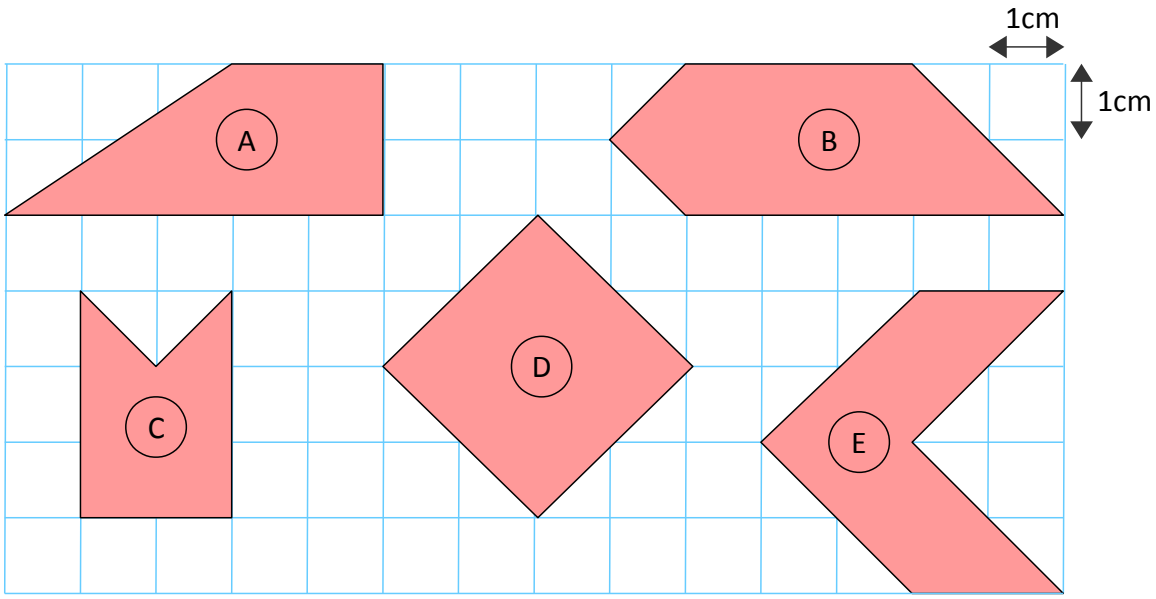
I drew a rectangle round the shape.
The area of the rectangle is $4 \times 5 = 20m^2$.
I then cut off the two triangles on the top corners.
The area of each triangle is $2 \times 2 \div 2 = 2m^2$.
So the total area is $20 - 2 \times 2 = 20 - 4 = 16m^2$.



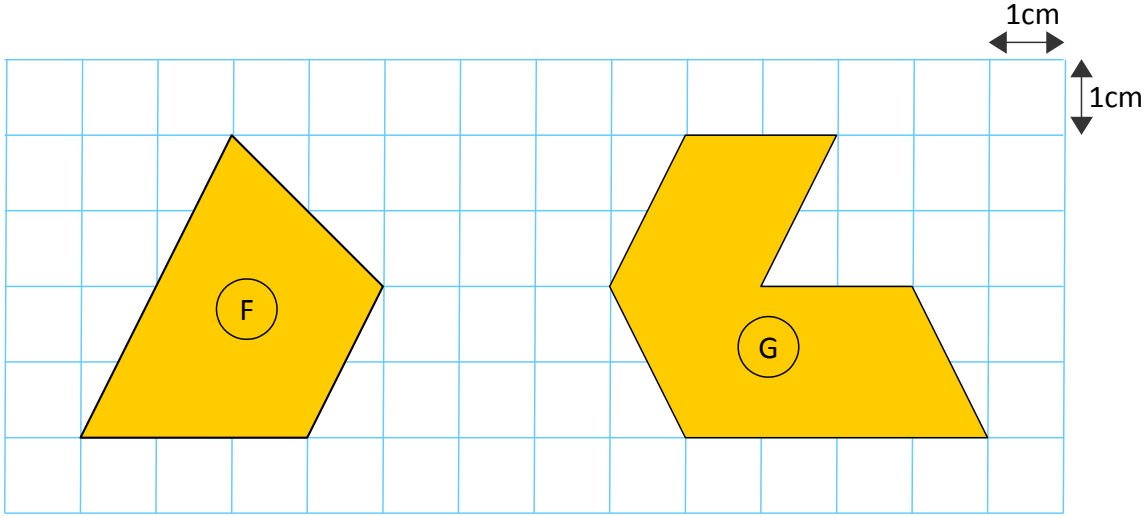
Compound shapes

Fitness check

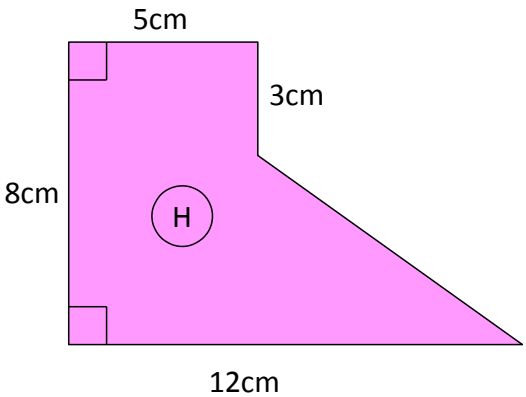
1. Calculate the areas of these shapes. Make sure you clearly show your calculations.

