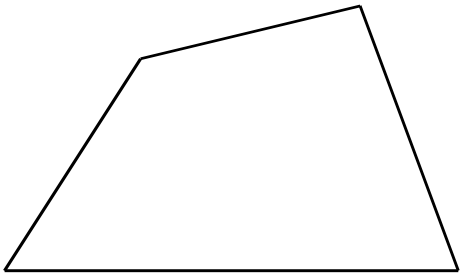
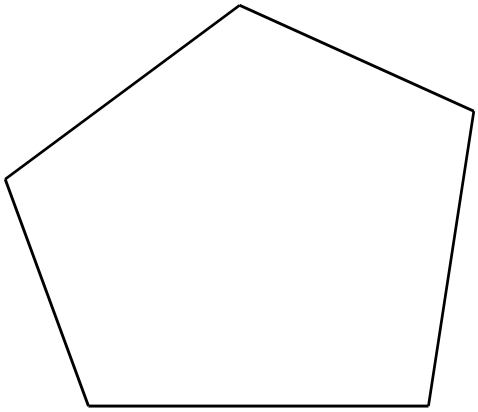
Let's think about the angle properties of a polygon?

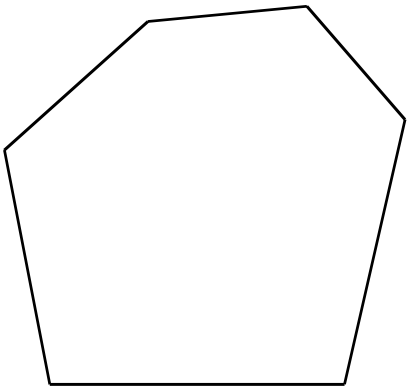
In each of the polygons below, think about different ways that can be used to find the sum of its angles.



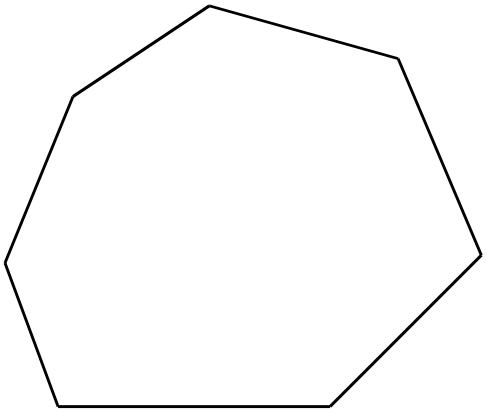
Quadrilateral



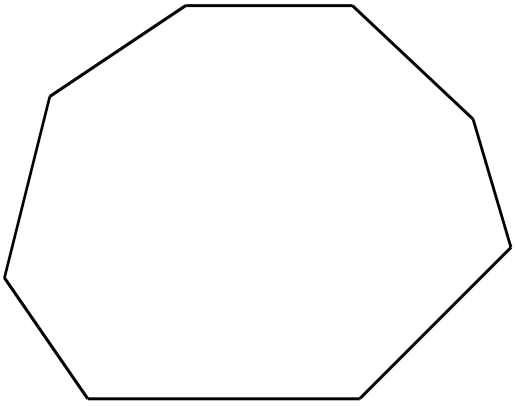
Pentagon



Hexagon



Septagon



Octagon

Polygons and angles

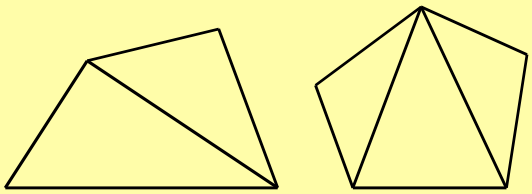
Interior and exterior angles of polygons



Jon

Jon's work

I divided the polygons into several triangles with diagonals from one vertex.



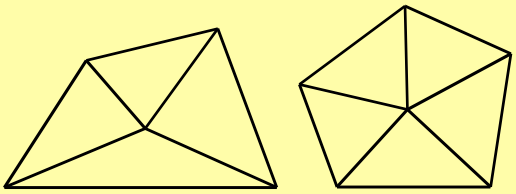
	Quadrilateral	Pentagon	Hexagon	Septagon	Octagon	...
Number of triangles	2	3				...
Calculation for the sum of angles	2 x 180°	3 x 180°				...



Alice

Alice's work

I divided the polygons into several triangles by connecting each vertex to a point inside the polygon.



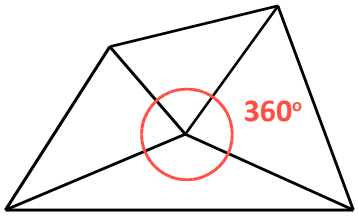
	Quadrilateral	Pentagon	Hexagon	Septagon	Octagon	...
Number of triangles	4	5				...
Calculation for the sum of angles	4 x 180° - 360°	5 x 180° - 360°				...

1 Why did Alice subtract 360°?



Matthew

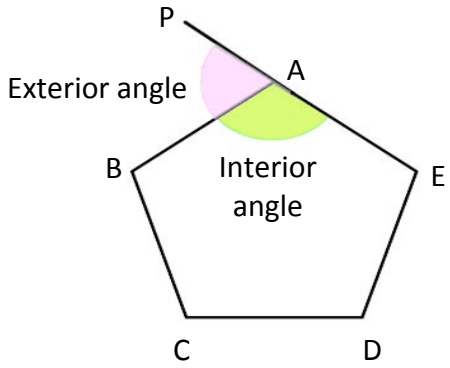
Alice created extra angles around her point inside the polygon.
She then had to subtract these extra angles.



Polygons and angles

When labelling a polygon, use its vertices in order as shown in this pentagon ABCDE.

As shown in the diagram, if we extend the line segment EA to a point P, then ∠BAP is called an **exterior angle**. Angles such as ∠BAE and ∠AED are called **interior angles**.



2 Write an algebraic expression to calculate the sum of the interior angles in an *n*-sided polygon based on Jon's idea.


Evie

Evie's explanation

The number of triangles is always 2 less than the number of sides. So if there are n sides, there are n − 2 triangles. So the sum of the interior angles would be (n − 2) x 180 or 180 x (n -2)

3 Write an algebraic expression to calculate the sum of the interior angles in an *n*-sided polygon based on Alice's idea.


Sarah

Sarah's explanation

If there are n sides, there are n triangles in Alice's method. So the sum of the interior angles would be n x 180° - 360° 180n − 360.

4 Can you explain why these two expressions are equivalent?.

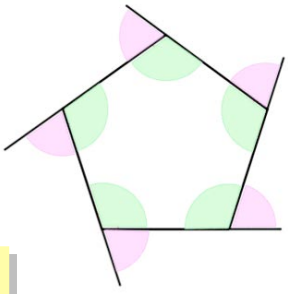
Fitness check

Calculate the sum of the interior angles for a polygon with

a) 4 sides b) 5 sides c) 6 sides d) 7 sides e) 8 sides



5 Can you work out the sum of the exterior angles for a polygon? Starting with a pentagon might help.



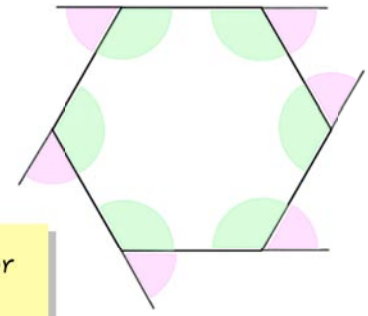


Matthew

Matthew's explanation

At any vertex the sum of the interior and exterior angles is 180°. As we have 5 vertices, the total sum of the interior and exterior angles is 5 x 180° = 900°.
But the interior angles add up to 3 x 180° = 540°.
So the exterior angles add up to 900° – 540° = 360°.

