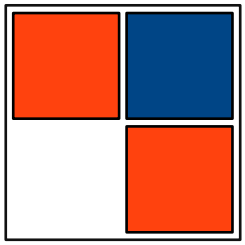
How many moves will it take?

Sliding puzzles have been a favourite pastime of children for a long time. The aim of the game is to slide the blue square from its starting position, in the top right hand corner, to the bottom left hand corner in the least number of moves. You may only slide squares up, down, left or right and not diagonally.

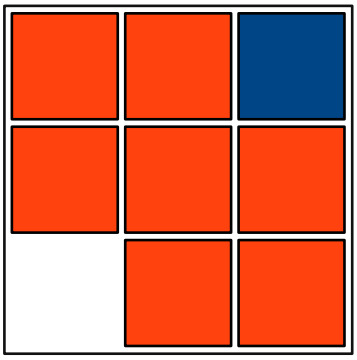


1 How many moves will it take to solve this puzzle?



Evie

That was easy. I wonder how many moves it will take if we start with a 3 x 3 board?



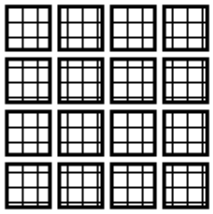
Translation

Moving a shape in a fixed direction and by a fixed direction is called a **translation** or slide. In the sliding puzzle games the direction is always one unit horizontally or vertically. The actual shape, in this case a square, remains unchanged in a translation.

Designing a city

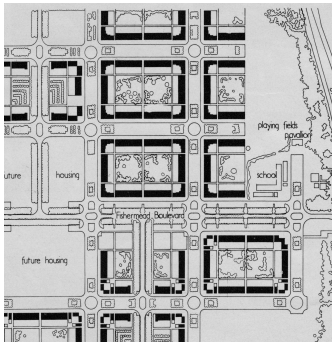
A lot of cities are laid out in a very simple grid pattern with streets almost solely running east-west and north-south. Some examples of this are San Francisco (shown below), Edinburgh, Milton Keynes and Jaipur, also popularly known as the Pink City, which is the capital and largest city of the Indian state of Rajasthan. Land developers naturally favour simple grid patterns with square or rectangular building lots as these are much easier to plan and construct.



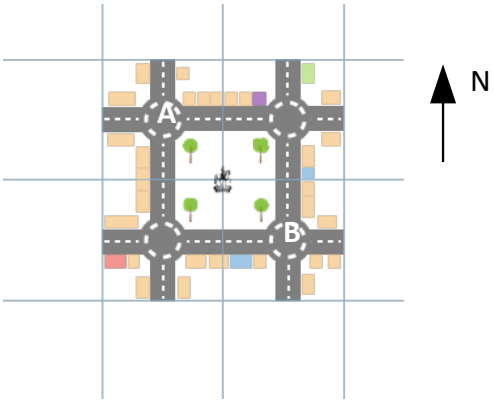


How many different routes are there?

The Fishermead Estate



When British city planners were designing Milton Keynes, they decided to divide the land into estates, all following a basic grid pattern. To ensure that buildings and facilities could be easily accessed by all residents, they modelled all the routes available to get from point A to point B.

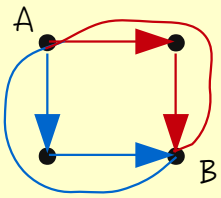


Look carefully this section of the map. If you are at roundabout A, how many ways are there of getting to roundabout B assuming you always travel East or South?

Sarah's working



Sarah

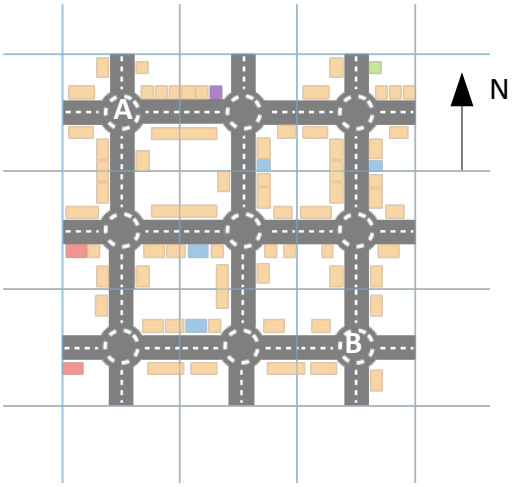


There are only two ways of getting from A to B.



Jon

It will be much more of a challenge if we use a 3 x 3 grid.

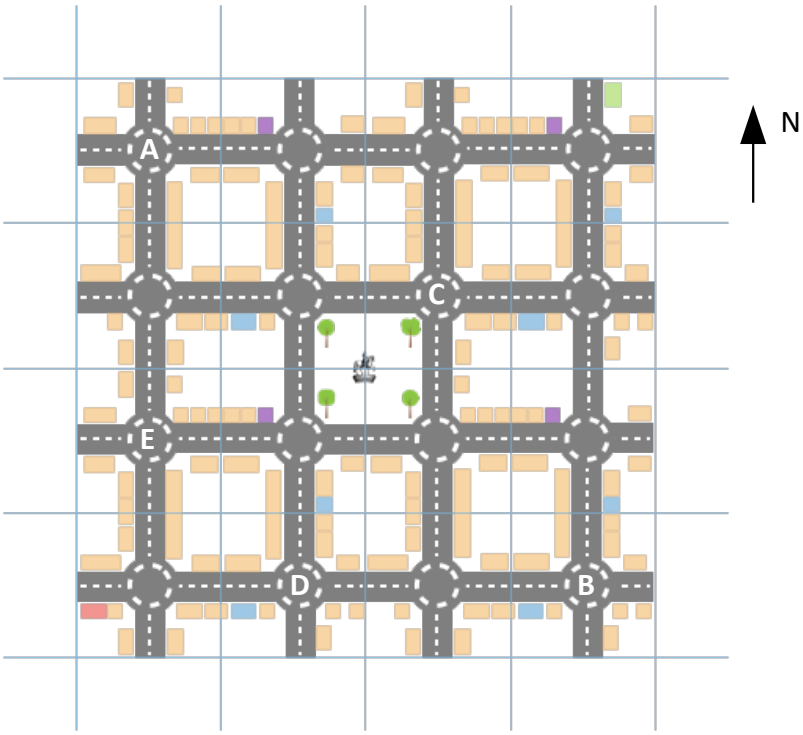


2 How many routes are there to get from A to B still assuming you always travel East or South?

How many different routes are there?

3

For the very determined only. How many routes are there to get from A to B still assuming you always travel East or South?



Translation notation

Translations are usually described by how many units you have moved horizontally and how many units you have moved vertically. For example, to move from A to C in the diagram above we move two units right and one unit down.

This can also be written as $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$. This is called **vector notation**.

To describe the move from D to C we could write $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ in vector notation.

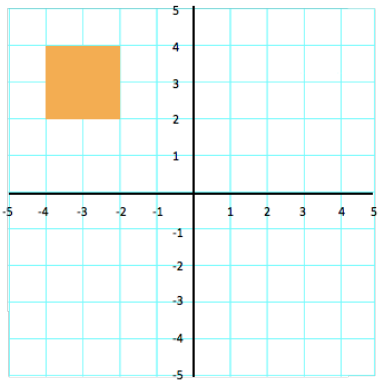
Fitness check

Describe in words and using vector notation how you could move from:

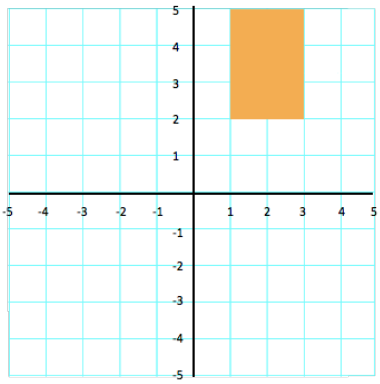
a) A to E
b) E to D
c) D to B
d) B to C
e) D to A
f) E to C
g) C to B
h) E to A

Translation

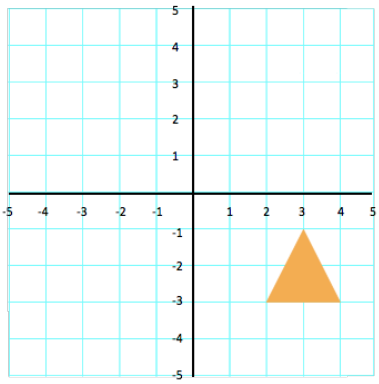
You will need worksheet Transformations_3b_1 for this activity.



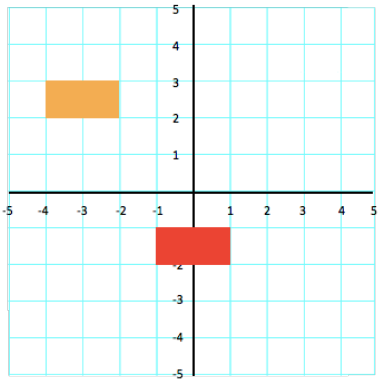
Translate 6 squares right and 6 squares down

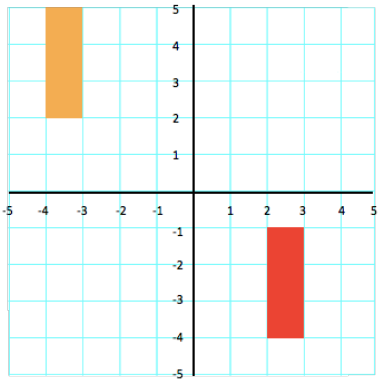


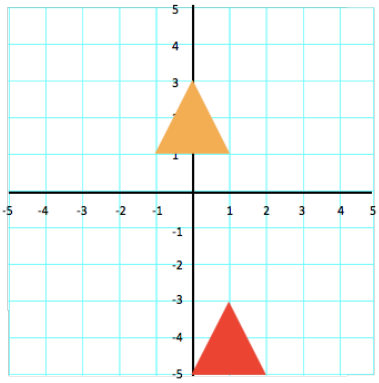
Translate 4 squares left and 7 squares down



Translate 4 squares left and 5 squares up

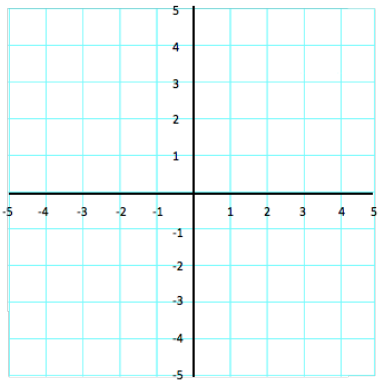


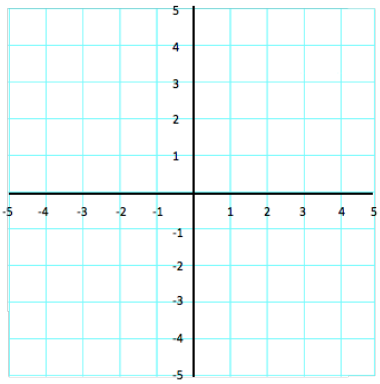


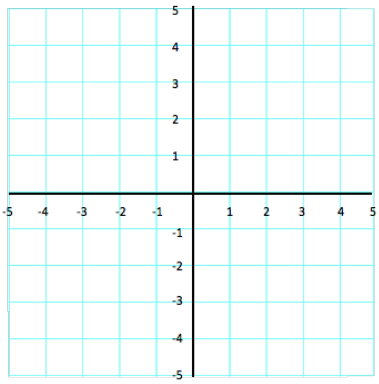


Write down the translation in vector form that moves the orange shape onto the red shape.
Then write down the translation in vector form that moves the red shape onto the orange shape.

4 Can you explain what happens to the translation vector in each case?



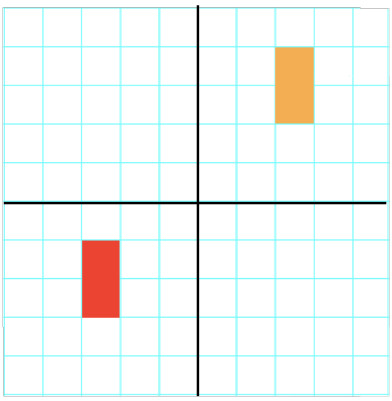
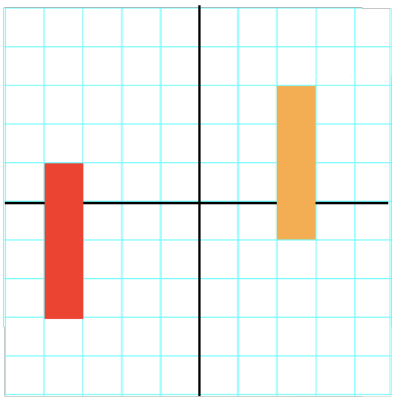
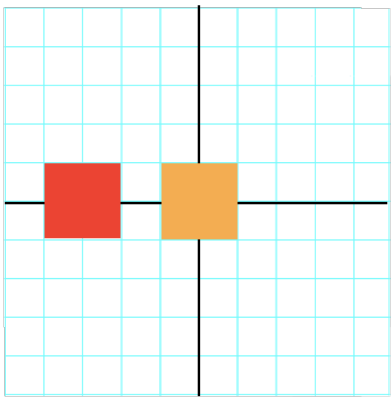
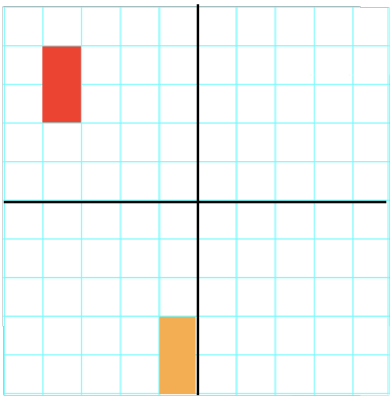
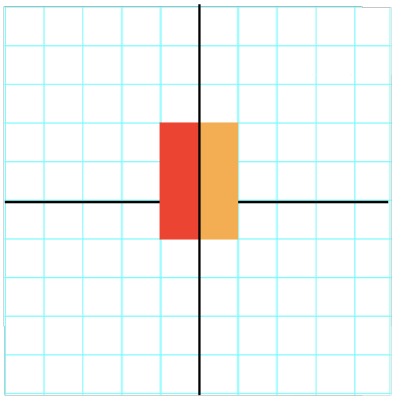
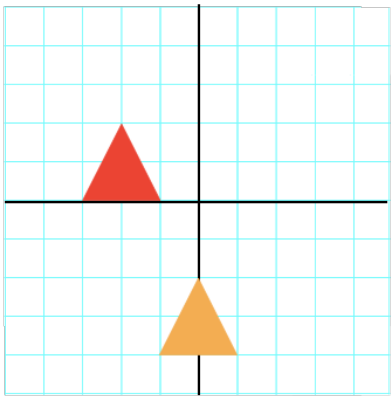
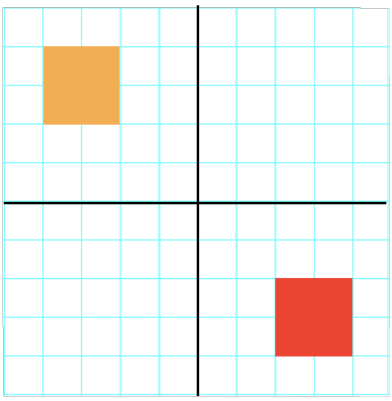
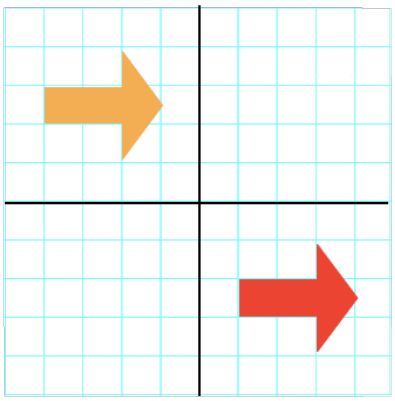
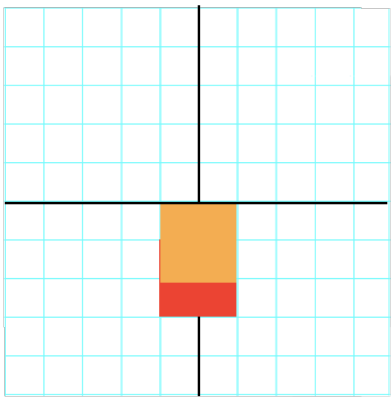




Question creation - Have fun creating problems for your friends to solve. Create your own translation challenges.