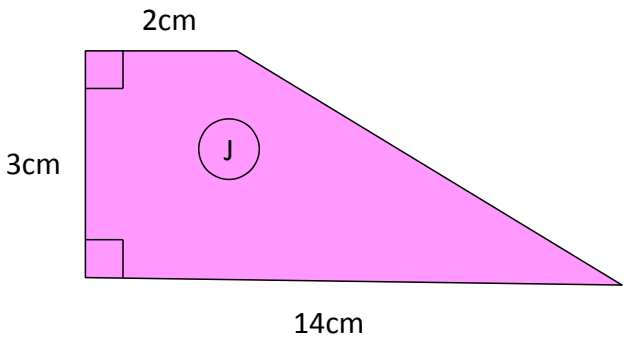


2. Calculate the areas of these shapes. Make sure you clearly show your calculations.

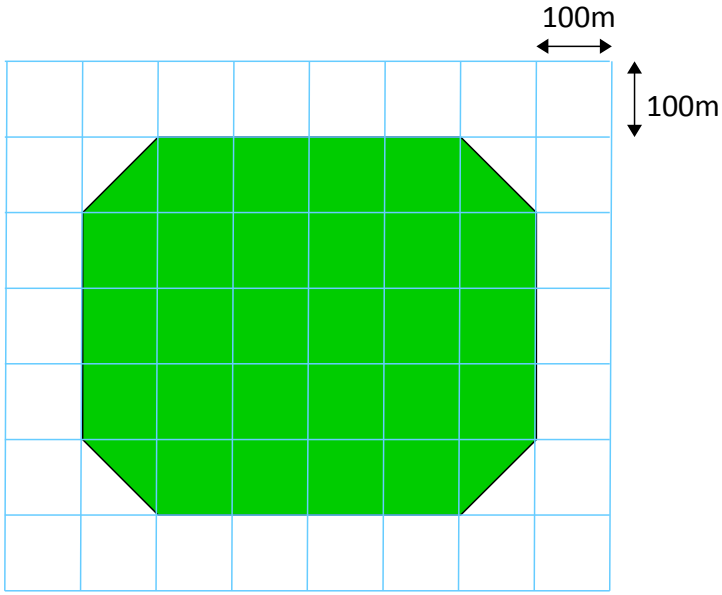


3. Calculate the areas of these shapes. Make sure you clearly show your calculations.





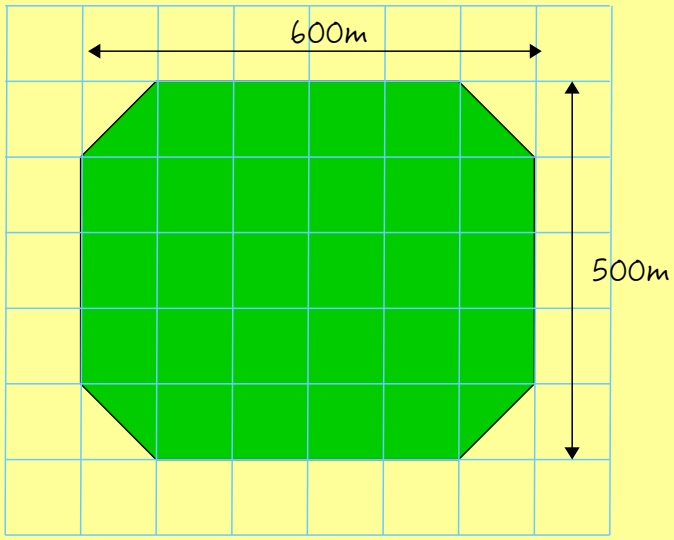
17 Calculate the area of this field




Matthew

Matthew's working

The area of one cut off corner
= 100 x 100 ÷ 2 = 5000m²
The area of 4 cut off corners
= 4 x5000 = 20 000m²
The area of the rectangle surrounding the field
= 600 x 500 = 300 000m²
So the area of the field
= 300 000 – 20 000
= 280 000m².



A new unit of measure

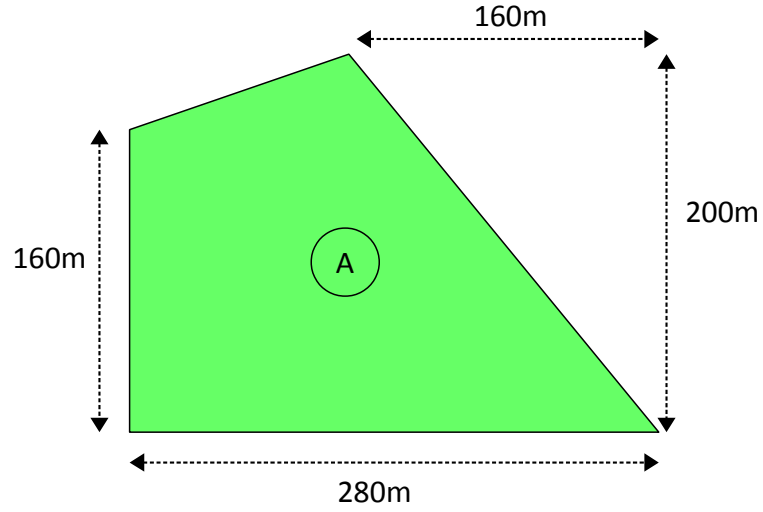
For large areas we often measure the area in **hectares** where

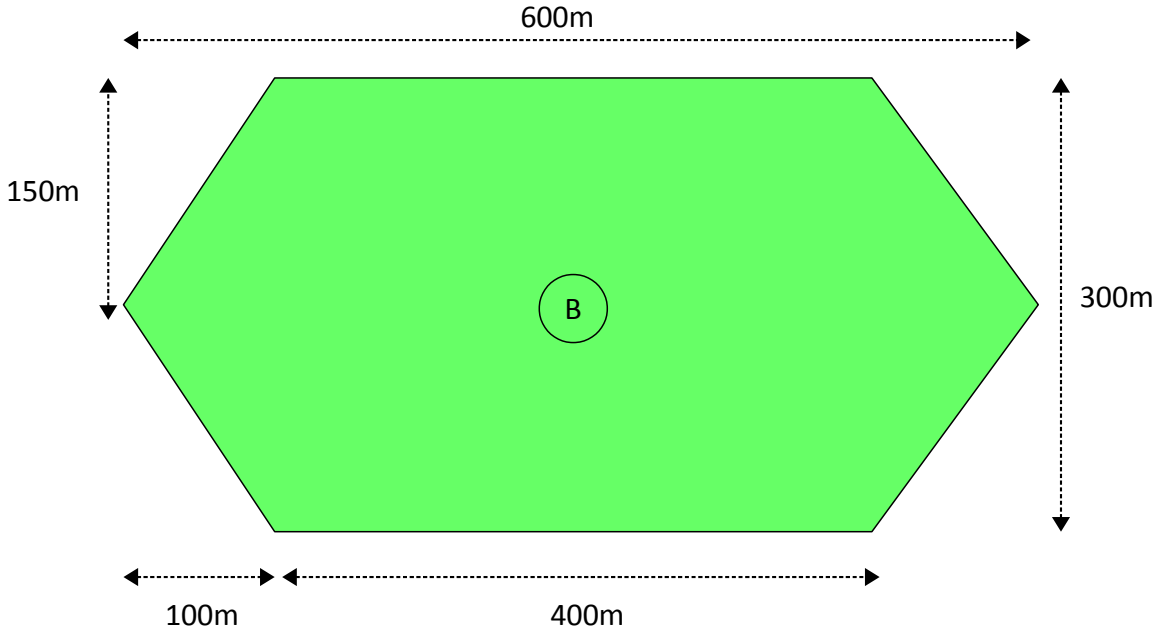
1 hectare is written as **1ha** and represents an area of 10 000m².

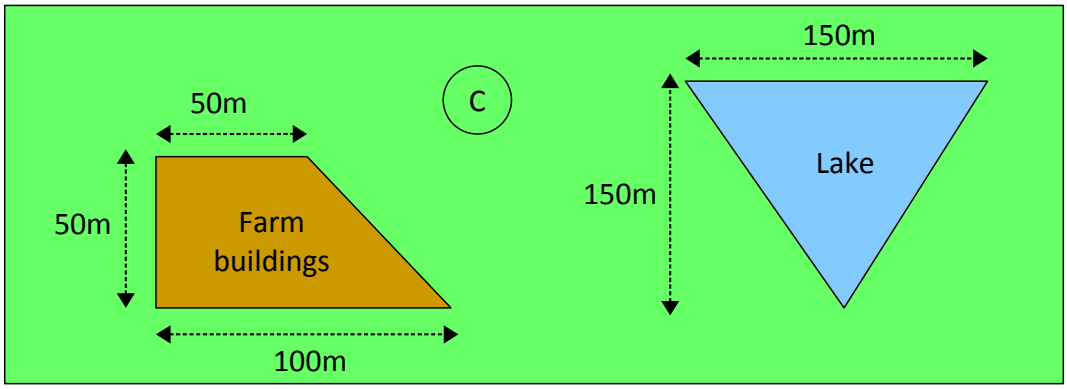
18 What is the area of the field in hectares?