6 Can you work out the sum of the exterior angles for a hexagon in the same way?.





Amber

Amber's explanation

As we have 6 vertices, the total sum of the interior and exterior angles is 6 x 180° = 1800°.
But the interior angles add up to 4 x 180° = 720°.
So the exterior angles add up to 1080° – 720° = 360°.
Which is the same a pentagon.

7 Can you explain why the exterior angles of a polygon will always add up to 360°.



Sarah

Sarah's explanation

If there are n sides, the sum of the interior and exterior angles will be n x 180° = 180n°.
The sum of the interior angles is 180n – 360.
The difference between these is 360°, so must be the sum of the exterior angles.

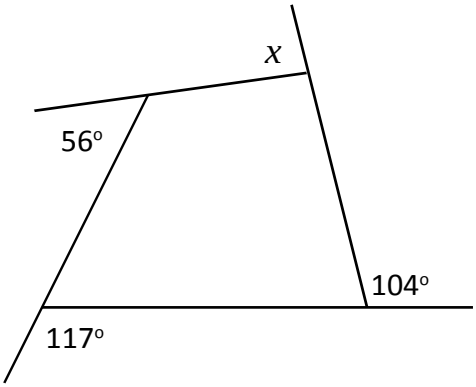
Two important results to remember

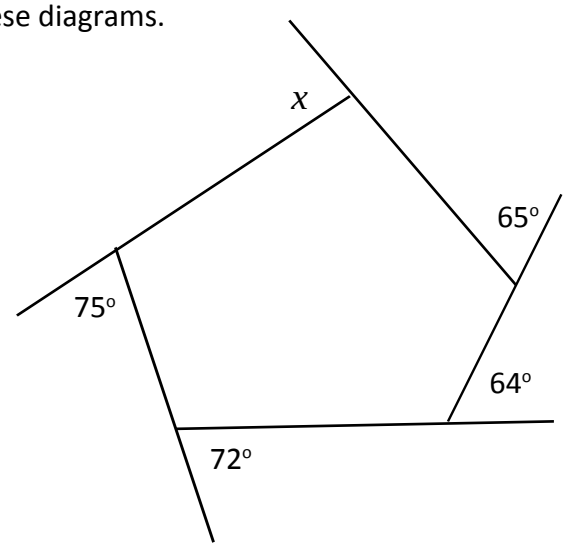
The sum of the interior angles of a n-sided polygon is 180° x (n – 2).
The sum of the exterior angles of a polygon is 360°.

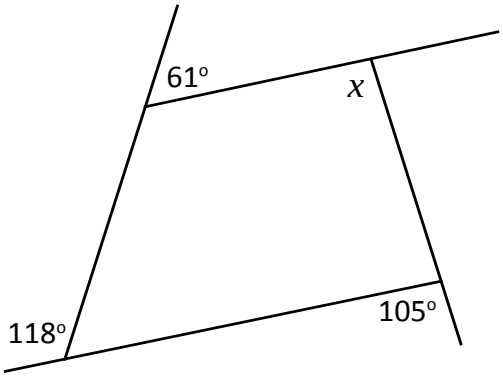
Polygons and angles

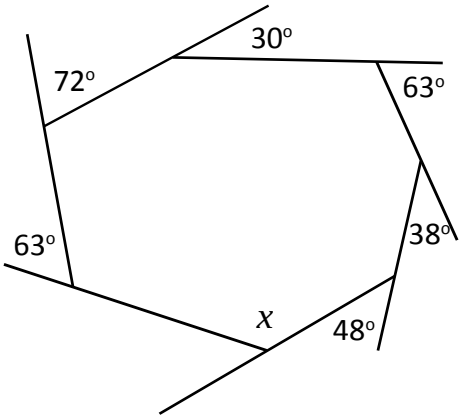
Exercise

1. Find the sum of the interior angles of a decagon (10-sided polygon)
2. Find the sum of the interior angles of a dodecagon (12-sided polygon)
3. Find the sum of the interior angles of a icosagon (20-sided polygon)
4. Find which the measure on an interior angle of a regular nonagon (9-sided polygon)
5. Find which polygon has a sum of interior angles equal to 2520°.
6. Find which regular polygon has an exterior angle equal to 15°.
7. Find which regular polygon has an interior angle equal to 160°.
8. By calculation, find the measure of $\angle x$ in each of these diagrams.









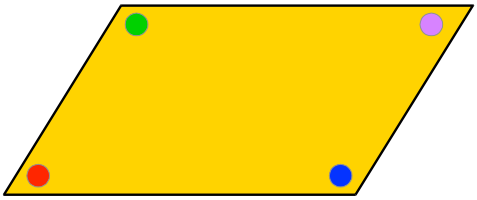
Question Creation



Question creation – Devise some missing angle problems for you and your friends to solve. Who can set the most challenging question?

Parallel lines and angles

Parallelograms



8 What do you know about parallelograms?.



Evie

It's a quadrilateral with two pairs of parallel sides.

Opposite sides are equal

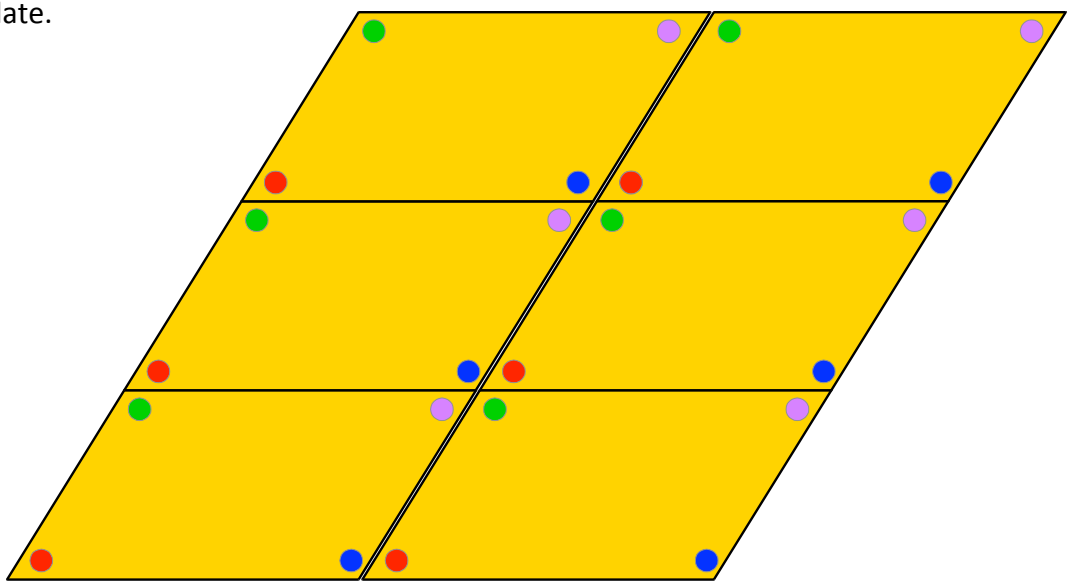


Sarah



Jon

As it's a quadrilateral it must tessellate.