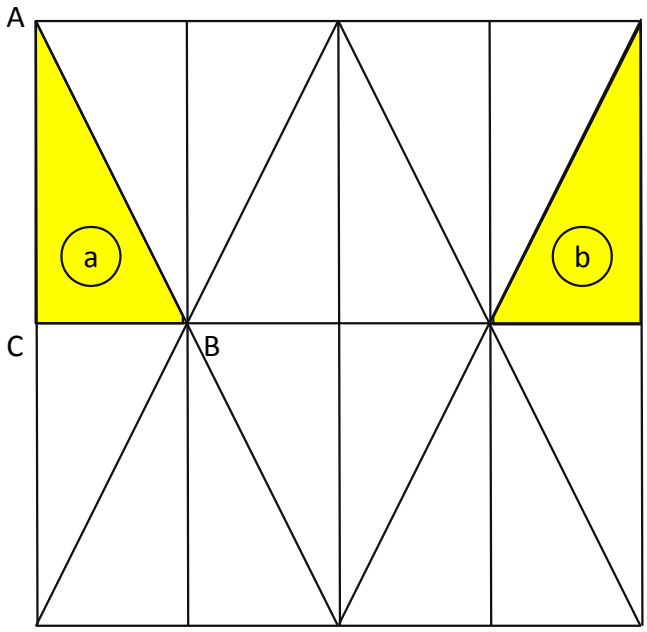
Sliding

Write the translation of the orange shape onto the red shape using vector notation You will need worksheet Translations_3b_2 for this activity.

Sliding and Flipping

Look carefully at this diagram. Triangle (a) can be moved to fit exactly onto triangle (b) .
Identify the points of triangle (b) as A', B' and C' that fit exactly with the points A, B and C on triangle (a) .



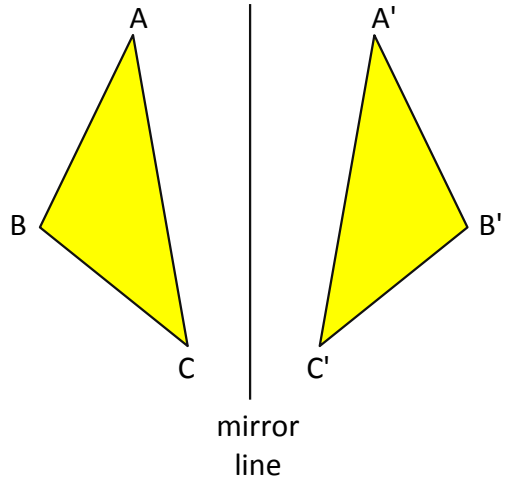
5

Can you explain how triangle (a) can be moved so that it fits exactly onto triangle (b) .

A transformation that requires moving a geometric figure by folding a plane on a given line is called **reflection**. The given line used for the transformation is called the **line of reflection** or **mirror line**.

Problem 1

On a sheet of tracing paper draw a scalene triangle ABC and a mirror line. Copy these into your exercise book. Now fold your paper along the mirror line and trace the new triangle into your book. Label your new triangle A'B'C'.



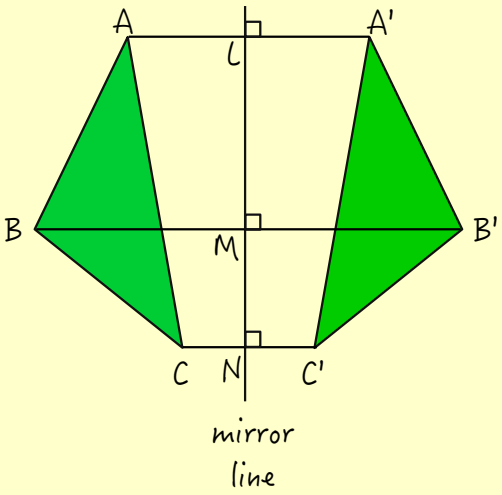
1. Draw line segments that connect corresponding vertices.
2. What kind of relationship is there between the line segments you have just drawn and the line of reflection?

Reflection

Evie's solution



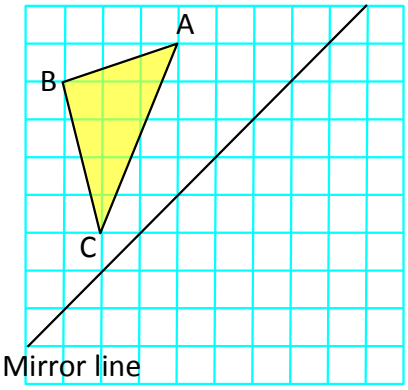
Evie

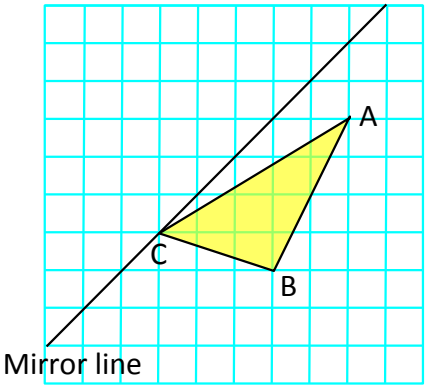


The mirror line is the perpendicular to AA', BB' and CC'.
L is the midpoint of AA', M is the mid point of BB' and N is the midpoint of CC'.
This means that the mirror line is the perpendicular bisector of AA', BB' and CC'.

Problem 2

In each of these two cases, draw a geometric figure that is the reflection of Δ ABC in the given mirror line.

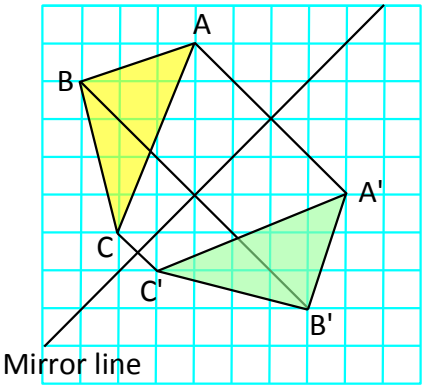




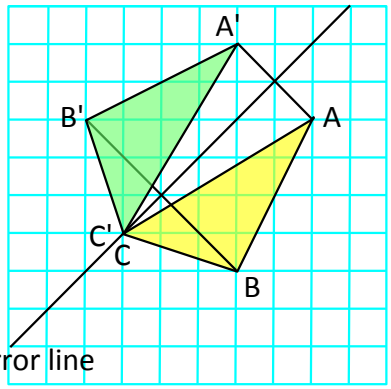
Jon's solution



Jon



Alice's solution



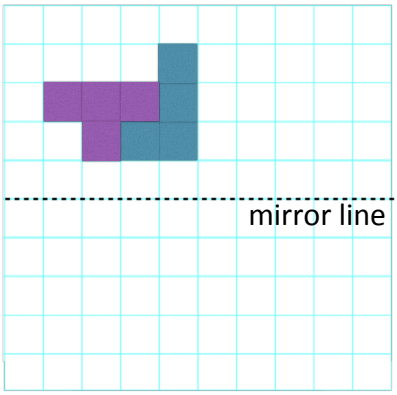


Alice

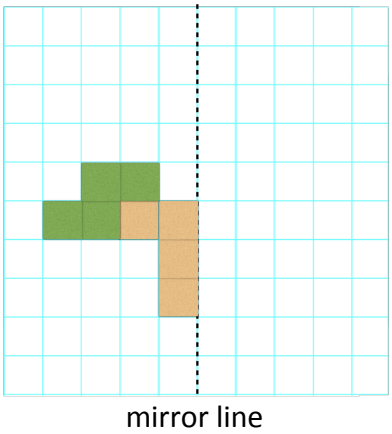
Flipping

Reflect these shapes in the given mirror line. You will need worksheet Transformations_3b_3.

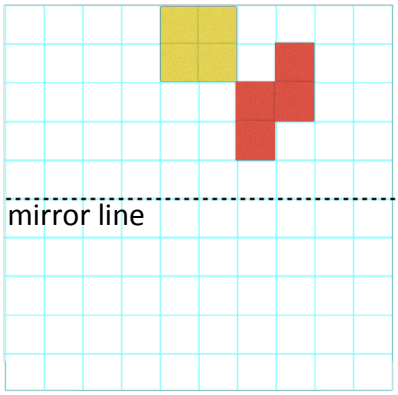
Question 1



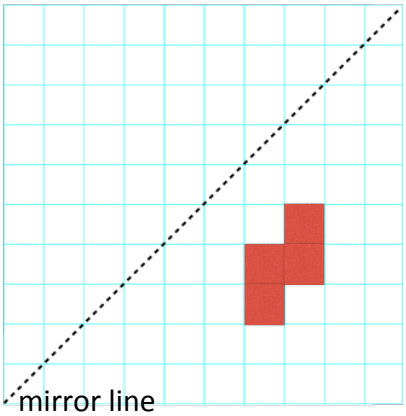
Question 2



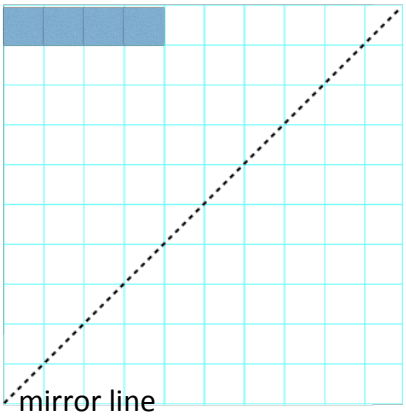
Question 3



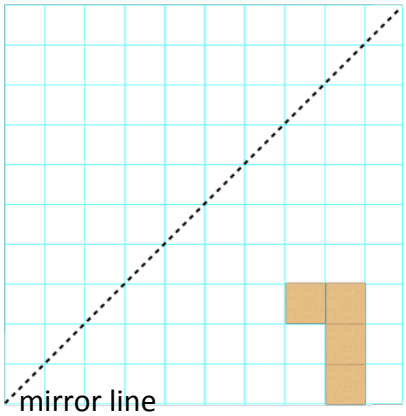
Question 4



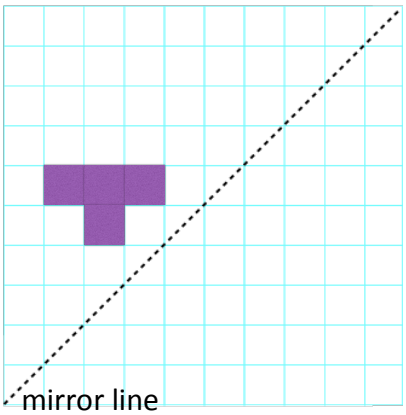
Question 5



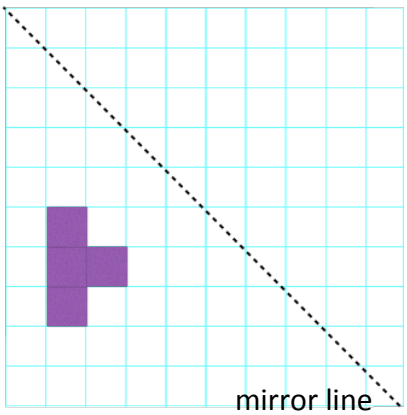
Question 6



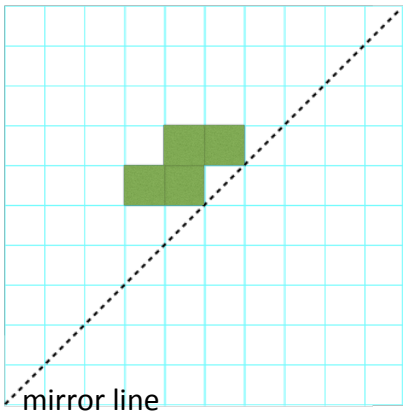
Question 7



Question 8

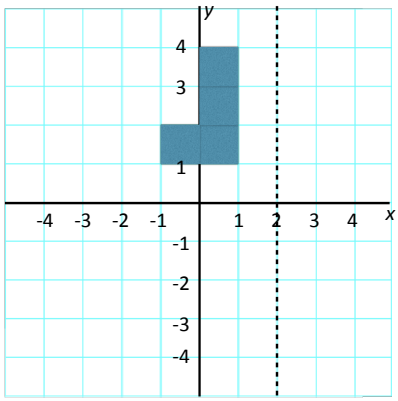


Question 9

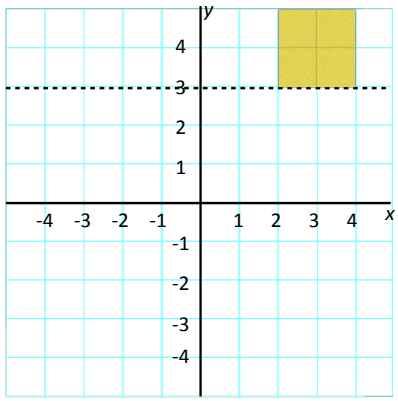


Flipping

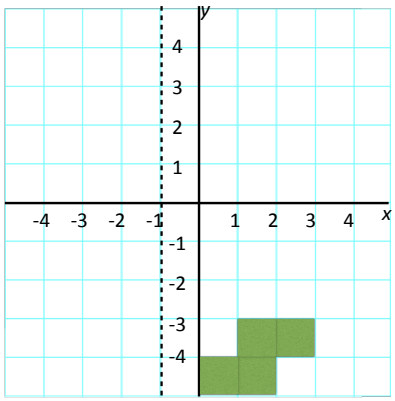
Reflect these shapes in the given mirror line. You will need worksheet Transformations_3b_4