Fitness check

Find the areas of each of these fields a) in m^2 b) in hectares

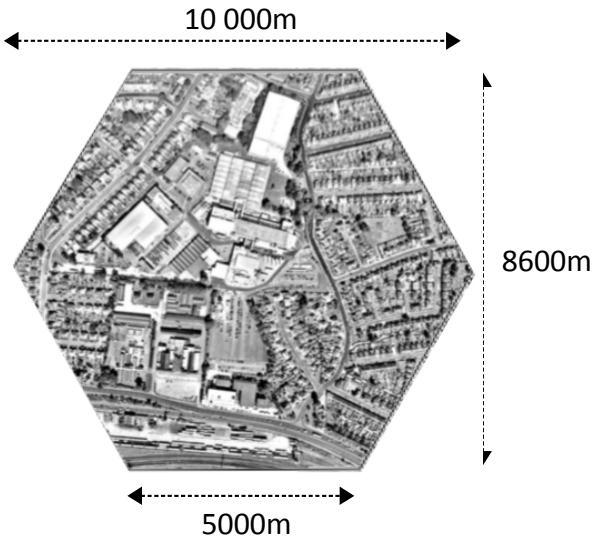






19

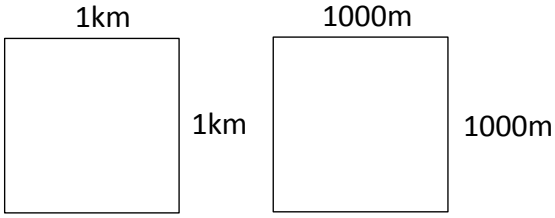
Evie's town is roughly shaped like a regular hexagon on her map. Think about how to express the area of the town.




Jon

To express a large area like the area of a town or county, we use a square with 1km sides as the unit

The area of a square with 1km sides is 1 square kilometre, and is expressed as $1km^2$

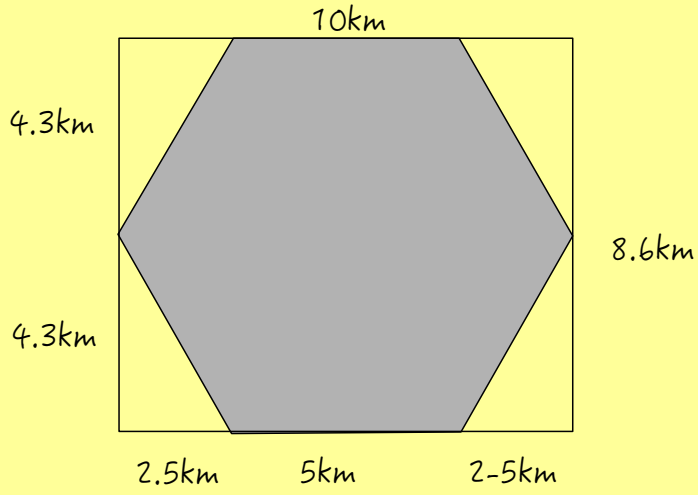


1. What is the approximate area of Evie's town in km^2?
2. How may m^2 is $1km^2$?
3. How many hectares (ha) in $1km^2$?


Evie

Evie's working

Area of the rectangle
$= 10 \times 8.6 = 86km^2$
Area of corner triangles
$= 4 \times 2.5 \times 4.3 \div 2$
$= 21.5km^2$
Area of town $= 86 - 21.5$
$= 64.5km^2$




Sarah

Sarah's working

1km = 1000m
$1km^2 = 1000 \times 1000 = 1\ 000\ 000m^2$
1 hectare $= 10000m^2$
$1km^2 = 1\ 000\ 000 \div 10\ 000 = 100$hectares

Units of area

Changing units

Convert

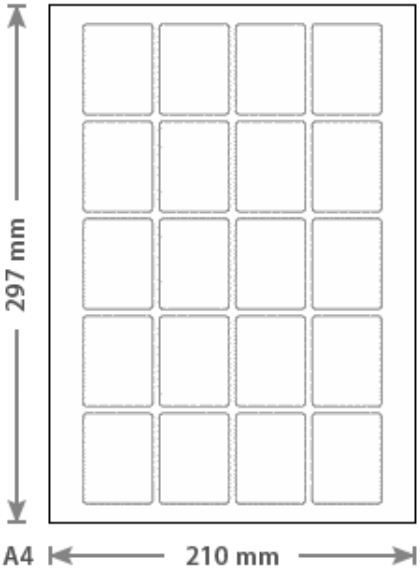
1. 8000cm² into m²
2. 6km² into m²
3. 6km² into hectares
4. 25 hectares into m²
5. 25 hectares into km²
6. 250 000m² into hectares

Measuring small areas

Labels come in all shapes and sizes. They can be bought in A4 sheets for easy printing. This sheet has 20 labels each measuring 40mm by 52mm.

What is the area covered by one label?

What do you think are the most appropriate units to measure this area?





Jon

To express a small area like the area of a label we need a smaller unit. Generally use a square with 1mm sides as the unit.

The area of a square with 1mm sides is 1 square millimetre and is expressed as 1mm²

1. What is the area of a label in mm²?
2. What is the area of a label in cm²?
3. How many mm² in 1cm²?



Amber

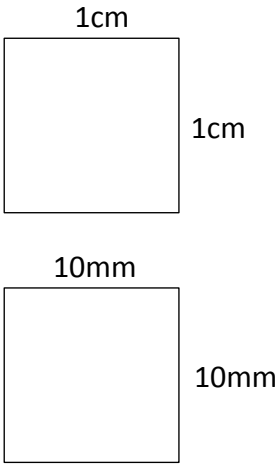
Amber's working

1. Area of a label = 40 x 52 = 2080mm²

2. Working in cm, area = 4.0 x 5.2 = 20.8cm²

3. So 1cm² = 10 x 10 =100mm²

So 1cm² is the same as 100mm².



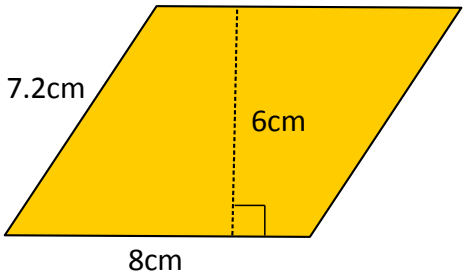
Changing units

Convert

1. 8000mm² into cm²
2. 20cm² into mm²
3. 2m² into mm²
4. 60 000 000mm² into cm²
5. 60 000 000mm² into m²
6. 400cm² into mm²

Geometry 2c: Mastery questions (area)