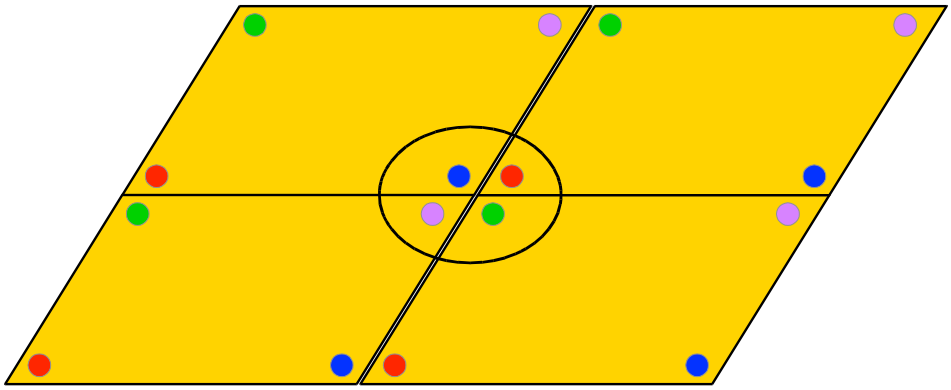


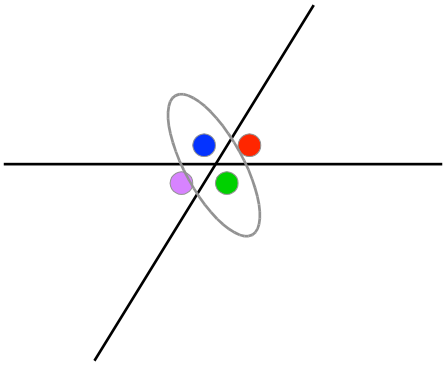
Matthew

And that helps you remember about the angles in a parallelogram.

9 What do you know about the angles in a parallelogram?

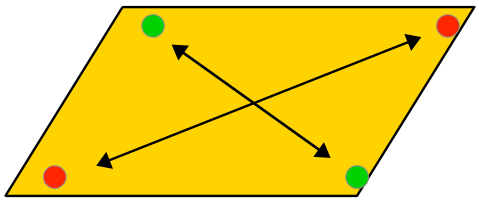
Angles in a parallelogram



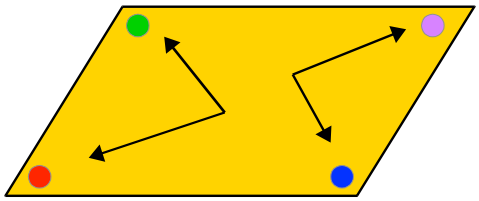


The green and blue angles are the equal as they are vertically opposite angles.
Similarly the red and purple angles are also equal for the same reason.
Blue angle and red angles add up to 180° as they form a straight angle.
Which means green angle and red angle must also add up to 180°.

In a parallelogram opposite angles are equal...



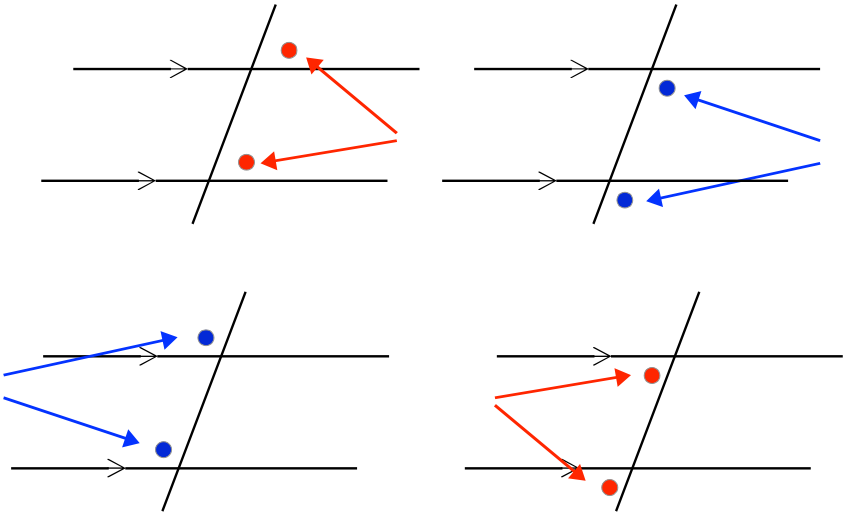
... and pairs of interior angles add up to 180°.
The correct terminology for this is the angles are **supplementary** (add up to 180°.



Angles and parallel lines

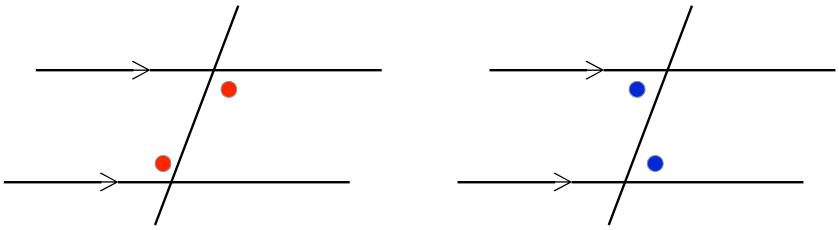
When a third line, known as a transversal, crosses two parallel lines, 4 pairs of **corresponding angles** are formed.

Corresponding angles

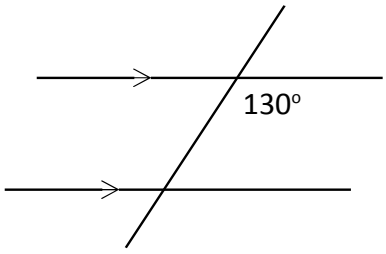


Sometimes it is helpful to think about equal interior angles that are on opposite sides of the transversal. These are known as **alternate interior angles** or just alternate angles for short.

Alternate angles



If you know one of the eight angles formed, you actually know all of them.