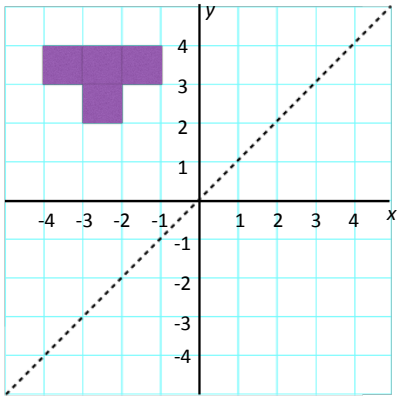
Question 1



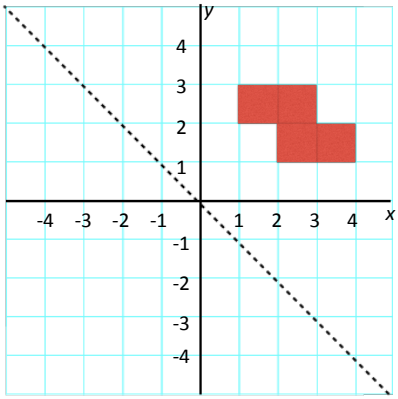
Question 2



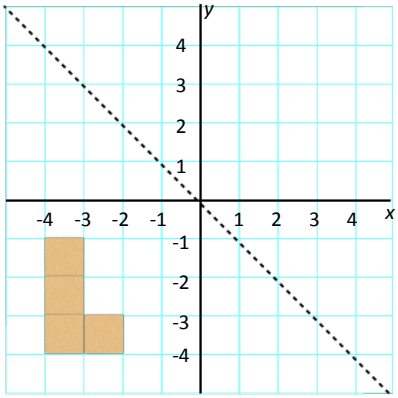
Question 3



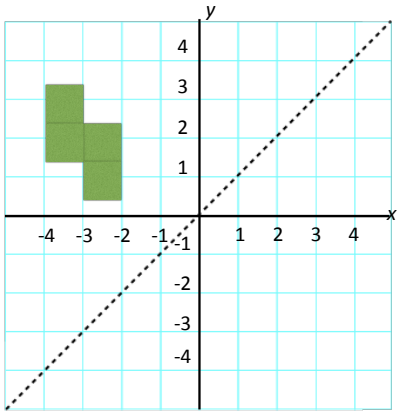
Question 4



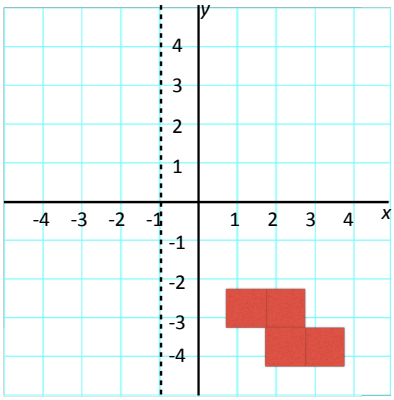
Question 5



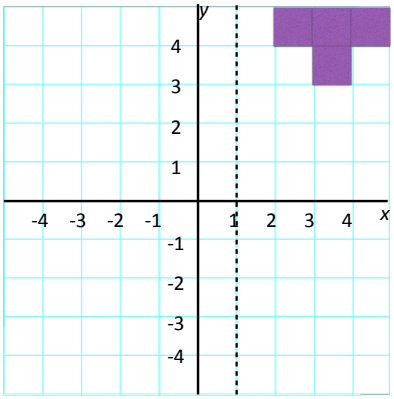
Question 6



Question 7



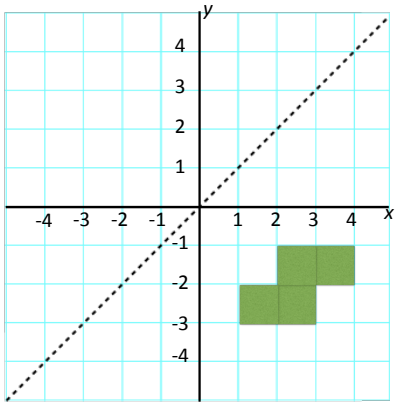
Question 8



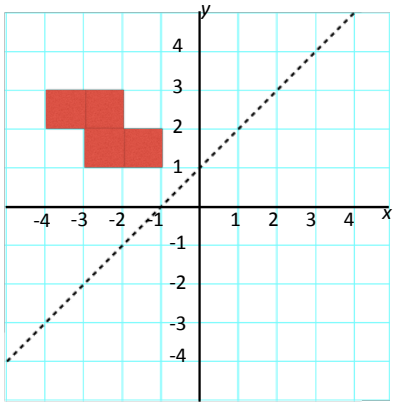
Question 9

Flipping

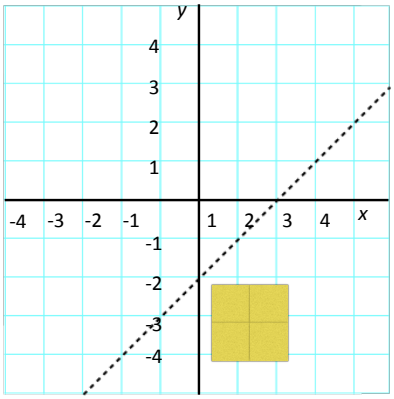
Reflect these shapes in the given mirror line. You will need worksheet Transformations_3b_5



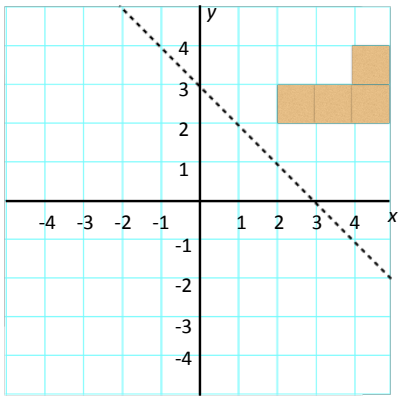
Question 10



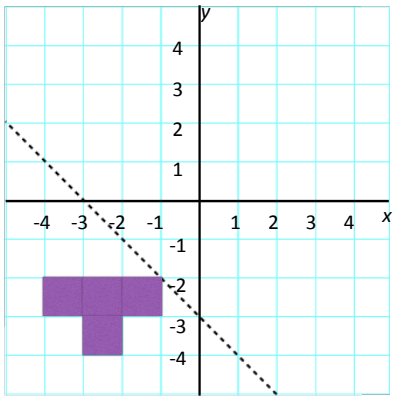
Question 11



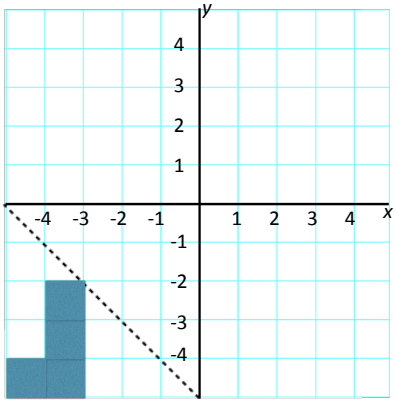
Question 12



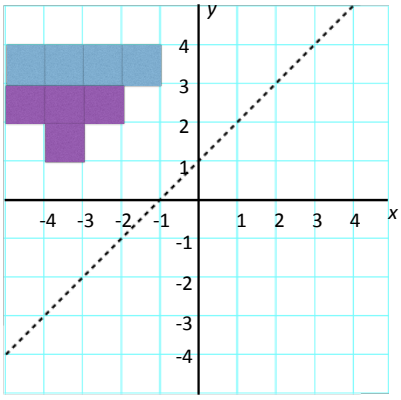
Question 13



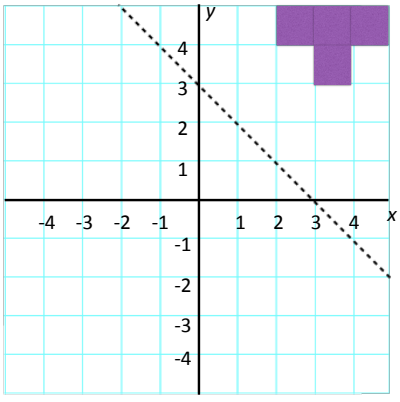
Question 14



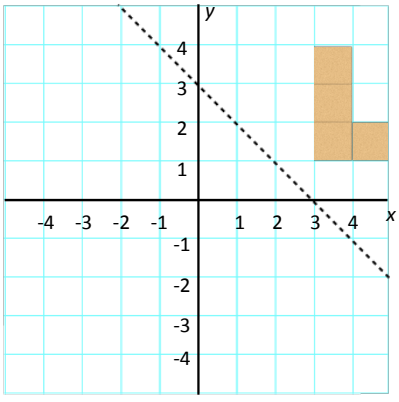
Question 15



Question 16

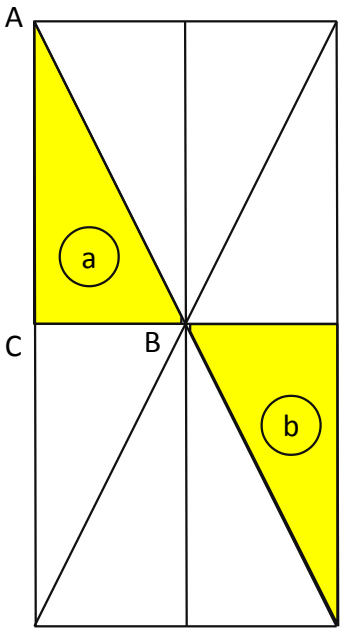


Question 17



Question 18

Rotation



In the diagram on the left, triangle (a) can be moved to fit exactly onto triangle (b)

1. Identify points of triangle (b) as A', B' and C' that fit exactly with points A, B and C on triangle (a).
2. How can triangle (a) be moved in a single transformation so that it can fit onto triangle (b)?

The transformation that rotates a geometric figure using a given point as the centre and a fixed size of rotation angle is called **rotation**.

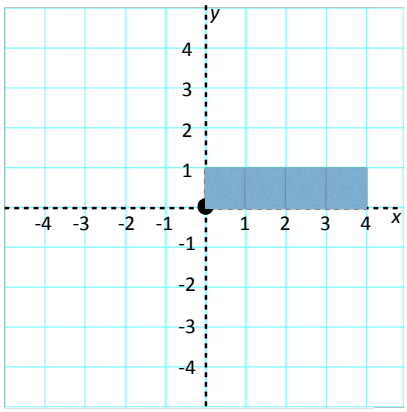
In this case B is the **centre of rotation** and the **angle of rotation** is 180° clockwise (although it could also be described as 180° anticlockwise).

It is a good idea to use tracing paper to help you to "see" what happens when we rotate shapes.

Fitness check

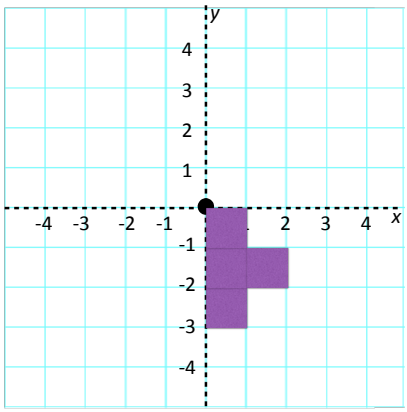
In each of the following questions, copy the diagram into your book and then follow the instructions.

Question 1



Rotate the shape 90° clockwise, through the origin (centre dot)

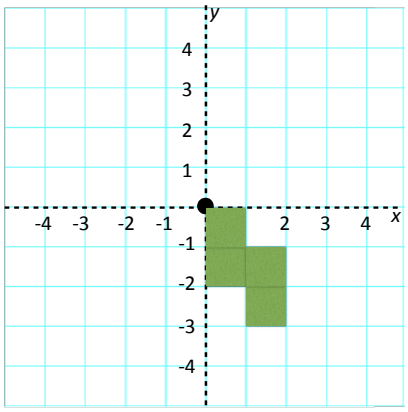
Question 2



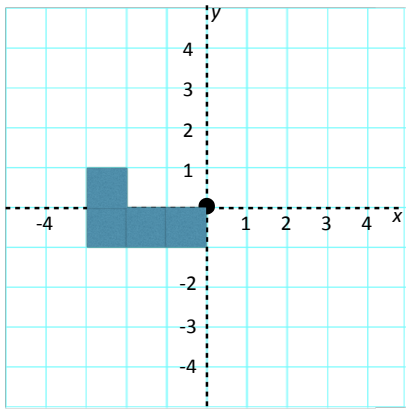
Rotate the shape 180° clockwise, through the origin (centre dot)

Rotation

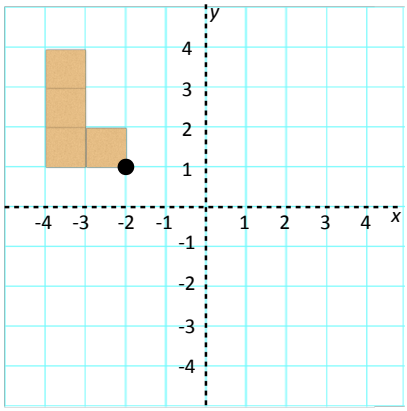
You will need worksheet Transformations_3b_6 for this activity.