1. What is the perimeter of this parallelogram?

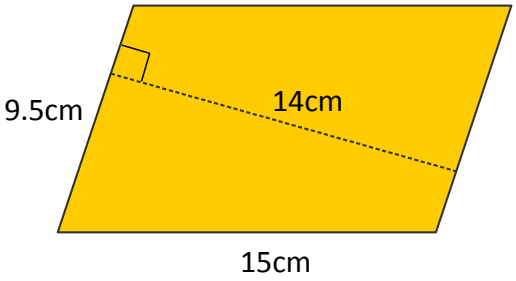


A	B	C	D
21.2cm	42.4cm	30.4cm	48cm

2. What is the area of the parallelogram in Q1?

A	B	C	D
21.2cm^2	42.4cm^2	30.4cm^2	48cm^2

3. What is the perimeter of this parallelogram?

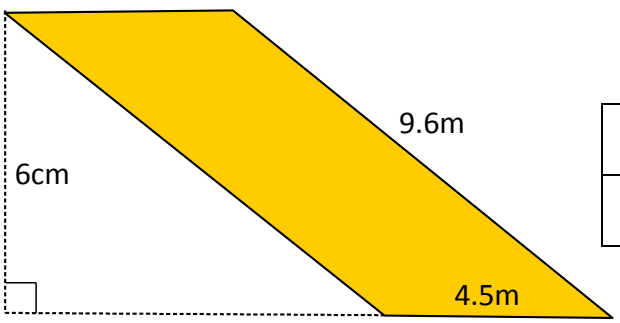


A	B	C	D
133cm	49cm	210cm	38.5cm

4. What is the area of the parallelogram in Q3?

A	B	C	D
133cm^2	49cm^2	210cm^2	38.5cm^2

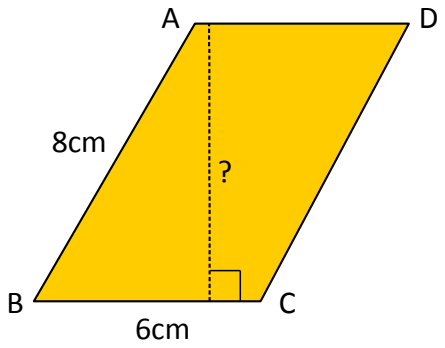
5. What is the area of this parallelogram?



A	B	C	D
27m^2	43.2m^2	57.6m^2	28.2m^2

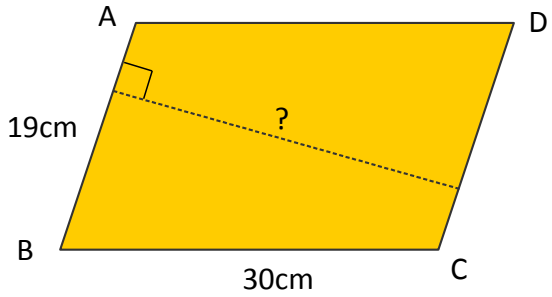
Geometry 2c: Mastery questions (area)

6. What is the 'height' of this parallelogram if BC is taken as the base and the area is 42cm^2?



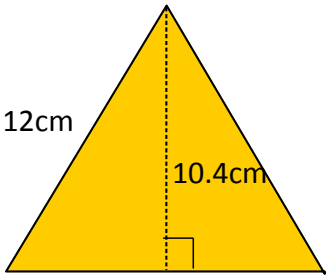
A	B	C	D
5.25cm	6.5cm	7cm	8cm

7. What is the height of this parallelogram if AB is taken as the base and the area is 532cm^2?



A	B	C	D
10.8cm	17.7cm	28cm	48cm

8. What is the perimeter of this equilateral triangle?

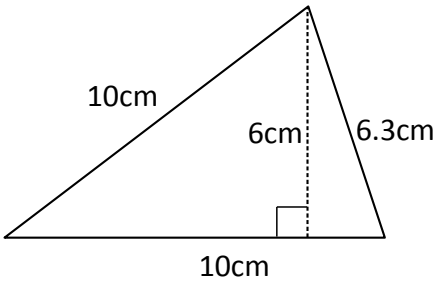


A	B	C	D
62.4cm	28.4cm	124.8cm	36cm

9. What is the area of the triangle in Q8?

A	B	C	D
62.4cm^2	28.4cm^2	124.8cm^2	36cm^2

10. What is the area of this triangle?

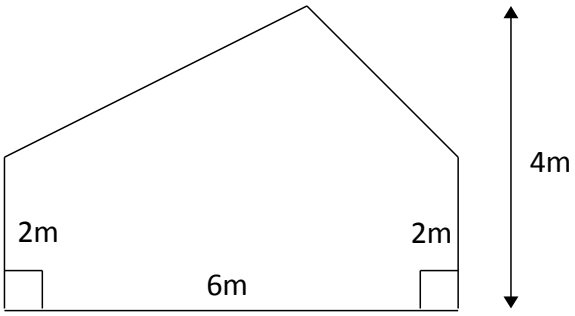


A	B	C	D
60cm^2	30cm^2	26.3cm^2	37.8cm^2

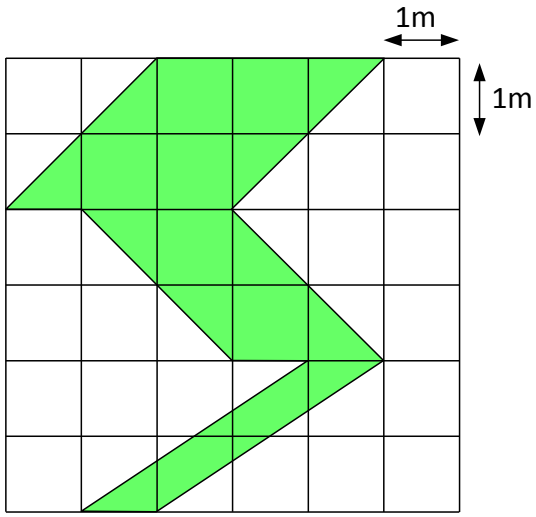
Geometry 2c: Mastery questions (area)

In question 11 – 13, you must show all of your working.

11. Calculate the area of this shape.



12. Calculate the area of the green shape

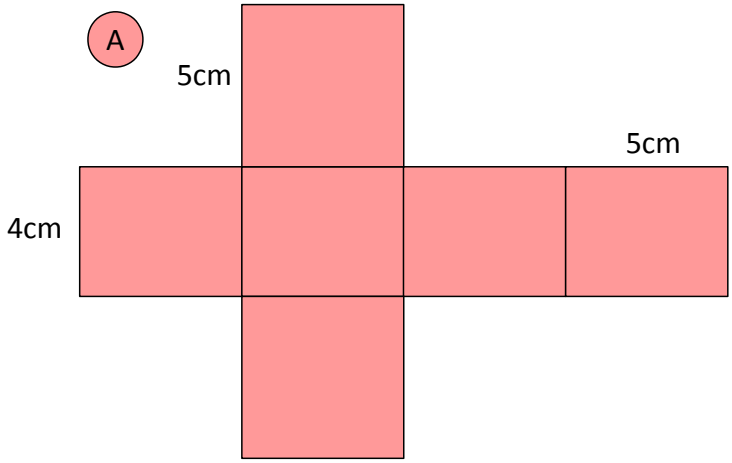


13. A rectangle has the same area as a triangle whose area is 22.5cm^2. What might the dimensions of the rectangle and the triangle be?