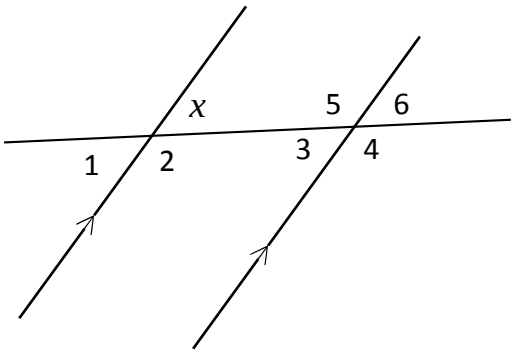


10 Find the 7 missing angles in the diagram above.

Exercise

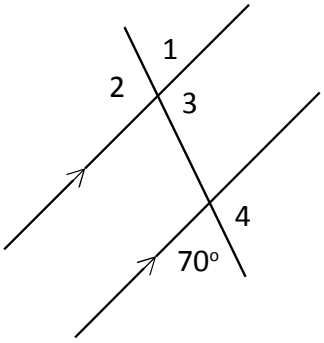
In the diagram which numbered angle is

1. alternate to x
2. corresponding to x
3. vertically opposite to x
4. supplementary to x

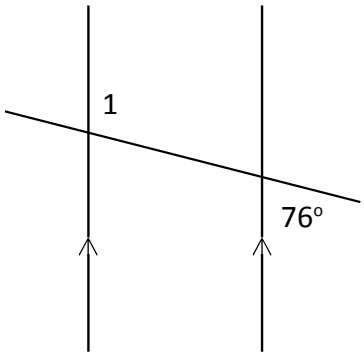


In each of these diagrams determine the size of the numbered angles

1

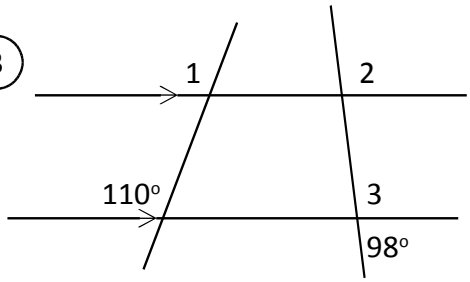


2

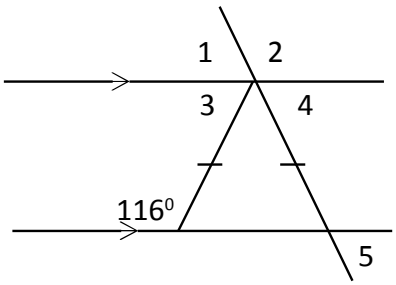


1

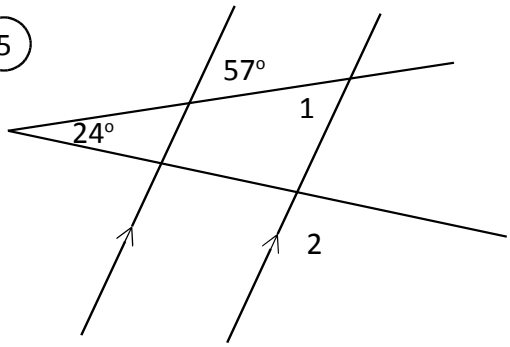
3



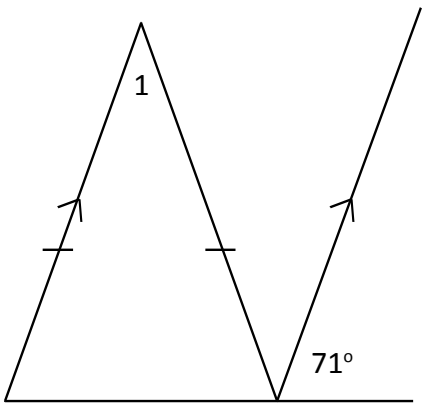
4



5



6



What do you know about constructions?

Using only a pencil, ruler and compass construct a triangle with side lengths 6 cm, 8 cm and 10 cm.

Compare it with your partner's triangle .

Are they exactly the same?

Using only a pencil, ruler and compass construct an isosceles triangle with one side length 6 cm and another 8 cm.

Compare it with your partner's triangle .

Are they exactly the same?

Construct an equilateral triangle with side length 7cm.

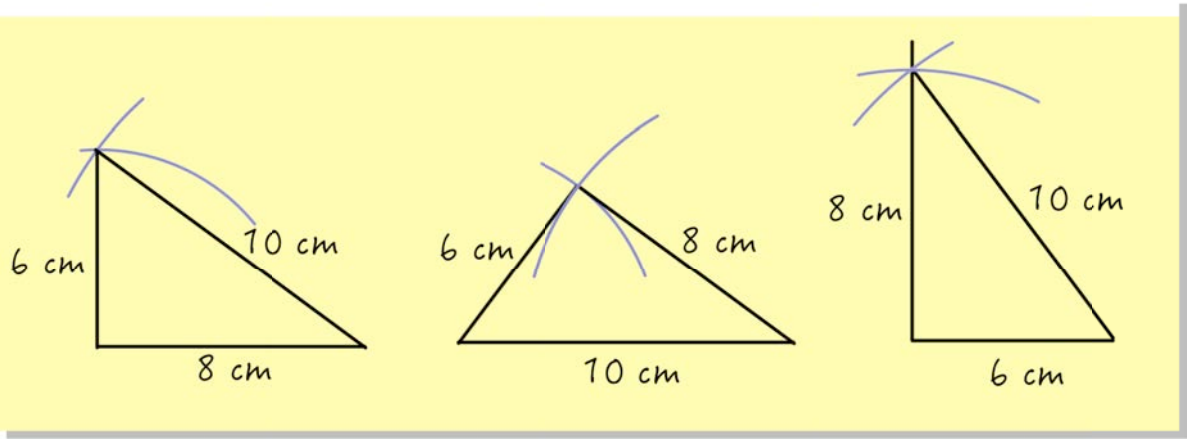
What can you say about the angles within the triangle?

Can you construct a congruent triangle to a given triangle –

without measuring any of the sides or angles?

Congruent shapes

When two shapes can fit on top of each other perfectly, we say they are congruent.
On the previous page, you were asked to construct a triangle with sides 6 cm, 8 cm and 10 cm.
As you can see below, the triangle could have been constructed in three different ways.



11 Are these triangles congruent?


Jon

I actually constructed all three triangles and then cut them out. The fit exactly and so they are congruent.

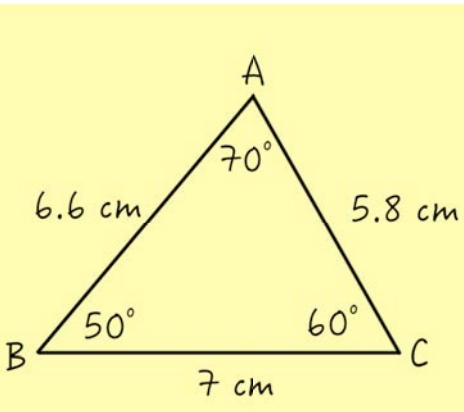
And that means not only are corresponding sides equal but so are corresponding angles!

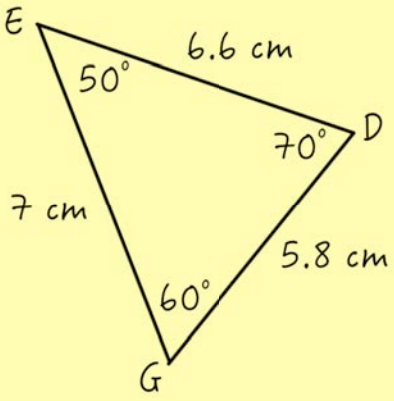

Alice

12 What does Alice mean by corresponding sides and angles?


Alice

Alice's explanation

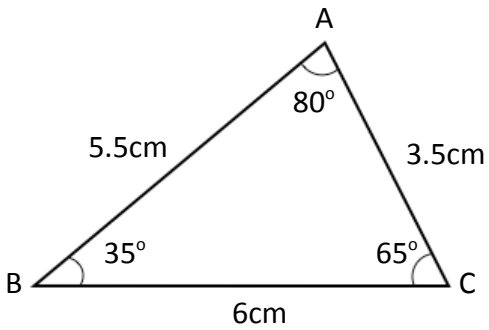




You have to match vertices, so D would fit on A, E would fit on B and G would fit on C. You can then see both sides and angles match up.
A good way of writing this would be to write triangles ABC / DEF are congruent.