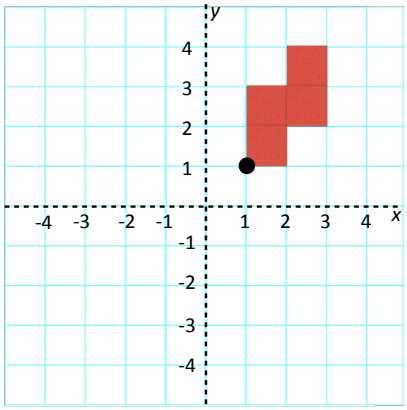
Rotate the shape 90° anticlockwise through the origin (centre dot)



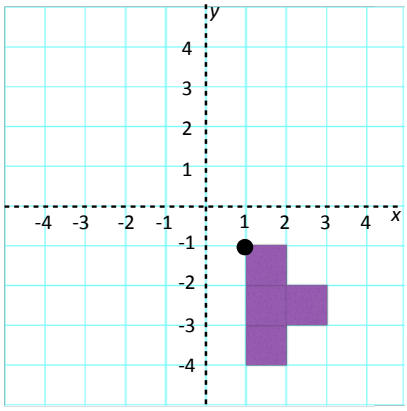
Rotate the shape 180° anticlockwise, through the origin (centre dot)



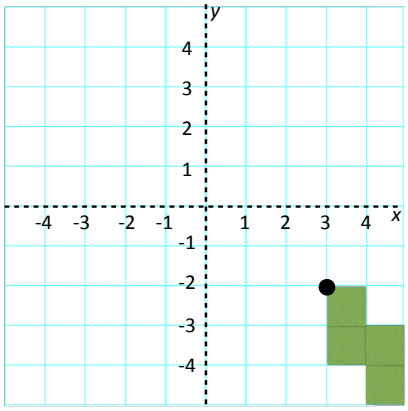
Rotate the shape 90° clockwise, about the centre (-2,1)



Rotate the shape 180° clockwise, about the centre (1,1)



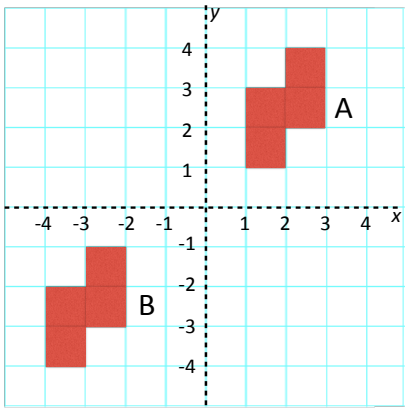
Rotate the shape 90° anticlockwise about the centre (1,-1)

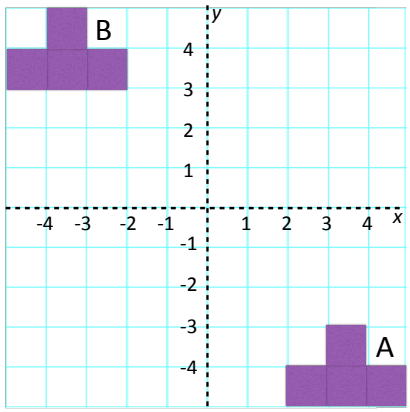


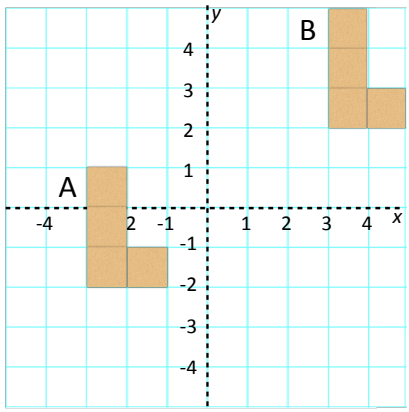
Rotate the shape 180° anticlockwise, about the centre (3,-2)

Rotation

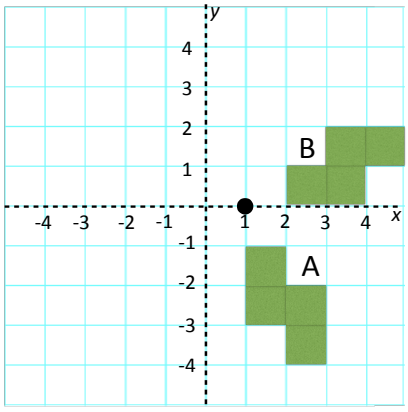
You will need worksheet Transformations_3b_7 for this activity.

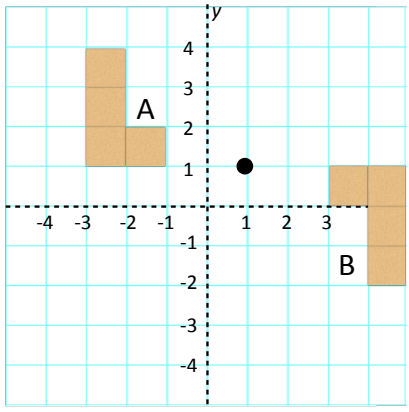


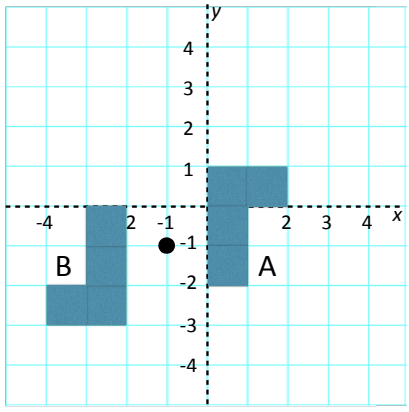




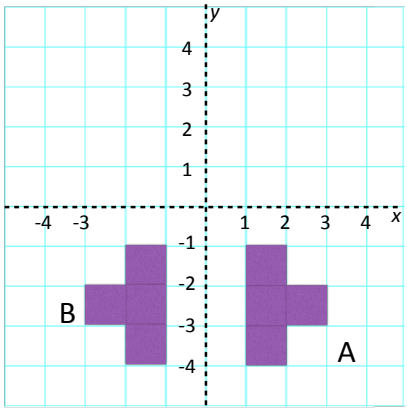
Describe fully the transformation that moves A to B

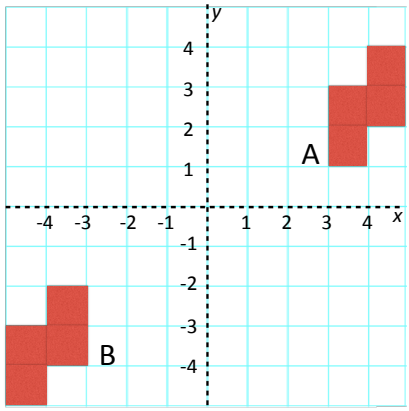


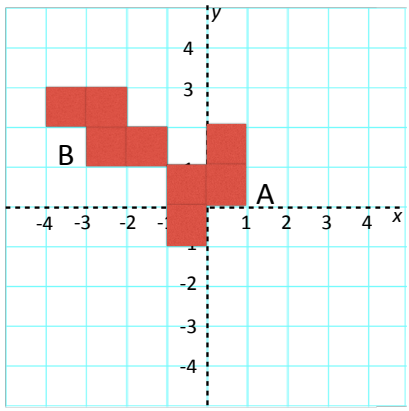




Describe fully the rotation that moves A to B



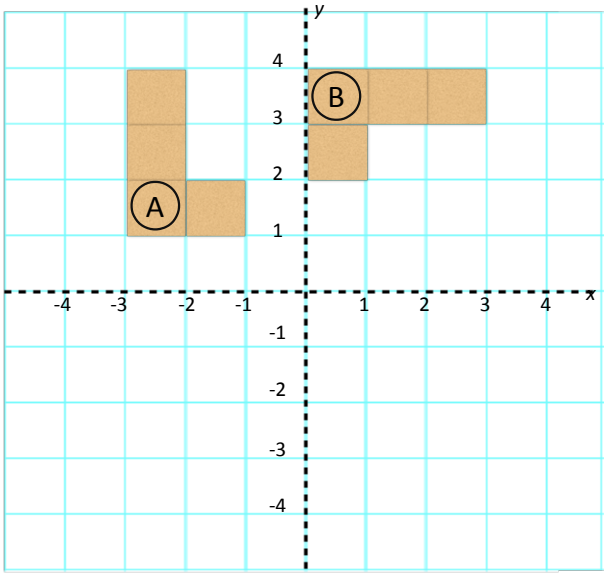


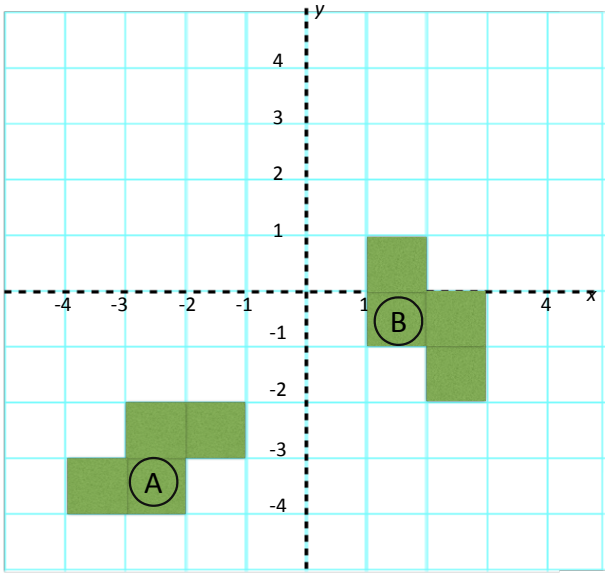


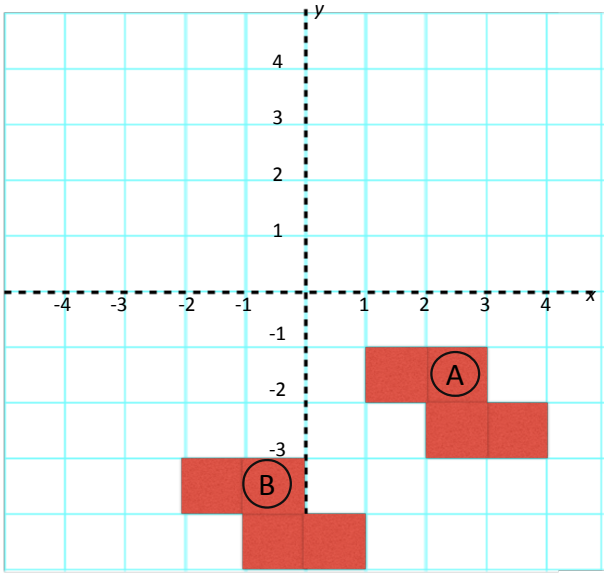
Describe fully the transformation that moves A to B

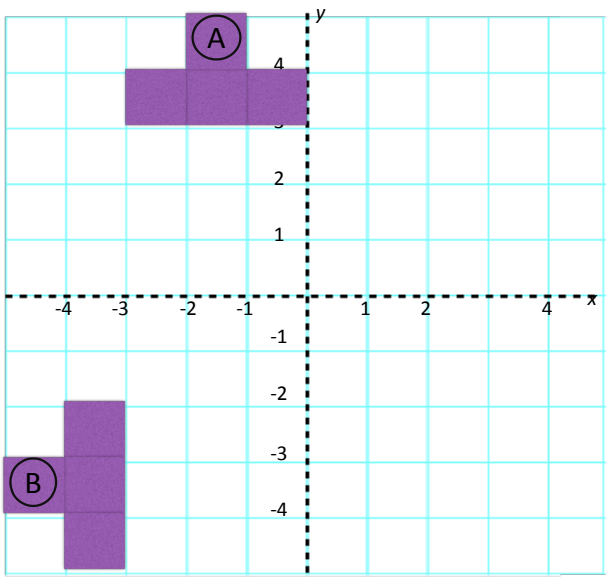
Where's the centre of rotation?

You will need worksheet Transformations_3b_8 for this activity. In each you need to find the centre of rotation and describe the rotation fully that moves shape A to shape B.







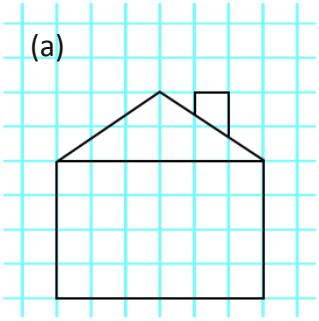


Enlargement

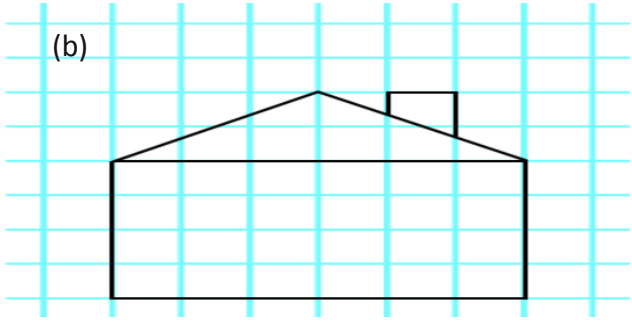


By using a photocopier, we changed the size of figure (a). Which of the following figures are similar to (a)?

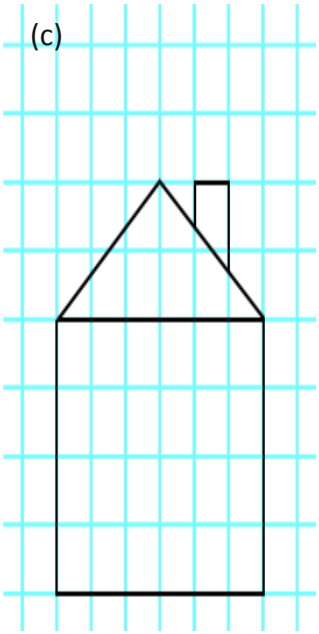
(a)



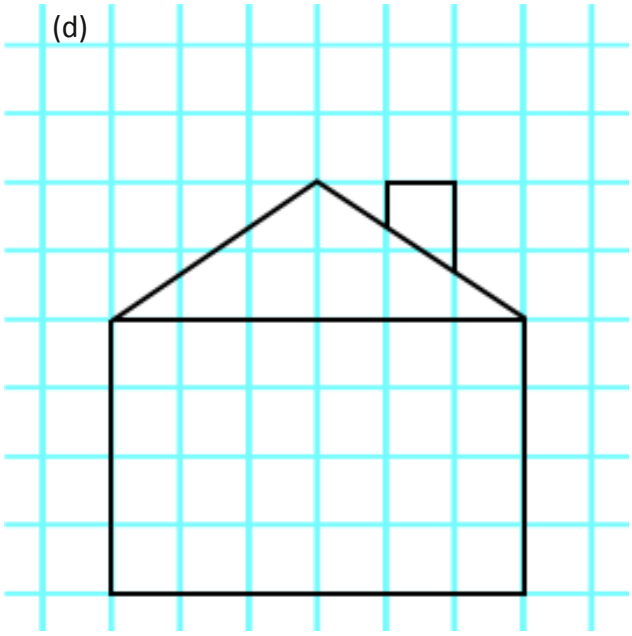
(b)



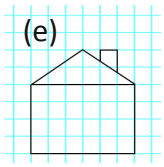
(c)



(d)



(e)





Alice

Are (b) and (c) different shapes?



Jon

Only the horizontal length is doubled in (b).



Amber

In (c) only the vertical length...