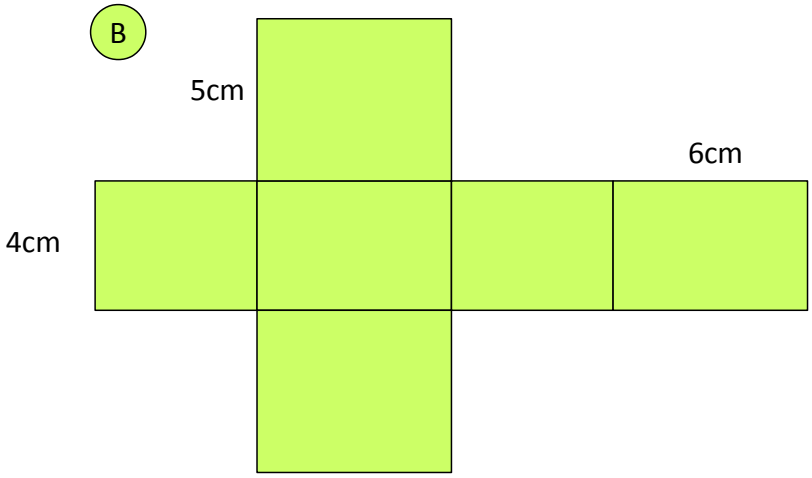
Let's make some cubes and cuboids

Draw the nets of these cubes and cuboids on card. Then, cut them out and make the cubes and cuboids.

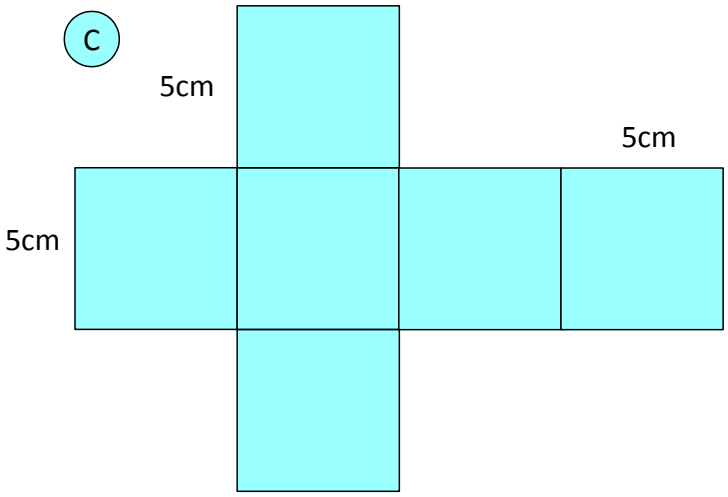
A



B



C



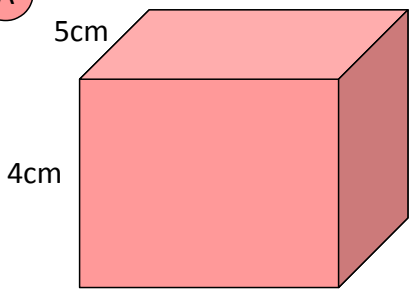
Let's think about the amount of space taken up by cubes and cuboids



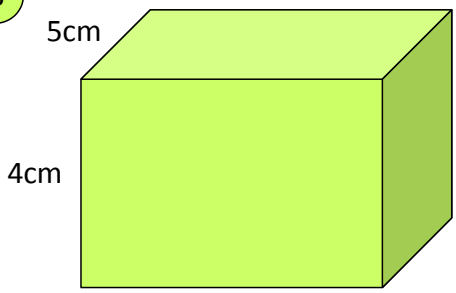
Alice

Which one takes up the most space?

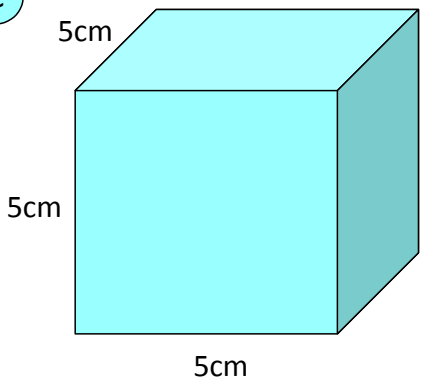
A



B



C





Jon

If you compare A and B, then ...



Sarah

If you compare A and C then ...



Matthew

But how do you compare B and C?

20

Which one takes up the most space, B or C and by how much.
Let's think about how to compare them.



Amber

When we measuring with water we use litres (l) or millilitres (ml).

When we thought about area we tried to fill it with 1cm squares. Maybe we can ...



Evie

Finding the volume of a cube



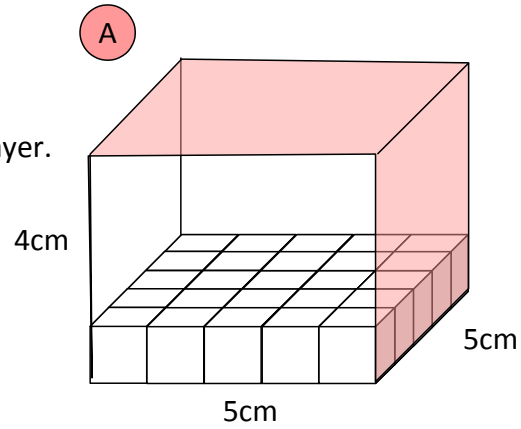
Taro

What if we worked out how many 1cm^3 cubes would fit in each box?



Sarah

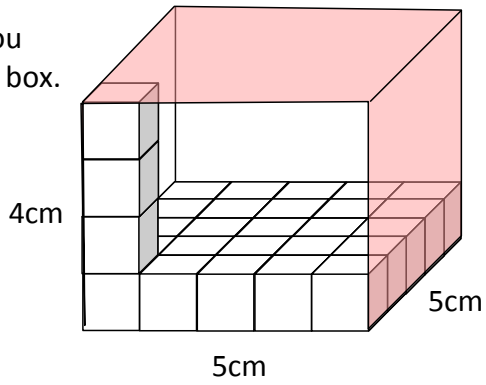
You can get 25 on the bottom layer.





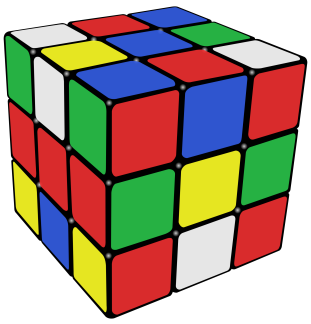
Jon

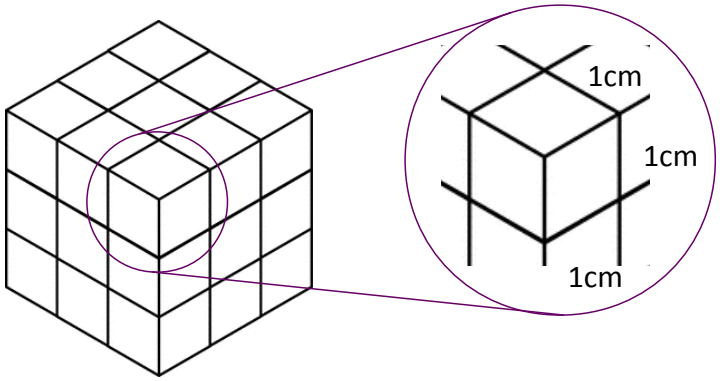
And you can make 4 layers, so you can get 4 x 25 = 100 cubes in the box.



We express the amount of space a cube or a cuboid takes up by the number of cubes of 1 cm edges that fit inside them. We call this the volume of the cuboid and it is measured in cm^3.