Think about how to draw triangle DEF so it will be congruent to triangle ABC.

Do you need all of this information to ensure your triangles are congruent?

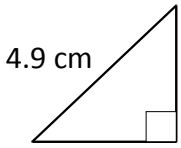


13 What information do you need to draw a congruent triangle?

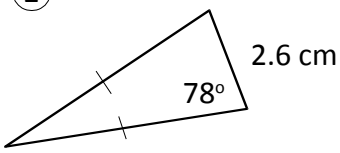
Power Builder

In each of these triangles decide if you have enough information to draw a congruent triangle.

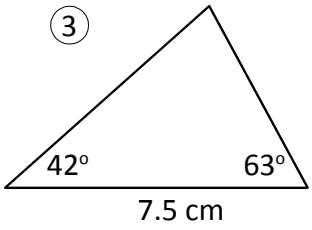
1



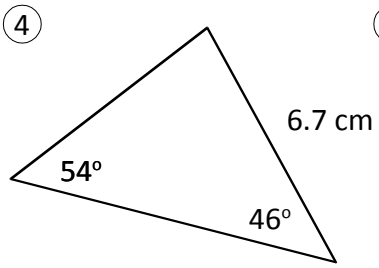
2



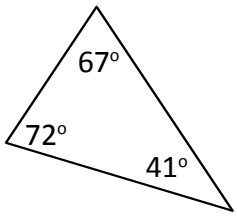
3



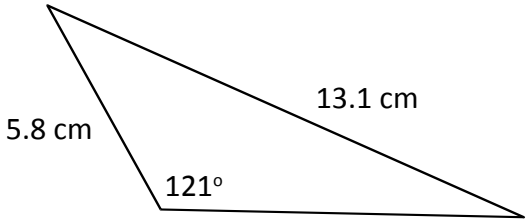
4



5



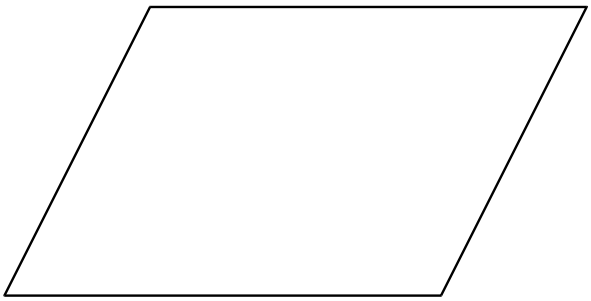
6



14 Does a diagonal split a parallelogram into two congruent triangles?

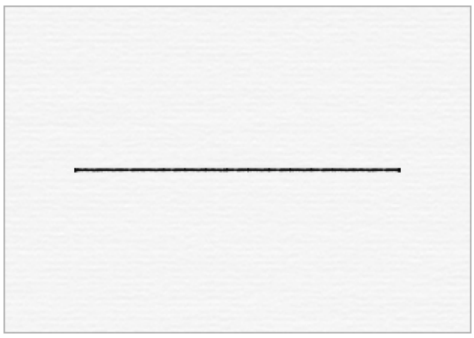


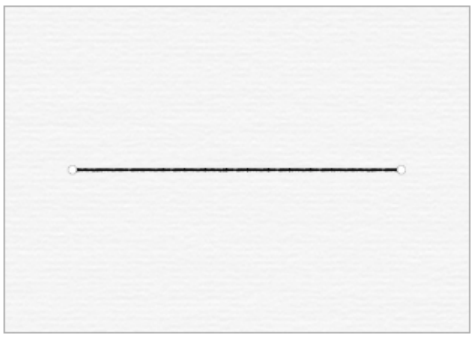
Evie



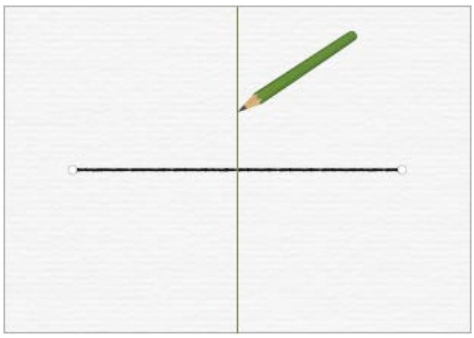
Bisecting a line by folding.

1. Draw a line on a piece of paper
2. Punch a small hole at each end of the line
3. Fold the paper over so that the holes line up with each other.
4. Open up the page and then draw a line on the crease.



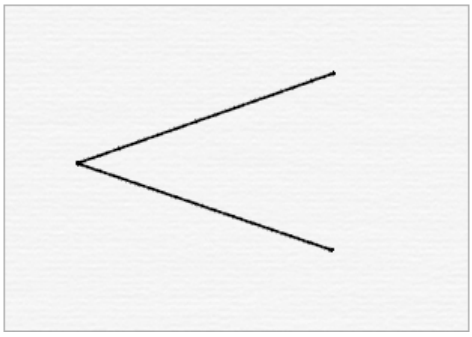


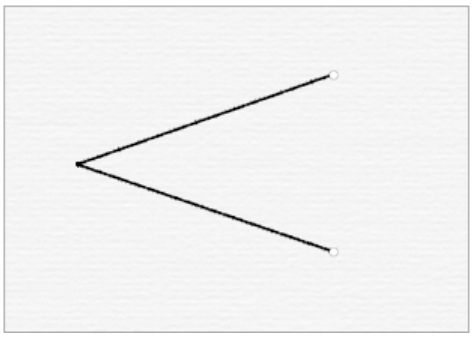




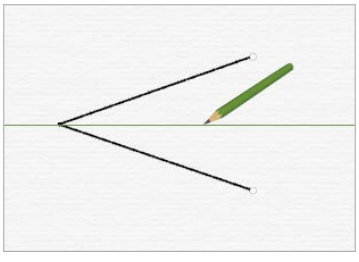
Bisecting an angle.

1. Draw your angle on a piece of paper using lines of equal length
2. Punch a small hole at each end of the line, and where they join.
3. Fold the paper over so that the two holes at the end of the lines line up with each other.
4. Open up the page and then draw a line on the crease.