Same Shape, Different Size...

Draw a figure below with both the vertical and horizontal lengths 3 times as long as those in figure (a) on the previous page and call it figure (x)



Sarah

The shape does not change when you make both the vertical and the horizontal lengths longer by the same proportion.

6 Investigate figures that have the same shapes but different sizes...

1. In figures (a) and (d), find the ratios of the lengths of corresponding sides.

2. In figures (a) and (d), investigate the measure of corresponding angles.

The ratio of corresponding sides (a) and (d) are all the same, 1 : 2. Also, the measures of corresponding angles are equal.



That's why they look like the same shape!

Jon

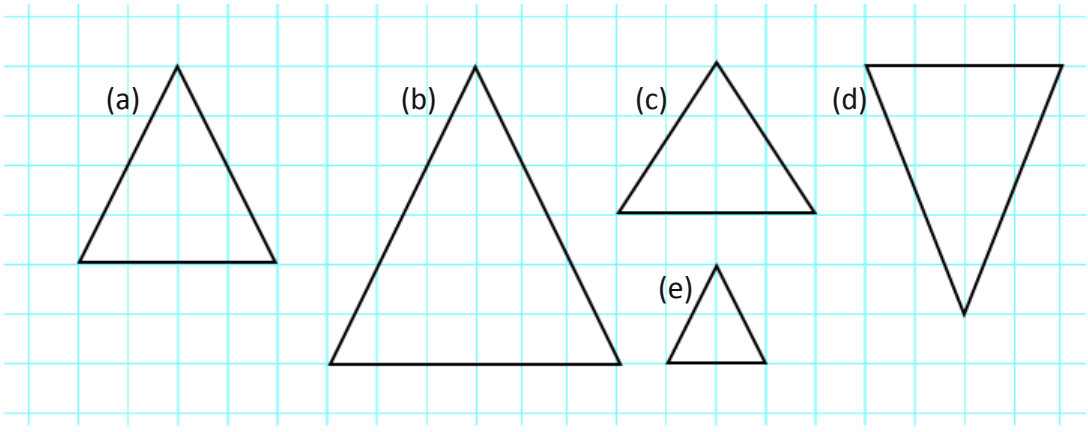
When a figure is made larger while the measures of the corresponding angles and the ratios of the lengths of corresponding sides are kept equal, the picture is called an **enlarged drawing.** When a figure is made smaller in the same way, it is called a **reduced drawing**.

When the lengths of the sides are made twice as long as the sides of the original figure, we say the shape has been enlarged by a scale factor of 2.

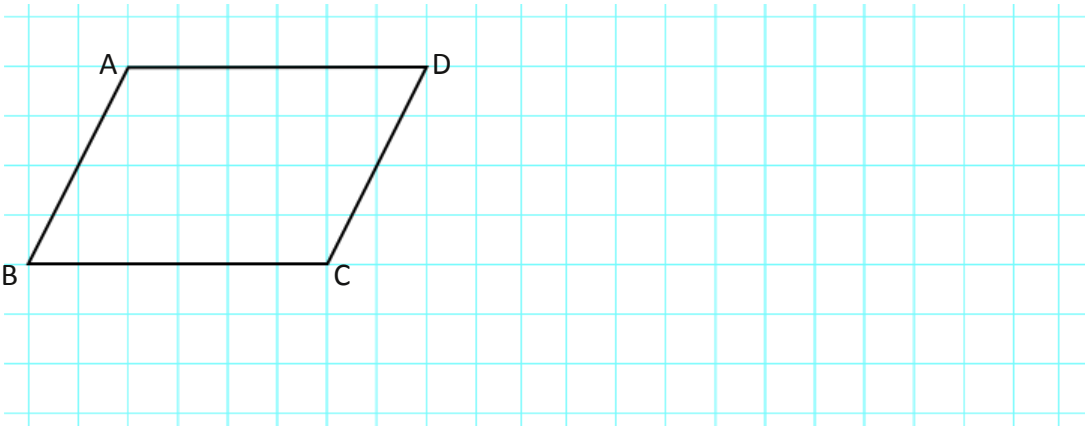
Also, when the lengths of the sides are made $\frac{1}{2}$ as long as the sides of the original figure, we say the shape has been enlarged by a scale factor of $\frac{1}{2}$

3. Check to see if figure (x) you drew previously is three times larger.

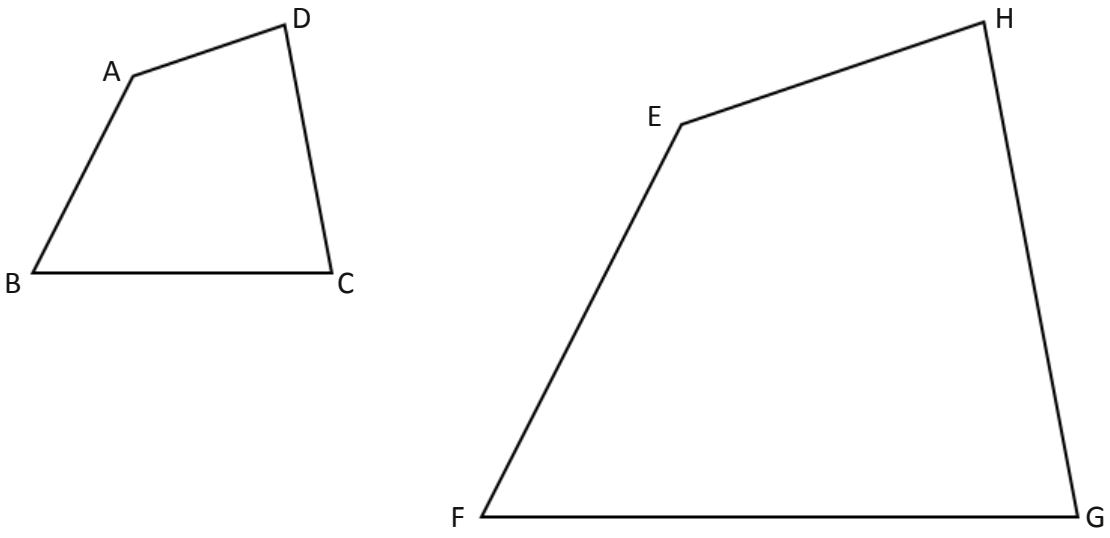
In the figures below, which triangles are enlarged or reduced drawings of triangle (a) ? Also, by how many times are they enlarged or reduced?



4. Draw a figure enlarged by a scale figure of three and a figure enlarged by the scale factor of $\frac{1}{2}$ of the parallelogram ABCD below.



5. Quadrilateral EFGH is an enlarged drawing of quadrilateral ABCD, by a scale factor of two.



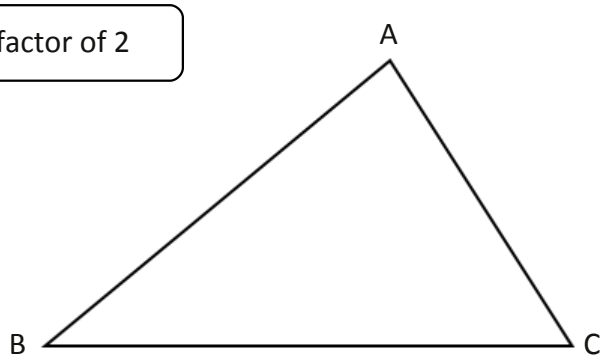
a. Which side corresponds to side AB, and how long is it?

b. Which angle corresponds to angle B, and what is its measure?

c. Which side corresponds to side FG, and how long is it?

d. Which angle corresponds to angle G, and what is its measure?

9 Draw triangle DEF, by enlarging ABC by a scale factor of 2



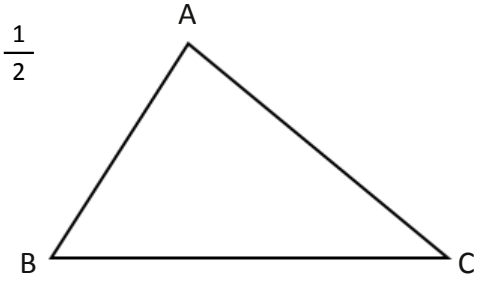
Lets think about how to draw an enlarged drawing...

E.

1. First draw side EF, which corresponds to side BC. How long should side EF be?

2. Next, to draw vertex D corresponding to vertex A, which side of angles of triangle ABC do you need to use?

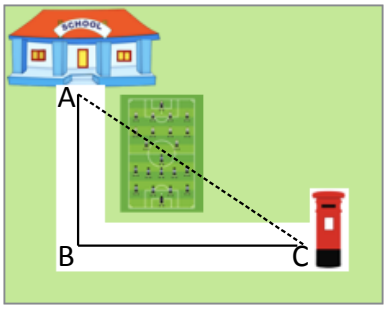
3. Enlarge the triangle below by a scale factor of $\frac{1}{2}$



Applications of reduced drawings

10

The figure on the right is a reduced drawing of the area around a school. The actual length of AB is 300m, and it is shown as 3cm on the reduced drawing. What is the direct distance from the school to the postbox? How about B the distance along the path?



Lets work out the actual distance using a reduced drawing.

1. Express the rate of reduction using a fraction. Also express the rate as a ratio.

				m			cm	
								3
3	0	0	0	0				

The rate in which an actual length is reduced is called a **scale**. A scale may be expressed in the following ways.

(A) $\frac{1}{10000}$ (B) 1 : 10000 (C) 0 100 200 300

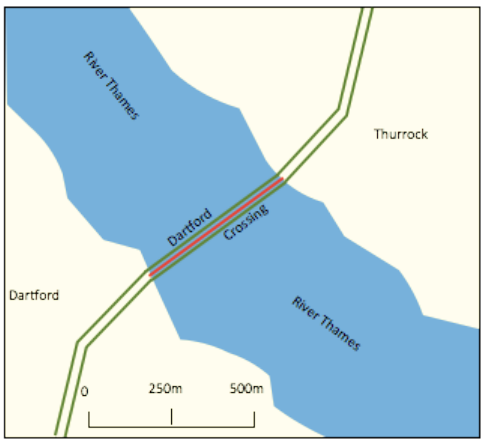
2. In the figure above, measure the length of BC and determine the actual length of BC.

   Also, what is the actual distance of the path from the school to the mailbox?

3. In the figure above, measure the length of AC and determine the actual distance from the school to the mailbox.

4. Find the length of the Dartford Crossing Bridge on the map below....





Applications of reduced drawings

1. A model of a camper van has a ratio of 1:50 to the actual van. Complete the following table.



Item	Model	Actual
Width of Tyres	?	50cm
Windscreen	?	200cm
Car Length	51cm	?
Wing Mirror	1.5cm	?
Steering Wheel	2.5cm	?

2. A model of the cargo ship 'Cielo Di Parigi' has a atio on 1:180 to the actual ship. Some of the dimensions are shown below, but some are not correct. Find the mistakes.



Item	Model	Actual
Ship Height	30cm	30m
Lifeboat Height	2cm	20m
Funnel Width	3cm	5.4m
Ship Length	1m	180m

3. If a radio controlled model yacht in the ratio of 1:30 is 16cm wide, how wide is the actual boat?



4. If the mast on the yacht is 3m, how high is the mast on the model?

How far is it from …. to …. as the crow flies?

As the crow flies

"As the crow flies" or "in a beeline" is an idiom for the shortest path between two points (on a map, disregarding the vagaries of intervening terrain). It is also known as the geodesic distance.

The map shown below shows most of the airports in the UK. Unfortunately there is no scale but we do know that it is 556 km from London to Glasgow as the crow flies.



As the crow flies, how far is it from ...

1. London to Oxford
2. Plymouth to Southampton
3. Cork to Aberdeen
4. Liverpool to Edinburgh
5. Bristol to York
6. Norwich to Canterbury
7. Portsmouth to Manchester
8. Leeds to Swansea
9. How many airports are within 200 km of Birmingham?
10. Amber says she has found two airports that are roughly 400 km apart.
    Which airports might she have chosen? Is there more than one option?

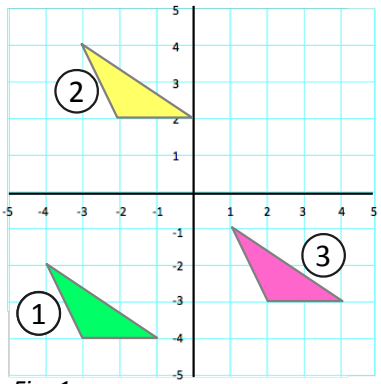


Fig. 1

1. Which vector correctly describes the single translation that moves triangle (1) to triangle (3) (*Fig. 1*)?

A	B	C	D
$\begin{pmatrix}4\\1\end{pmatrix}$	$\begin{pmatrix}5\\1\end{pmatrix}$	$\begin{pmatrix}-5\\-1\end{pmatrix}$	$\begin{pmatrix}1\\5\end{pmatrix}$

2. Which vector correctly describes the single translation that moves triangle (2) to triangle (3) (*Fig. 1*)?

A	B	C	D
$\begin{pmatrix}5\\4\end{pmatrix}$	$\begin{pmatrix}4\\-5\end{pmatrix}$	$\begin{pmatrix}-4\\-5\end{pmatrix}$	$\begin{pmatrix}-4\\5\end{pmatrix}$

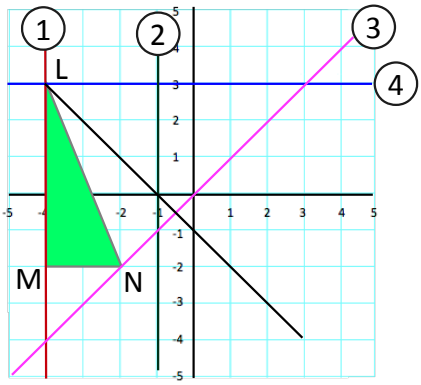


Fig. 2

3. Reflecting the triangle LMN (Fig. 2) in which line leaves exactly two vertices unchanged (invariant) ?

A	B	C	D
(1)	(2)	(3)	(4)

4. If the LMN (Fig. 2)is reflected in the line (3) , what are the coordinates of L'?

A	B	C	D
(-4, 3)	(4, -3)	(3, -4)	(4, -3)

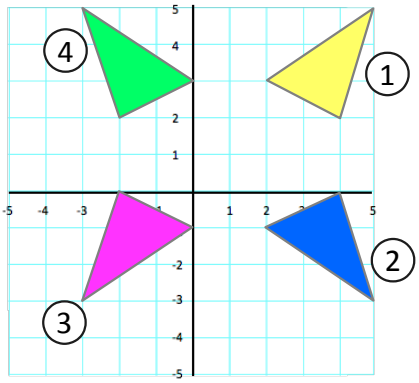


Fig. 3

5. If the triangle (1) is reflected in the line $x = 1$, which triangle is the image of triangle (1) (*Fig. 3*)?

A	B	C	D
(1)	(2)	(3)	(4)