For example, a Rubic cube is made up of 27 identical cubes and so would have a volume of 27cm^3 if the cubes all had 1cm sides.







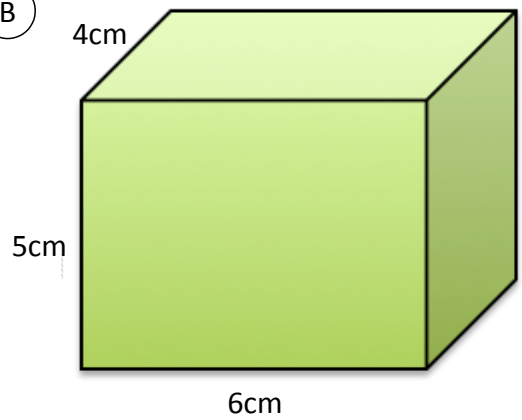
Evie

And so the volume of box A is 100cm^3.

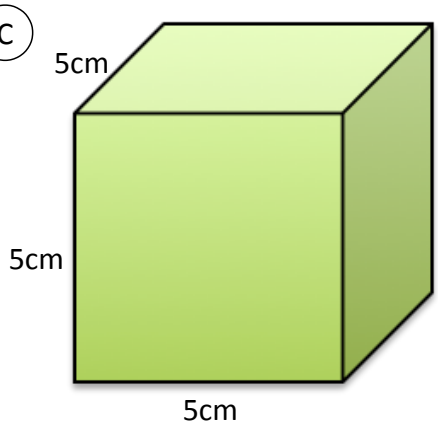
21 Using the same idea, work out the volume of boxes B and C and then put the boxes in order of size by volume.

Finding the volume of a cube

B



C



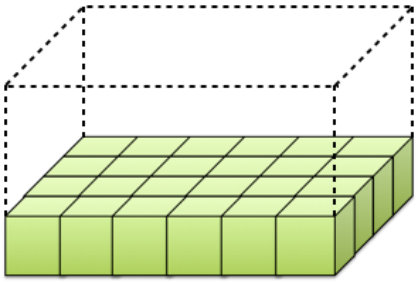


Jon

Let's think about how to calculate volume...

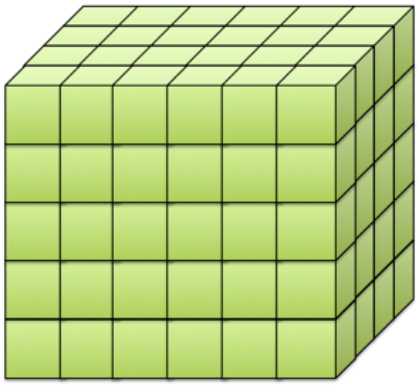
Let's investigate how many 1cm^3 cubes will fit in B.

1. Start by thinking about how many 1cm^3 cubes will fit in the bottom layer.



☐ x ☐ = ☐ (pieces)

2. How many layers will fit inside?



☐ (layers)

3. Calculate the number of 1cm^3 cubes

The number of 1cm^3 that will fit in B is 120^3 because... **4 x 6 x 5 = 120** So the volume of B is 120cm^3

Now calculate the volume of C.

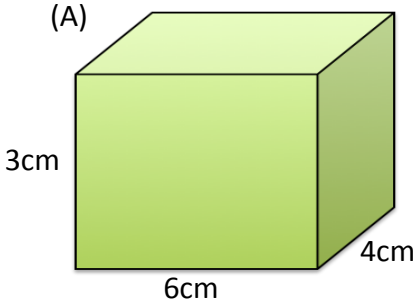
Cubes and Cuboids

Which one takes up the most space?

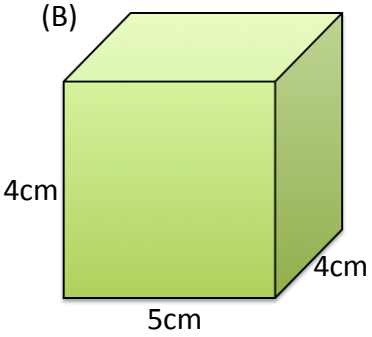
22

Using the ideas from the previous pages, put these 5 cubes in size order by volume starting with the smallest.

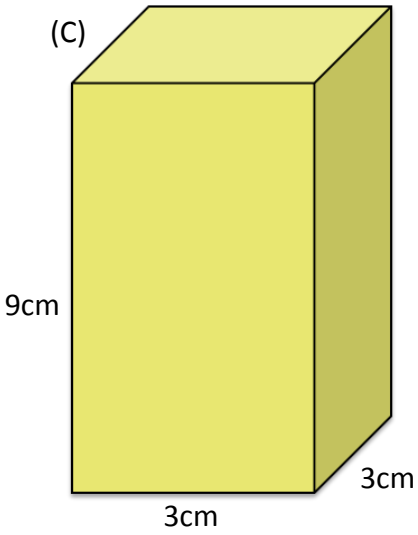
(A)



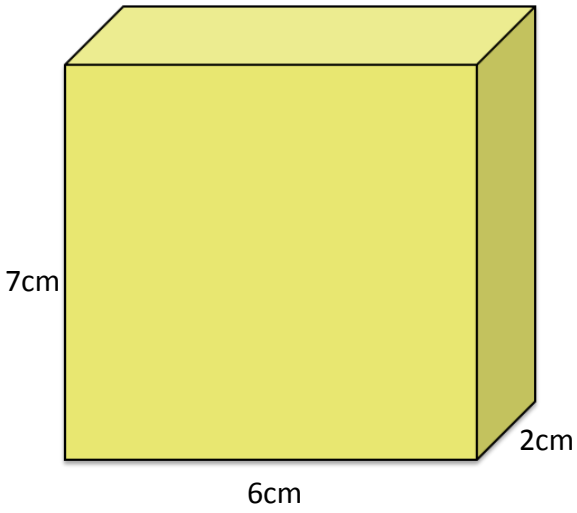
(B)



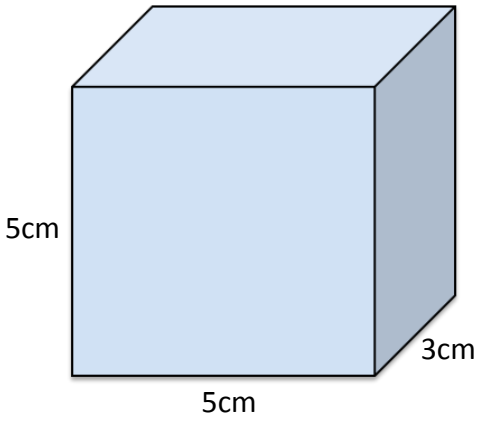
(C)



(D)



(E)



Cuboids

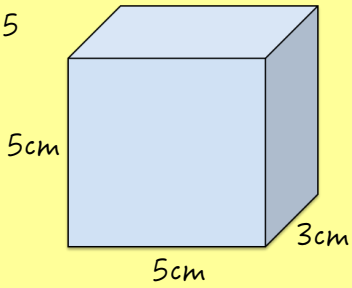


Alice

I've thought of an easy way of calculating the volume of a cube or a cuboid.