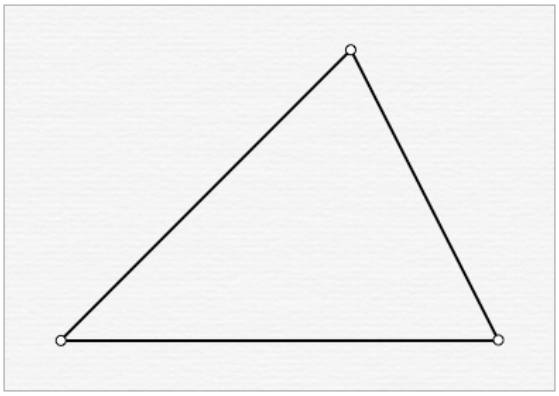


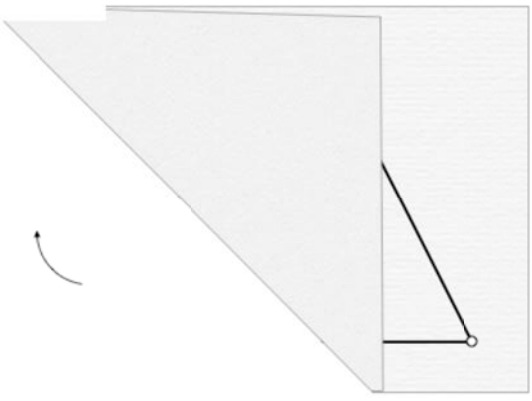
Bisecting the sides of a triangle

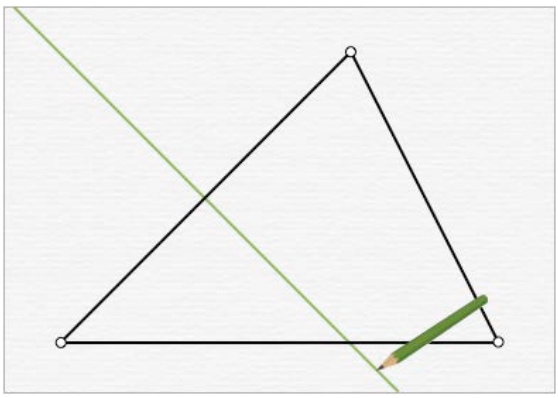


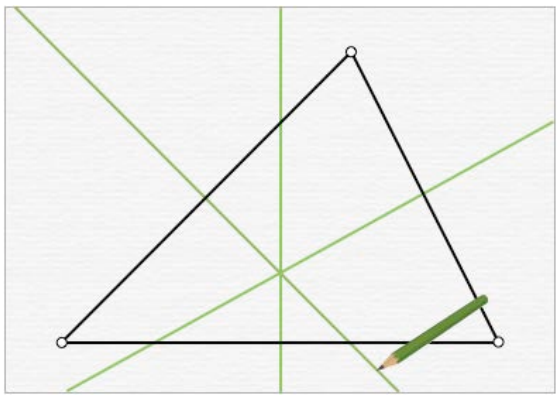
1. Start with an A4 sheet of paper
2. Place it on your desk, in the portrait position.
3. Fold the bottom edge to the top edge and crease.
4. Cut the paper in half along the fold.
5. Take one of the halves and position in the landscape position.
6. Roughly, in the centre of the paper, construct a scalene triangle with sides 10cm, 11cm and 8cm.
7. At each vertex punch a small hole.
8. Now bisect each side by folding in half and by matching the holes.







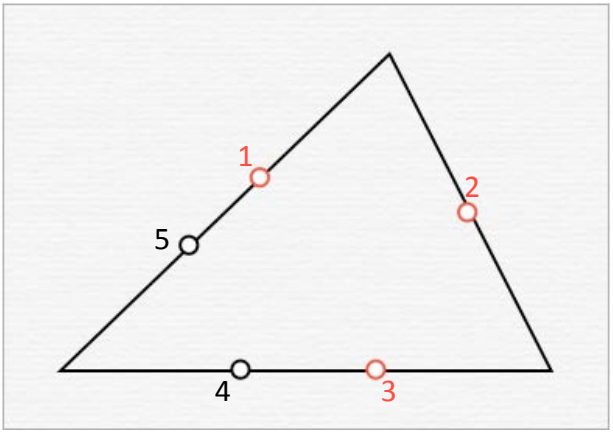




Bisecting the angles in a triangle

Now use the other half of your A4 paper
Position it in the the landscape position.

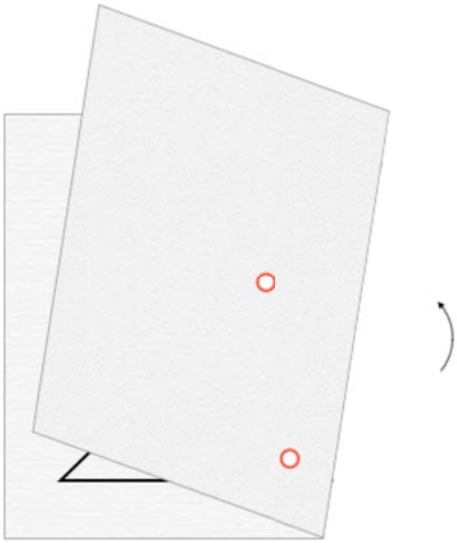
Just as last time, roughly in the centre of the paper, construct a scalene triangle with sides 10cm, 11cm and 8cm.

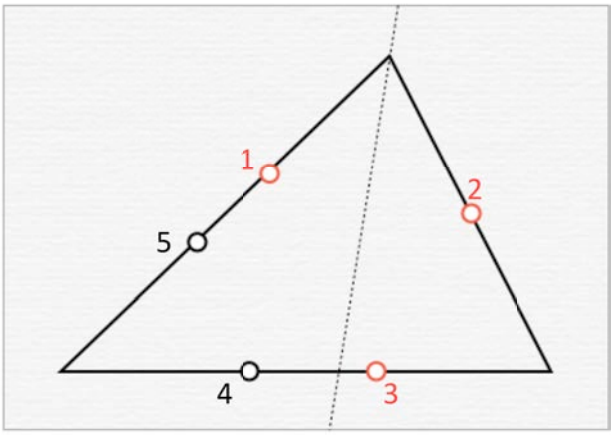


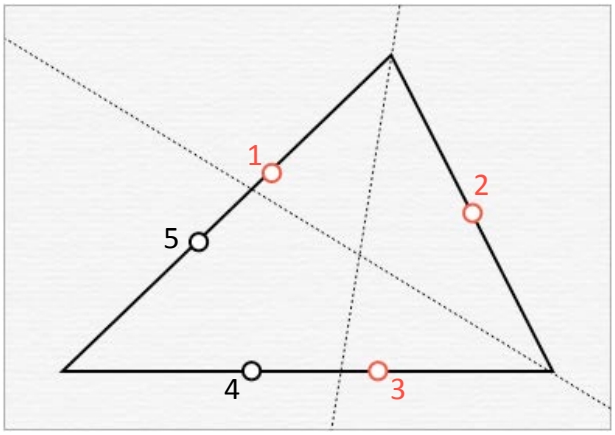
On each side punch a hole 4 cm from each vertex. This means you will have 2 holes on the 11 cm and 10 cm sides but only one on the eight cm side.

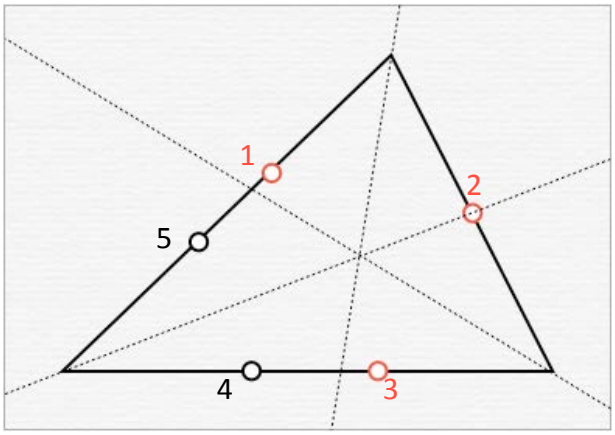
Now bisect each angle by matching up the appropriate holes and crease the paper. For example match hole 1 and 2, then crease the paper.
Now unfold and match holes 2 and 3, crease and unfold.
Now match holes 4 and 5, crease and unfold.