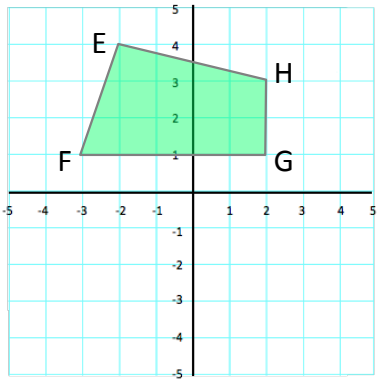


Fig. 4

6. If the shape EFGH is reflected in the line $y = -1$, what are the coordinates of F' (*Fig. 4*)?

A	B	C	D
(1, 1)	(6, 1)	(3, -1)	(-3, -3)

Geometry 3b: Mastery questions

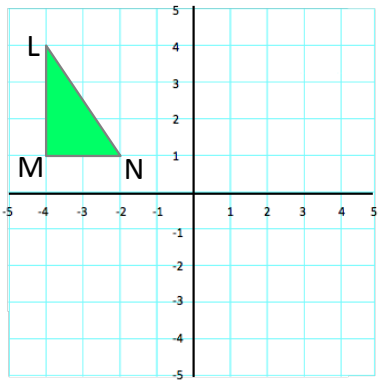


Fig. 5

7. If the triangle LMN (*Fig.5*) is rotated about the origin through 180°, what are the coordinates of M'?

A	B	C	D
(-4, -1)	(4, -1)	(2, -1)	(-2, -1)

8. If the triangle LMN (*Fig.5*) is rotated 90° anticlockwise about the origin, what are the coordinates of N'?

A	B	C	D
(1, 2)	(-2, -1)	(2, 1)	(-1, -2)

9. If the triangle LMN (*Fig. 5*) is rotated 90° clockwise about N. What are the coordinates of L'?

A	B	C	D
(1, 3)	(-2, 3)	(-2, -2)	(0, -2)

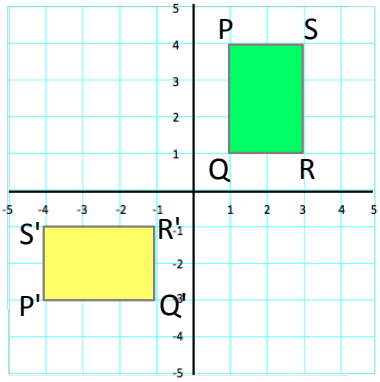


Fig. 6

10. The rectangle PQRS (Fig. 6) has been rotated through 90° anticlockwise onto the shape P'Q'R'S'. What are the coordinates of the centre of rotation?

A	B	C	D
(1, -1)	(3, -2)	(1, -2)	(2, -2)

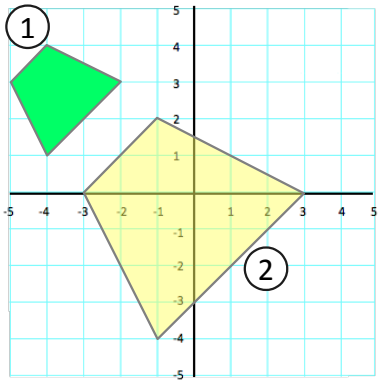


Fig. 7

11. Shape (1) has been transformed into shape (2) (*Fig. 7*). How is this transformation best described?

A	A translation.	B	An enlargement, scale factor 2.
C	An enlargement, scale factor 0.5.	D	A translation and an enlargement.

12. A model bus has a scale of 1: 40. If the height of the bus is 4.4m, how tall is the model?

A	B	C	D
1.1 m	10 cm	0.1 m	11 cm

13. The scale on a map is 1 cm represents 20 miles. If the distance between Plymouth and Norwich on the map is 16 cm, how far is it?

A	B	C	D
80 miles	800 miles	320 miles	32 miles

14. A scale of 1:100000 is equivalent to:

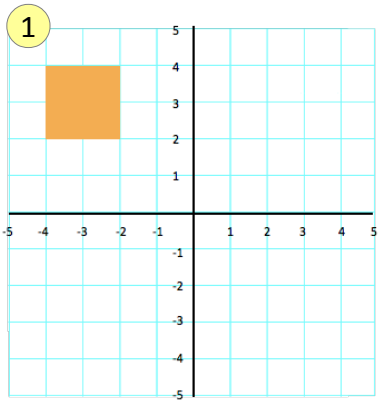
A	1 cm to 1 m	B	1 cm to 1 km
C	1 mm to 1 m	D	1 mm to 1 km

15. The actual distance between London and Tokyo is approximately 9500 km. If the scale of a map is 1: 80 000 000, what is the approximate distance between the two cities on the map?

A	B	C	D
11 cm	110 cm	12 cm	120 cm

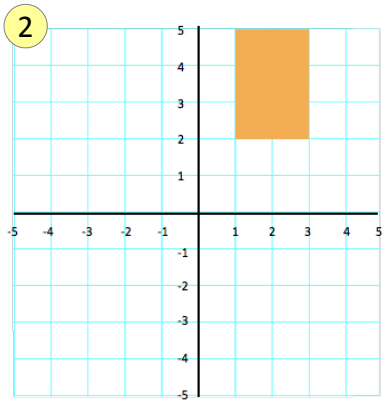
Worksheet Transformations_3b_1

1



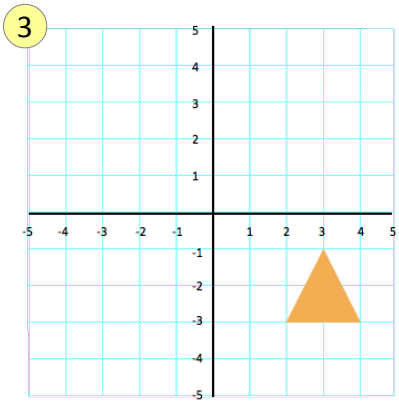
Translate 6 squares right and 6 squares down

2



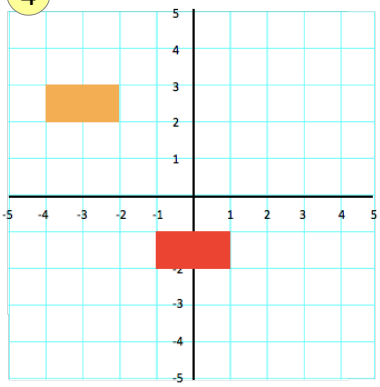
Translate 4 squares left and 7 squares down

3

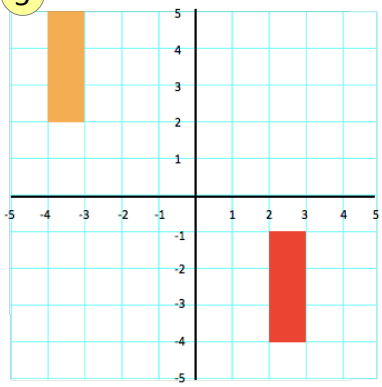


Translate 4 squares left and 5 squares up

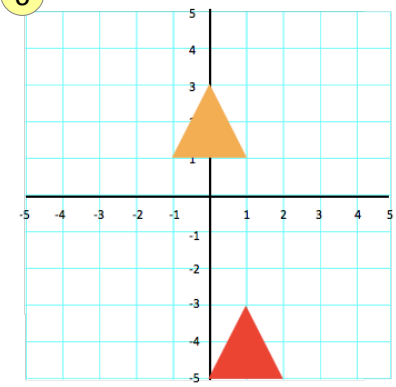
4



5



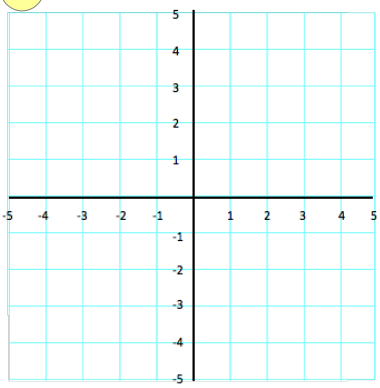
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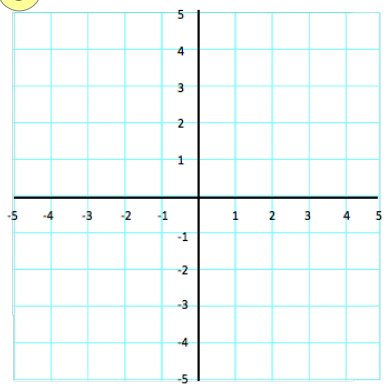
Write down the translation in vector form that moves the orange shape onto the red shape.
Then write down the translation in vector form that moves the red shape onto the orange shape.

Question creation - Have fun creating problems for your friends to solve. Create your own transformation challenges.

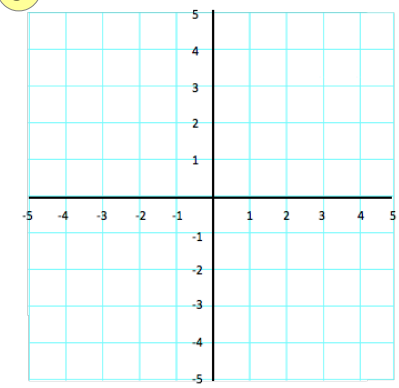
7



8



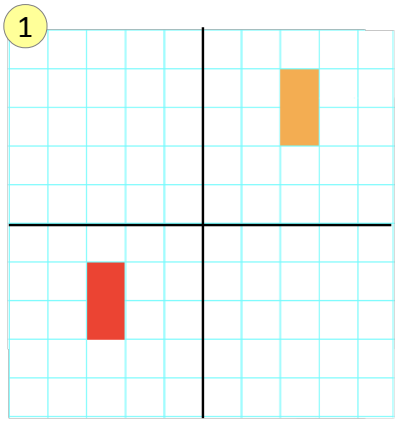
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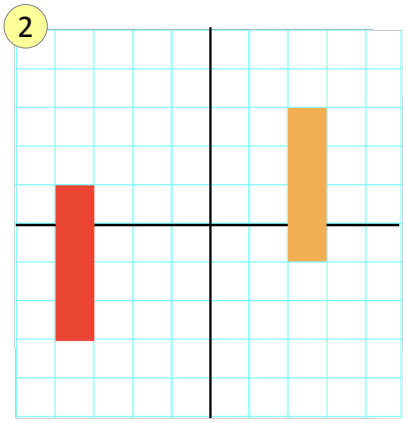
Worksheet Transformations_3b_2

Write the translation of the orange shape onto the red shape using vector notation.

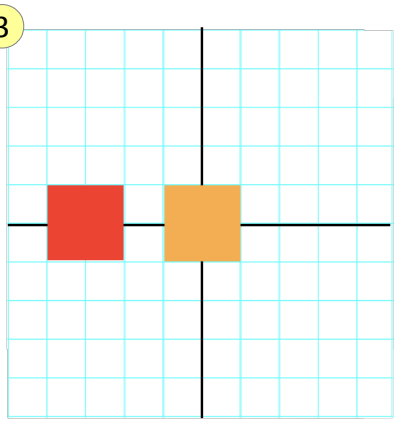
1



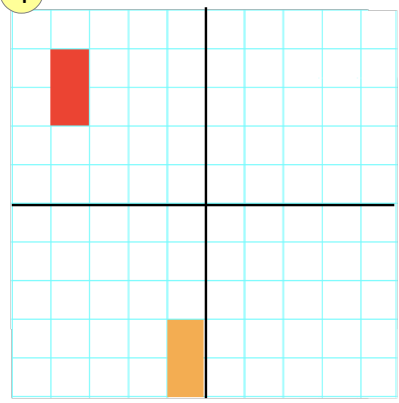
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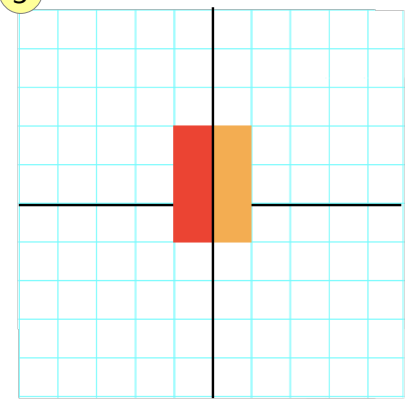
3



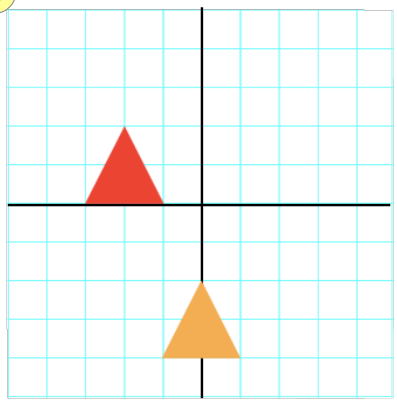
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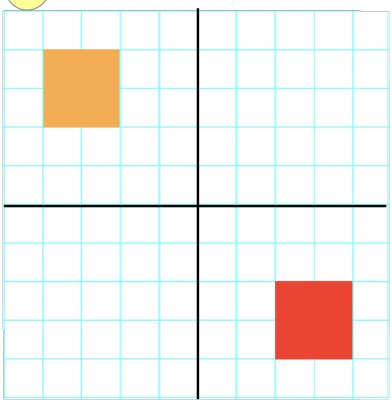
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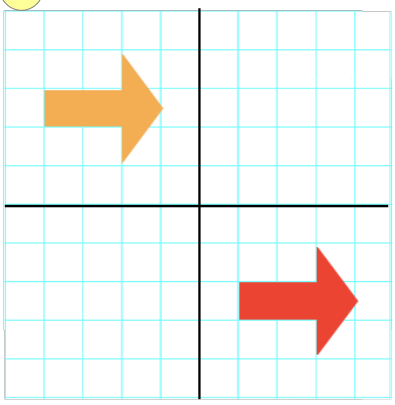
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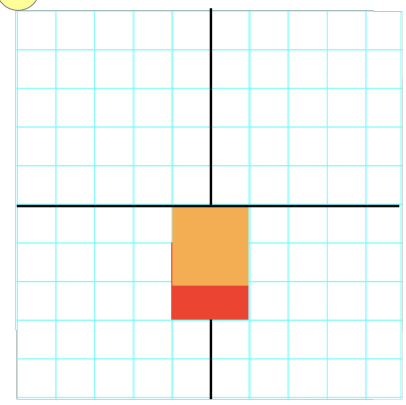
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8



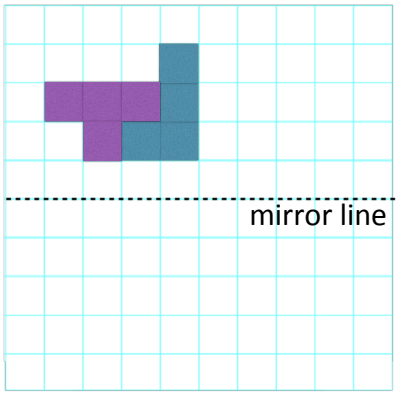
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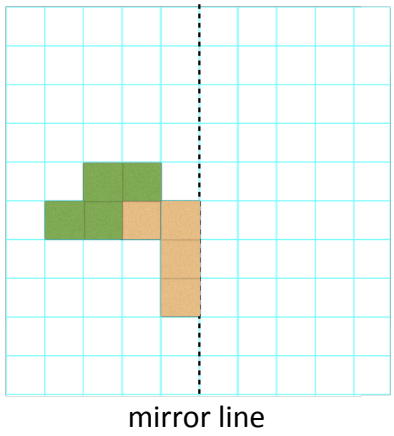
Worksheet Transformations_3b_3

Reflect these shapes in the given mirror line.