Alice's idea

Calculate the area of the base - that's 5 x 3 = 15cm^2 (which is also how many cubes will fit on the base) and then multiply by the height (which is the number of layers).
So the volume is 15 x 5 = 75cm^3.





Amber

I've made a table of my results

Amber's table

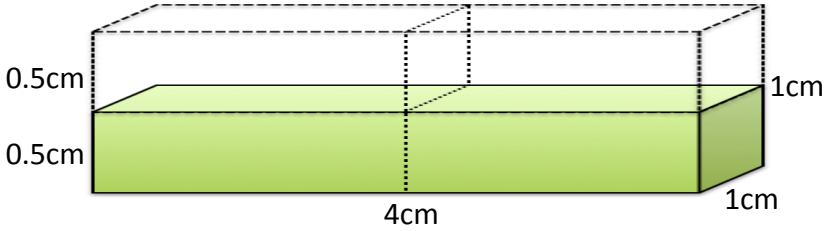
Shape	Base	Base area	Height	Volume
A	4 x 6	24cm^2	3	72cm^3
B	5 x 4	20cm^2	4	80cm^3
C	3 x 3	9cm^2	9	81cm^3
D	6 x 2	12cm^2	7	84cm^3
E	5 x 3	15cm^2	5	75cm^3

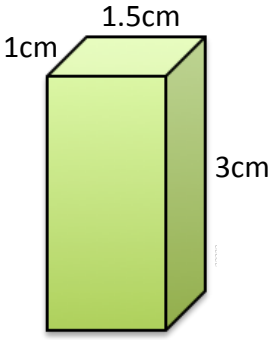


Taro

And so the smallest is A, followed by E, then B, then C and finally D is the biggest by volume.

23 Using Alice's idea calculate the volume of these two cuboids







Matthew

Matthew's notes

The volume of cubes and cuboids can be calculated using the following steps..

Measure the length, width and height
Multiply the three numbers that express the lengths of the three edges

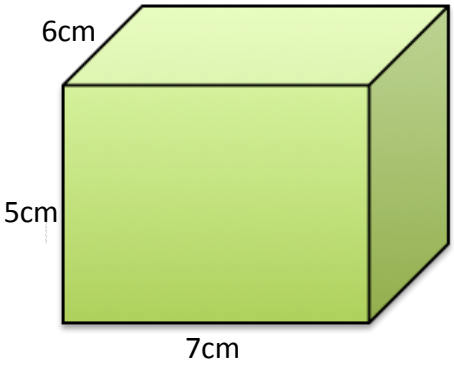
The volume of cubes can be calculated using the following formulas...

Volume of cuboids = length x width x height
Volume of cubes = edge x edge x edge

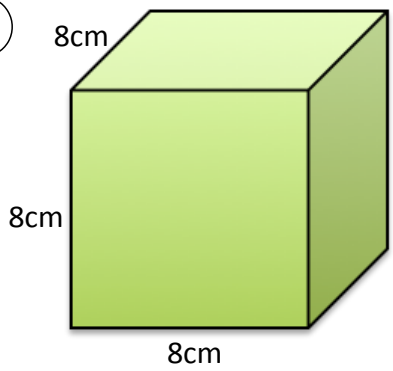
24 Can you explain why Matthew's and Alice's ideas always give the same answer?

Use Matthew's method to find the volumes of these shapes.

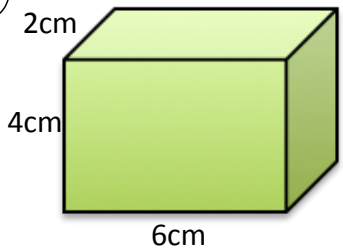
A



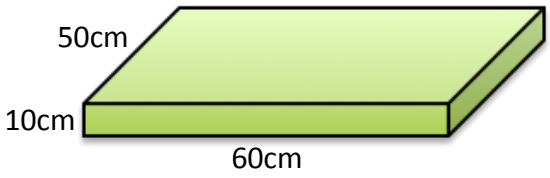
B



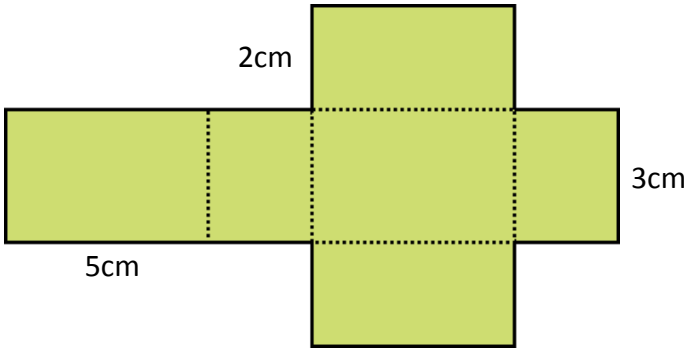
C

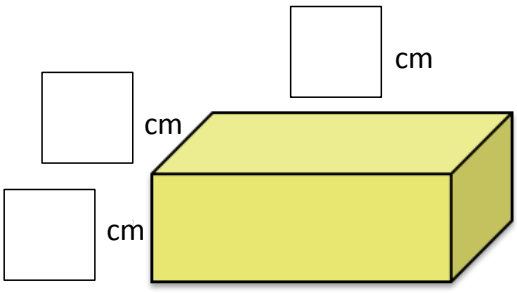


D



The picture below is a net of a cuboid. Find the volume of this cuboid.

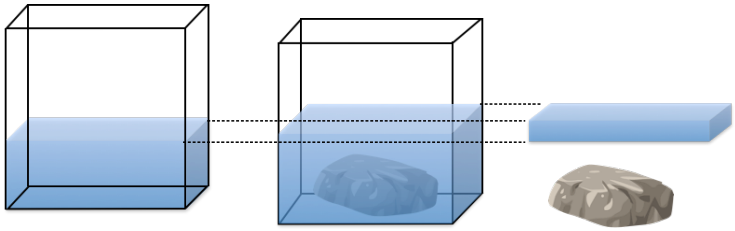




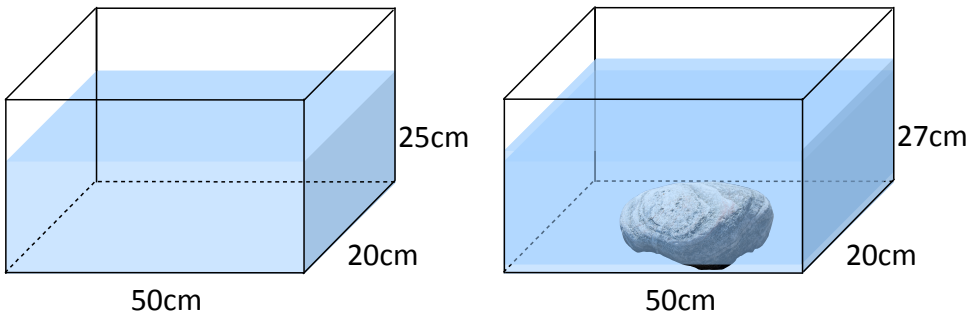
Maths Story: How to find the volume of a rock...

We can determine the volume of irregular shapes like rocks and eggs using the following method.

Submerge the rock in a tank of water. When the rock is submerged, the water level goes up by the volume of the rock. Therefore, if we calculate the volume of the water that increased, we know the volume of the rock.