The three angle bisectors should meet at a point!





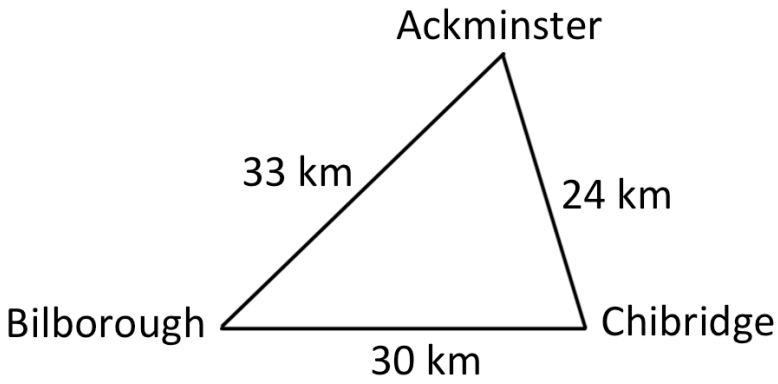




Two classic problems

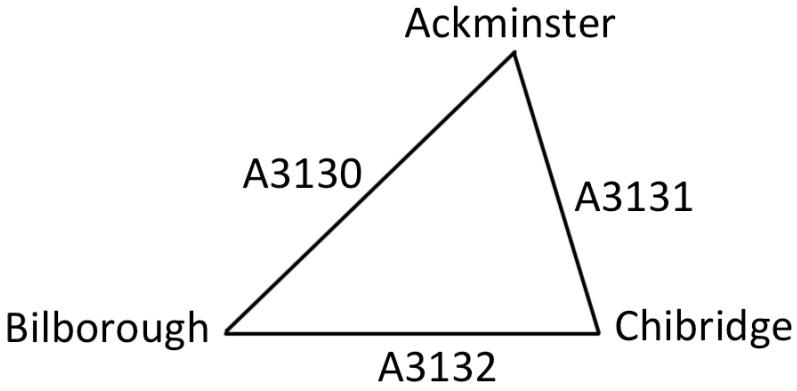
The Hospital Problem

Three neighbouring towns, Ackminster, Bilborough and Chibridge decided to combine resources and build a local hospital. Each town, however, wanted an assurance that it would be equidistant from each of the three town centres. Based on the map below, sketch your proposed location for the new hospital.



The Megastore Problem

At the next joint town meeting, the councils agreed that they would also build a new megastore to serve all three towns. They insisted that it had to be an equal distance from the main roads joining the towns. They gave planning permission to construct three new roads but they all had to be of equal length. On a new diagram, sketch your proposed location of the new Megastore.



15

What is the same and what is different about these two problems?

Classic Constructions

The problems on the previous page can be solved by classic compass and straight edge constructions. This type of problem has fascinated mathematicians for over 2500 years.
By a straight edge, we mean an uncalibrated ruler which can only be used for drawing straight lines not measuring.
Of course, in the two previous problems you do need to start from an accurate representation of the problem

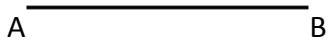
In this section, you will study the seven basic compass and straight edge constructions.

In each case, try to work out how you might do it for yourself before discussing your solutions with the rest of the group. Remember you can not use your ruler to measure length, only for drawing straight lines.

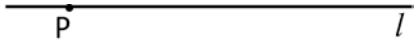
Construction 1 Copy a line segment

Given the line segment AB, a line l and a point P, locate a point Q on l such that PQ = AB

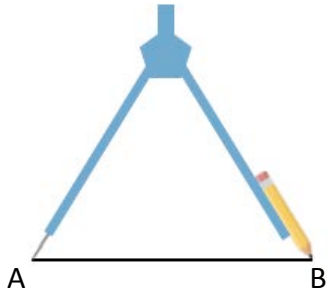
(a)



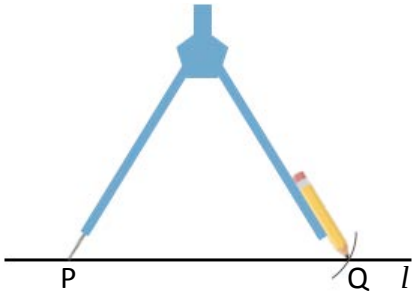
(b)



(c)



(d)



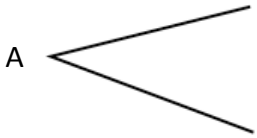
Procedure

1. Open the compass to length AB
2. With the point of the compass at mark P, mark off an arc of radius AB that intersects line l. Let Q be the point of intersection of the arc and line. Then PQ = AB

Construction 2 Copy an angle

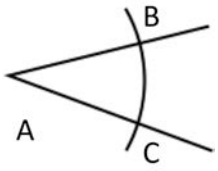
Given an ∠A and a line segment PQ, locate a point S such that ∠SPQ = ∠A.

(1)

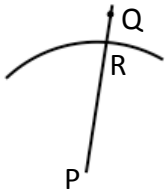




(2a)

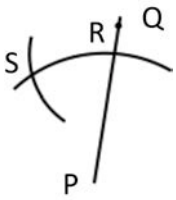


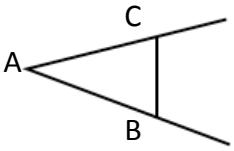
(2b)

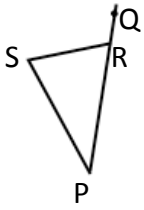


Procedure

1. With the point of the compass at A, construct an arc that intersects with each side of angle A. Call the intersection points B and C.
2. With the compass open to the radius AB, place the point at P and construct an arc of radius AB that intersects line segment PQ. Call the point of intersection point R.
3. Place the point of the compass at C and open the compass to radius CB. Then, with the same compass opening, put the point of the compass at R and construct an arc of radius CB intersecting the first arc. Call the point of intersection of the two arcs point S.
4. Construct a line segment PS. Then ∠BAC = ∠SPR.







Construction 3 Construct the perpendicular bisector of a line segment.

Given a line segment AB, construct a line l so that l is perpendicular to AB and intersects AB at the mid point of AB.

Procedure

1. Place the compass point at A and open the compass to a radius r such that r > $\frac{1}{2}$AB; that is r is more than one-half the distance AB. Construct arcs of radius r on each side of AB.