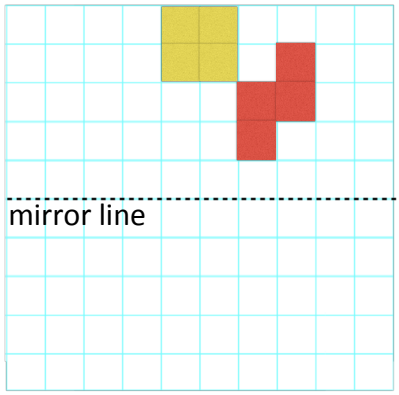
1 Question 1



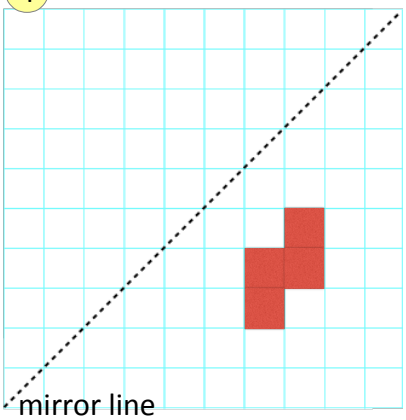
2 Question 2



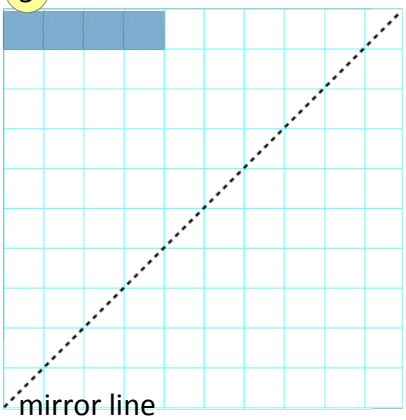
3 Question 3



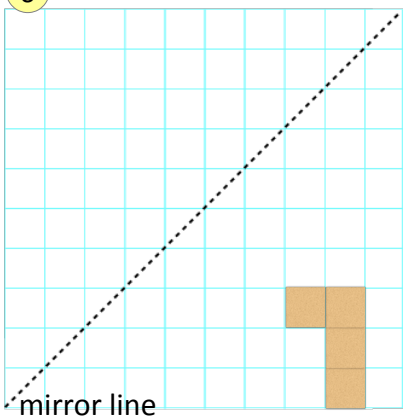
4 Question 4



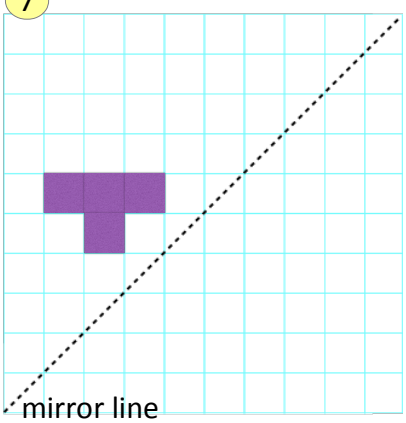
5 Question 5



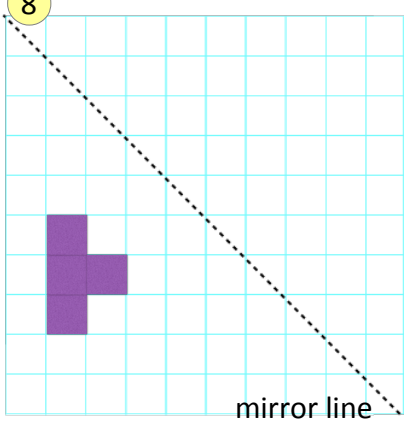
6 Question 6



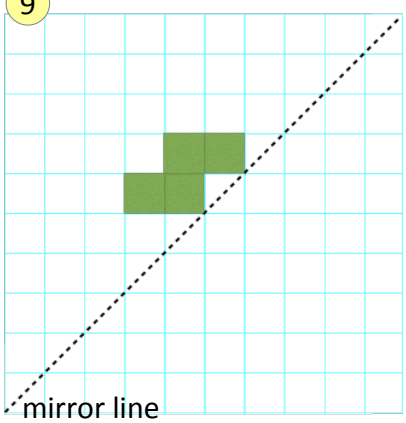
7 Question 7



8 Question 8

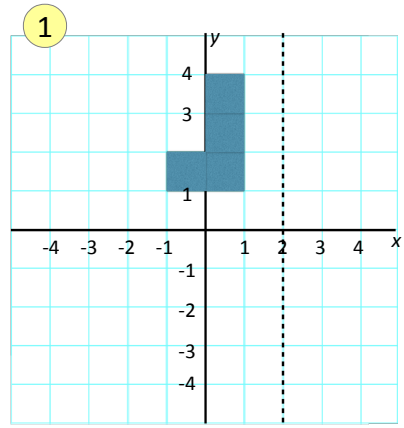


9 Question 9

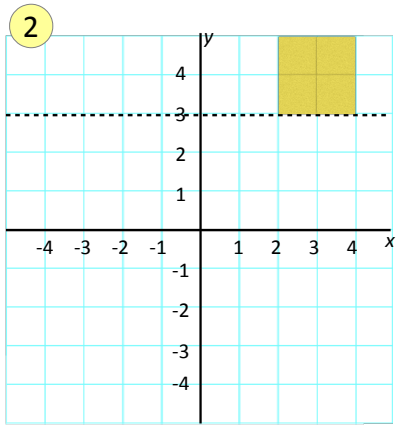


Worksheet Transformations_3b_4

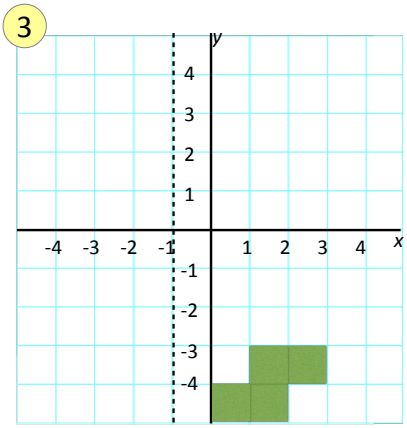
Reflect these shapes in the given mirror line.



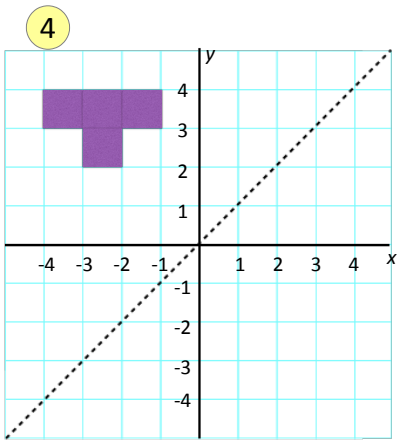
Question 1



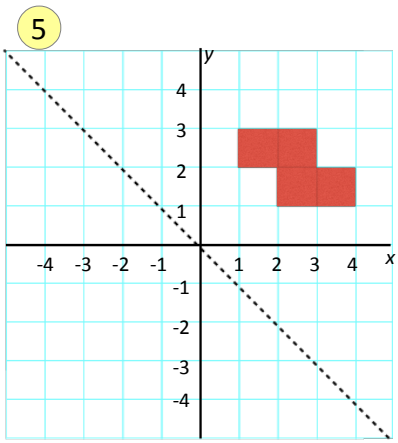
Question 2



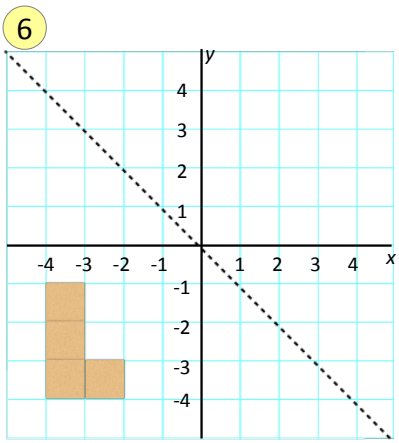
Question 3



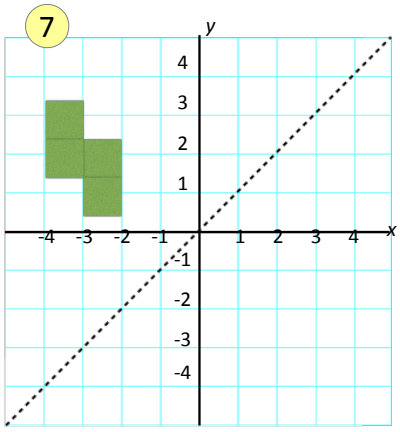
Question 4



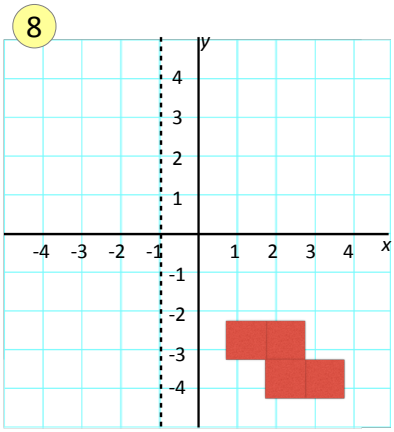
Question 5



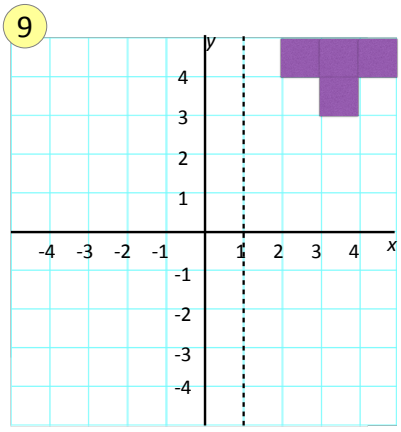
Question 6



Question 7



Question 8

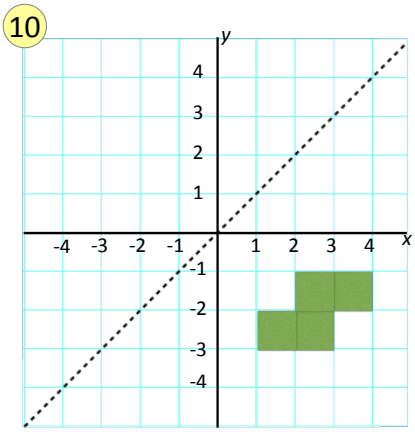


Question 9

Worksheet Transformations_3b_5

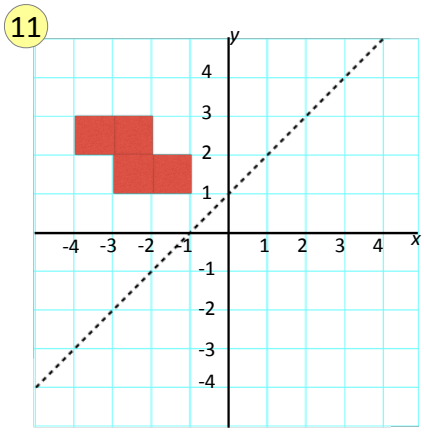
Reflect these shapes in the given mirror line.