1. For example, suppose you have a fish tank as shown below, filled to a depth of 25cm.



   After placing the rock in the tank, the depth of the water rose to 27cm. What is the volume of the rock?

2. What difference would it make in the previous question if the depth of water rose by 4cm instead?

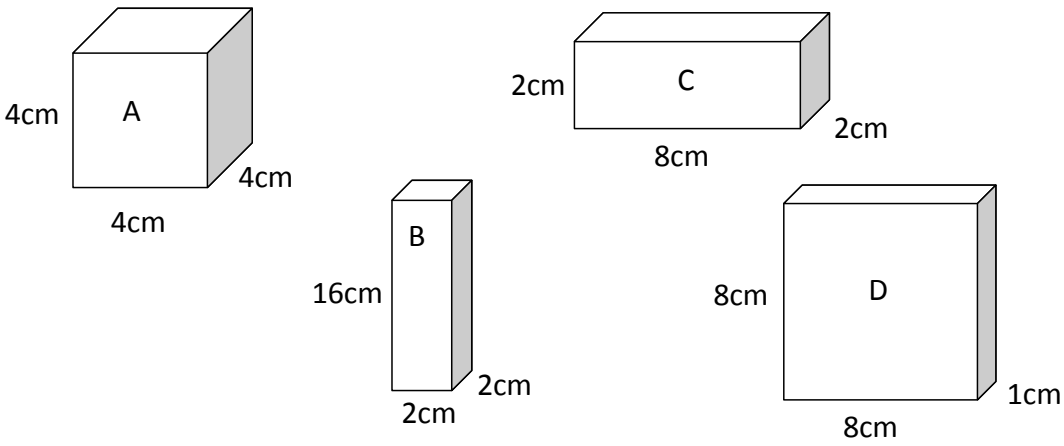
3. If the total height of the tank is 40cm, how much water would the tank hold in litres?

4. Going back to Q1, how many similar shaped rocks could be put in the tank before it overflowed?

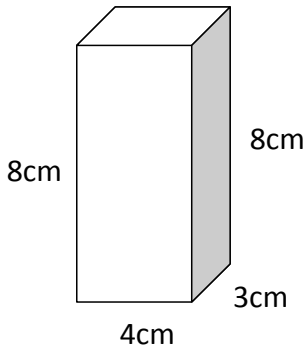
5. Estimate the number of litres it would take to fill a rectangular swimming pool if the length is 50m, the width 24m and the depth of water is 2m.

Geometry 2c: Mastery questions (volume)

1. Which of the following cubes and cuboids does not have the same volume as the other three?

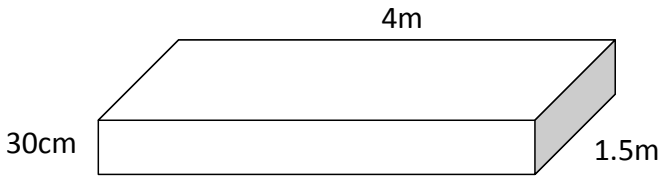


2. The dimensions of a cuboid are given on the diagram below. What is the volume of the cuboid?



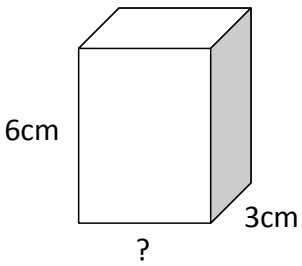
A	B	C	D
$32cm^3$	$96cm^3$	$256cm^3$	$768cm^3$

3. A sandpit has the dimensions as shown. What is the volume of the sandpit?



A	B	C	D
$180cm^3$	$180m^3$	$18m^3$	$1.8m^3$

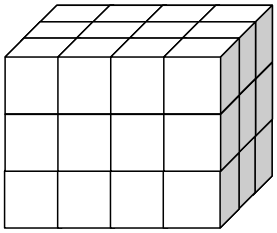
4. A small box has a volume of $72cm^3$. What is the value of the missing length?



A	B	C	D
4cm	12cm	13cm	18cm

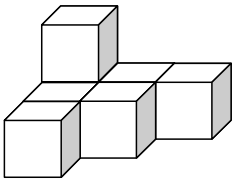
Geometry 2c: Mastery questions (volume)

5. How many cubes are required to make this solid below?



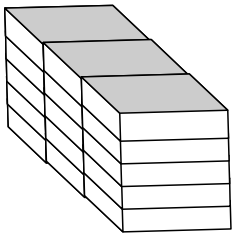
A	B	C	D
33	36	66	72

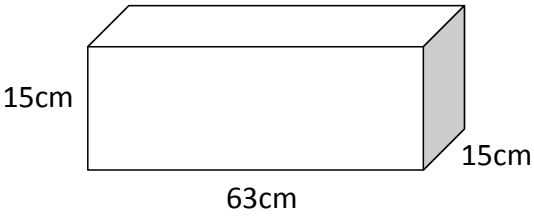
6. How many more cubes would be needed to make this a cube of side length 4?



A	B	C	D
20	64	9	57

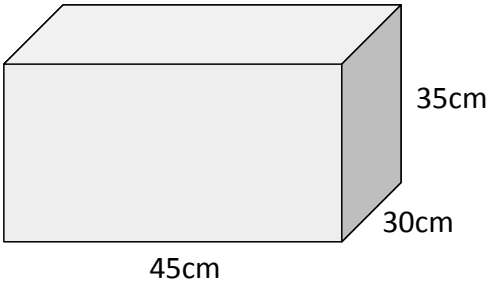
7. 15 identical books are tightly packed into a large box for transportation.
The dimension of the box are shown in the diagram. What is the volume of one book?





A	B	C	D
14175cm^3	2835cm^3	945cm^3	225cm^3

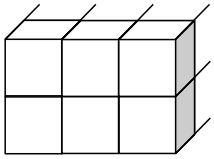
8. The dimensions of a fish tank are shown below. What is the capacity of the tank in litres?



A	B	C	D
47.25L	472.5L	4725L	47250L

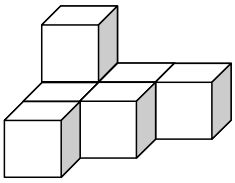
Geometry 2c: Mastery questions (volume)

9. A rectangular box is made out of cubes. The end of the box looks like this. Which one of these answers could not be the volume of the box?



A	B	C	D
24	39	54	72

10. How many more cubes would be needed to make this a cube of side length 4?



A	B	C	D
20	64	9	57

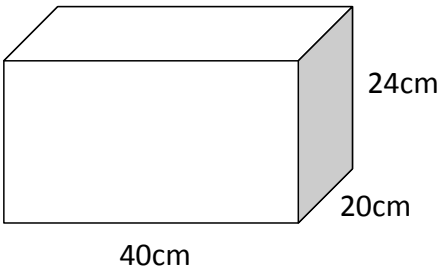
In Q11 – 12, you must show all of your working.

11. The tank shown below is filled to two thirds of its capacity.

    How deep is the water in the tank?

    A large stone is then put into the tank and the level of water goes up by 2cm.

    What is the volume of the stone?



12. Small packets of raisins are sold in rectangular boxes as shown.

    They are then tightly packed in larger rectangular shipping boxes each containing 16 packets.

    Work out the dimensions of three possible shipping boxes.