10



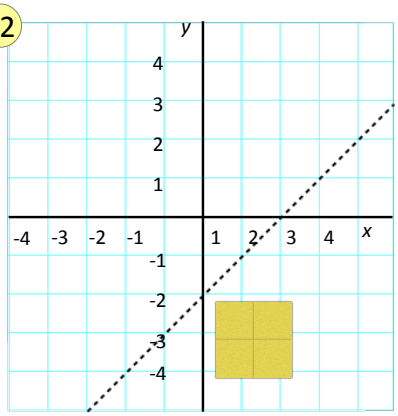
Question 10

11



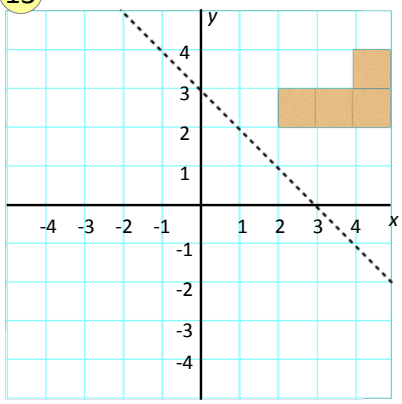
Question 11

12



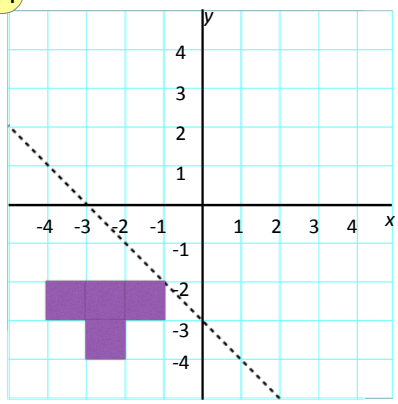
Question 12

13



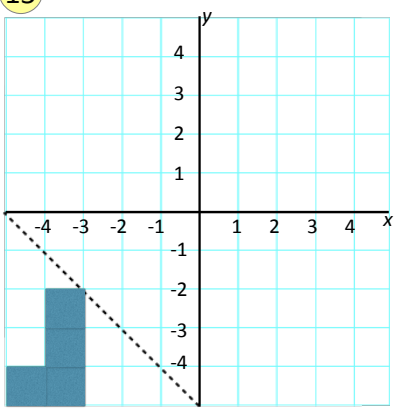
Question 13

14



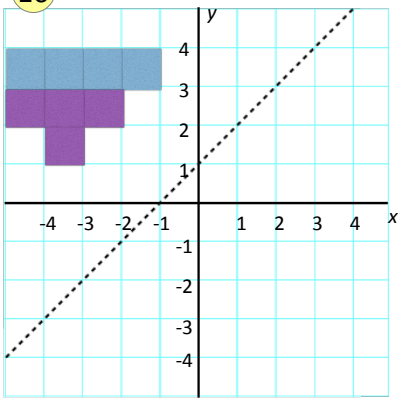
Question 14

15



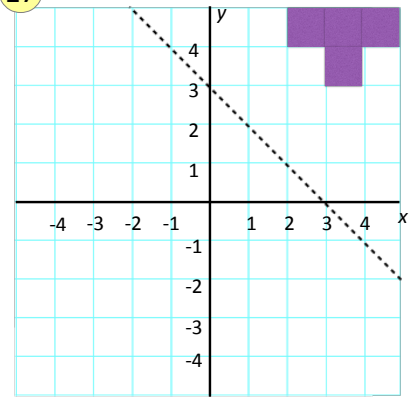
Question 15

16



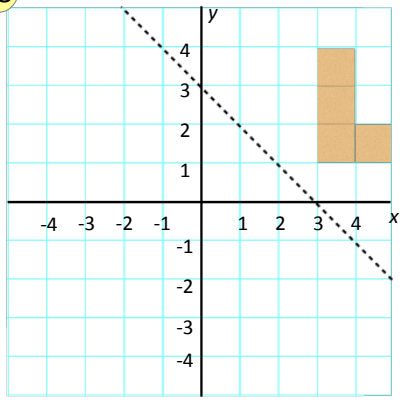
Question 16

17



Question 17

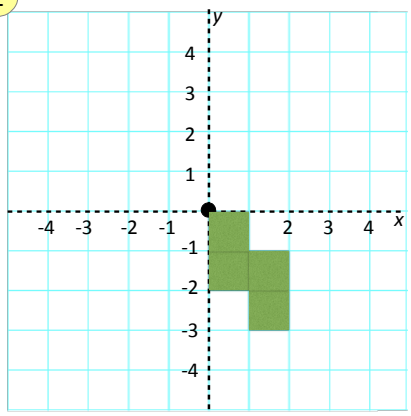
18



Question 18

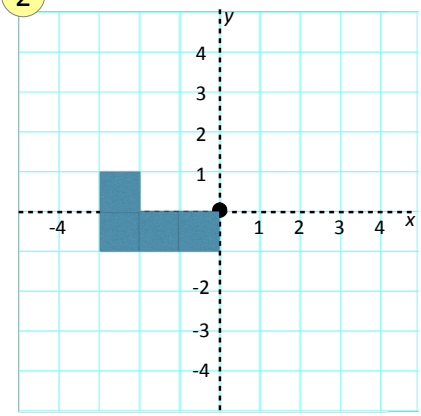
Worksheet Transformations_3b_6 Rotations

1



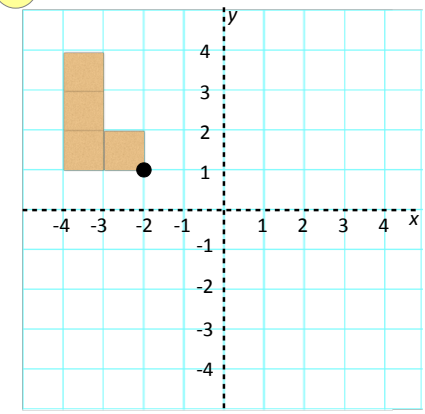
Rotate the shape 90° anticlockwise through the origin (centre dot)

2



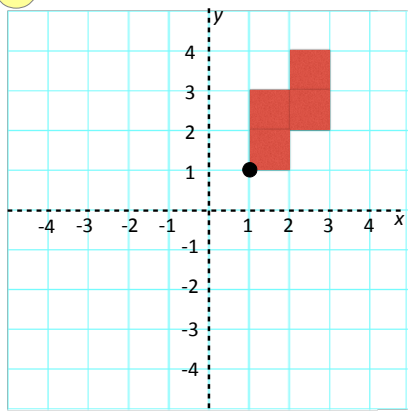
Rotate the shape 180° anticlockwise, through the origin (centre dot)

3



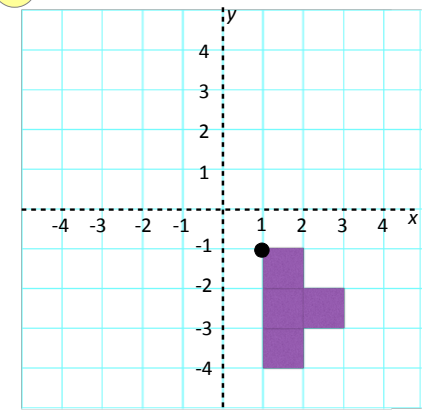
Rotate the shape 90° clockwise, about the centre

4



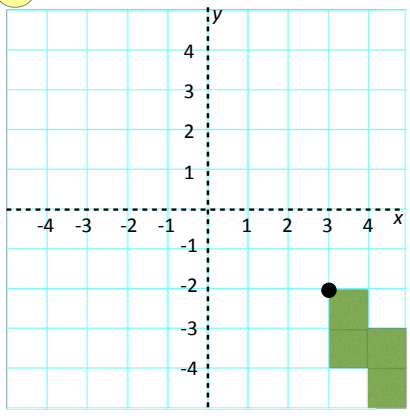
Rotate the shape 180° clockwise, about the centre

5



Rotate the shape 90° anticlockwise about the centre

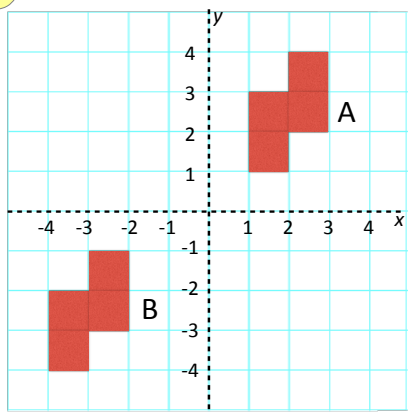
6



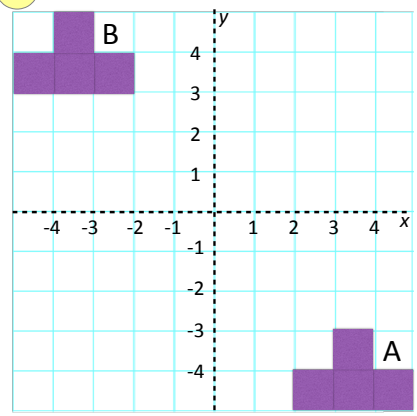
Rotate the shape 180° anticlockwise, about the centre

Worksheet Transformations_3b_7

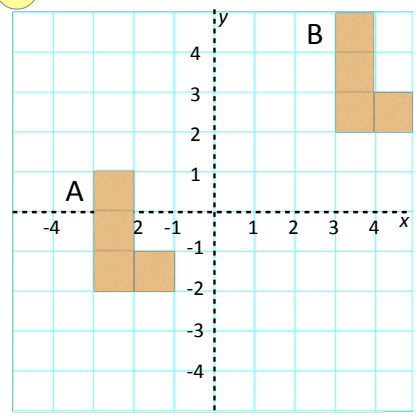
1



2

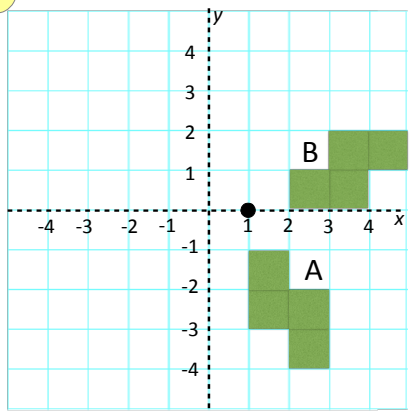


3

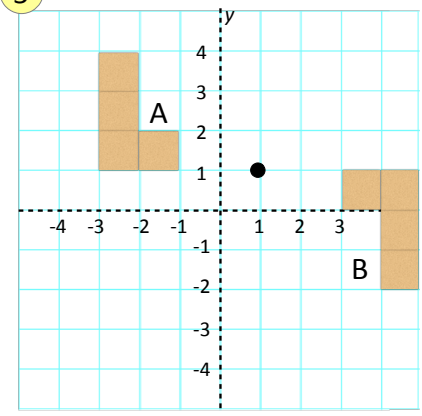


Describe fully the transformation that moves A to B

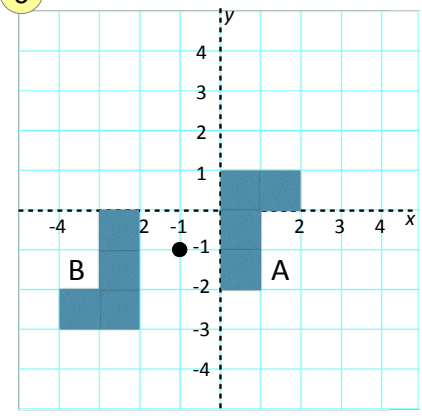
4



5

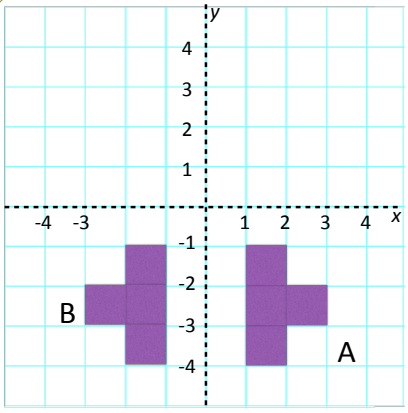


6

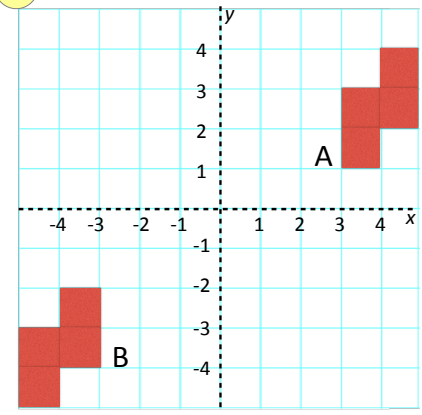


Describe fully the rotation that moves A to B

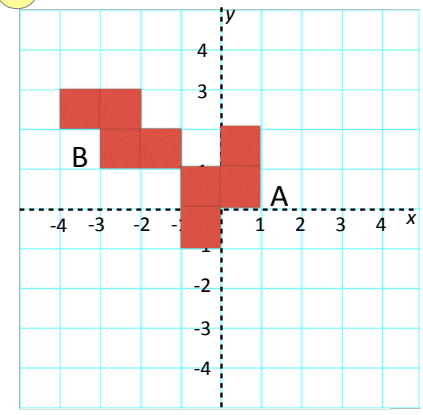
7



8



9

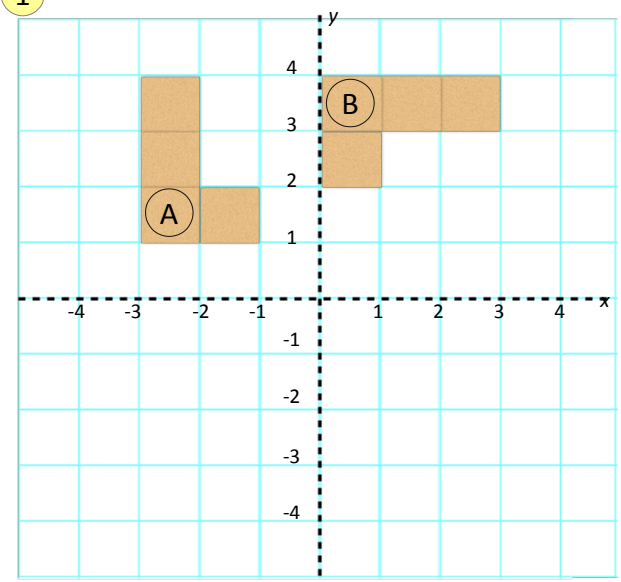


Describe fully the transformation that moves A to B

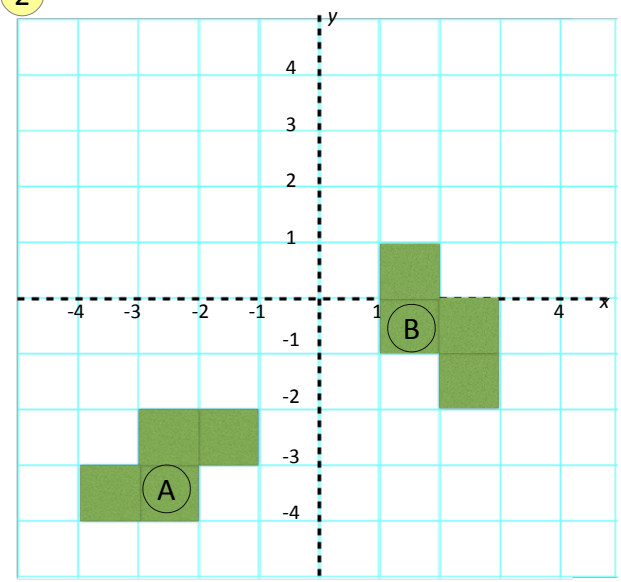
Worksheet Transformations_3b_8

In each you need to find the centre of rotation and describe the rotation fully that moves shape A to shape B.

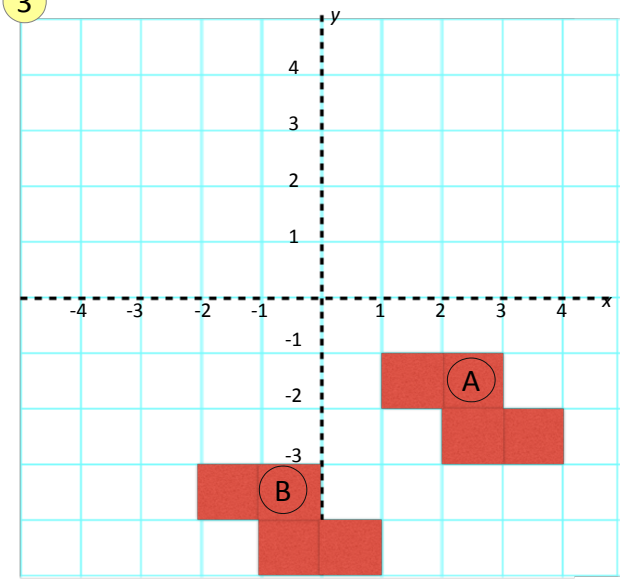
1



2



3



4