(a)



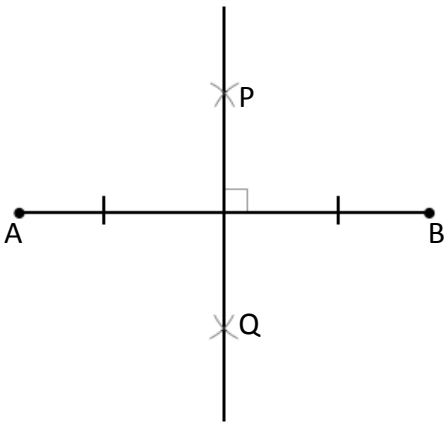
(b)



2. With the same compass opening, r, place the point of the compass at B and swing arcs of radius r on each side of AB

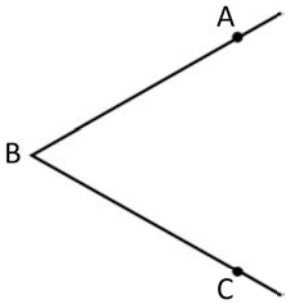
3. Let P be the intersection of the arcs on one side of AB, and let Q be the intersection of the arcs on the other side of AB. Construct line PQ, then line PQ is the perpendicular bisector of line segment AB.

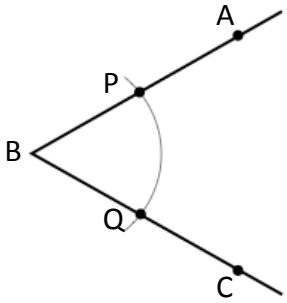
(c)



Construction 4 Bisect an angle

Given an angle ∠ABC, construct a line BR such that ∠ABR = ∠CBR,

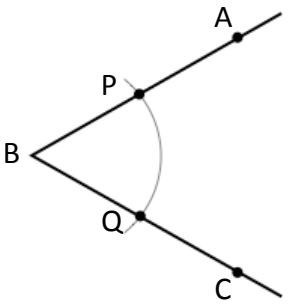




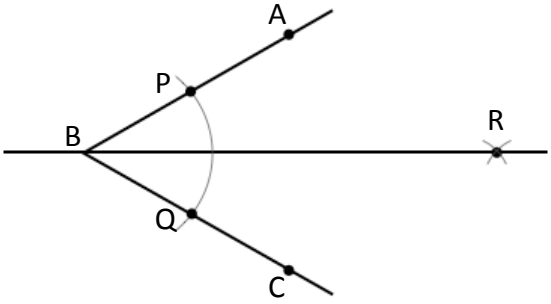
1. Put the compass point at B and construct an arc that intersects sides BA and BC. Label the points of intersection P and Q, respectively

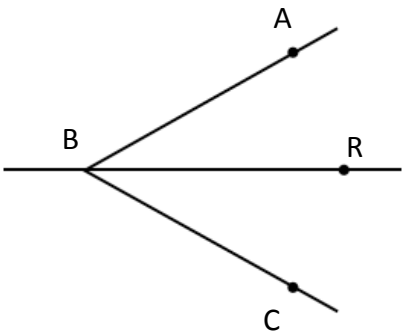
2. Put the compass point at P and then at Q, and construct intersecting arcs of the same radius. Label the intersection of the arcs R.

3. Construct line BR, the angle bisector of angle ABC.



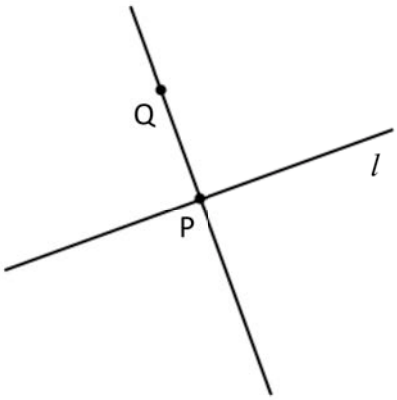


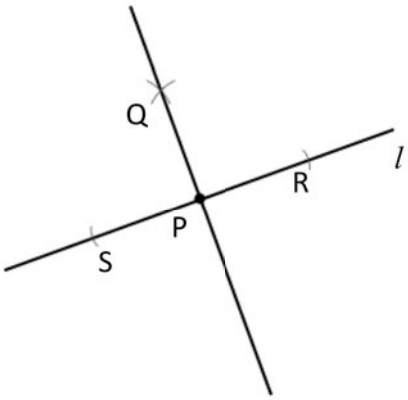




Construction 5 Construct a line perpendicular to a given line through a specified point on the line.

Given a point P on l construct QP such that QP is perpendicular to l.



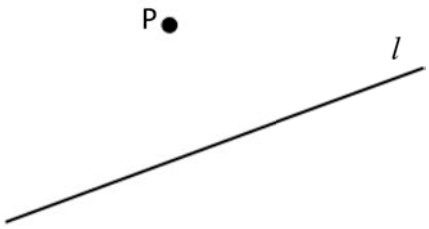


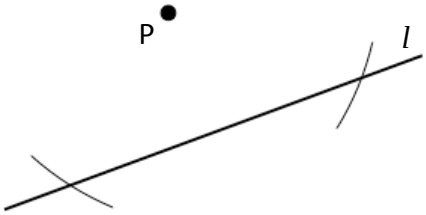
1. Place the point of the compass at P and construct arcs of a circle that intersect line l at points S and R

2. Bisect the straight angle SPR using construction 4. Let point Q be the intersection of the arcs constructed in step 2 of construction 4.

3. The line, QP, is perpendicular to line l.

Construction 6 Construct a line perpendicular to a given line through a specific point not on a line.
(This is also a construction of the shortest distance from a point to a given line.)

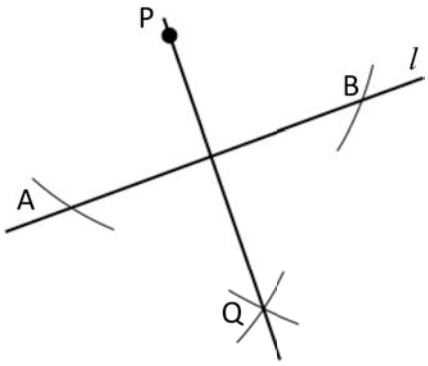


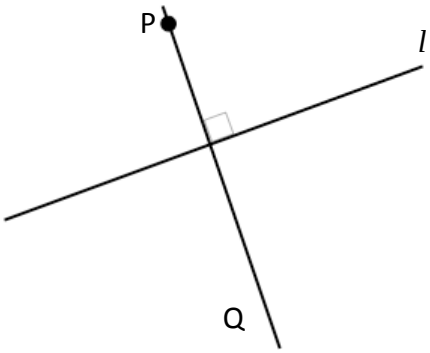


1. Place the point of the compass at P and construct an arc that intersects line l in two points. Label the points of intersection A and B.

2. With the same compass opening as in step 1, place the point of the compass at A, and then at B, to construct intersecting arcs on the side of l that does not contain P. Label the point of intersection of the arcs point Q.

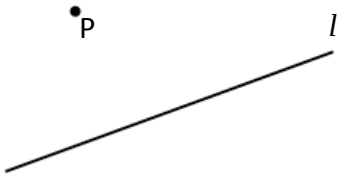
3. Construct line PQ. The PQ contains P and is perpendicular to l.

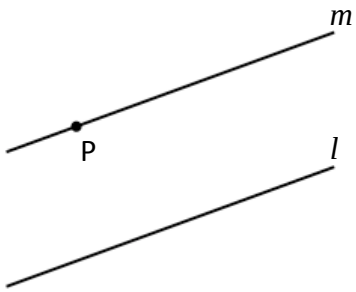




Construction 7 Construct a given line through a specific point not on the line.

Given a line l and a point P, not on l, construct a line m such that m is parallel to l and P lies on m.



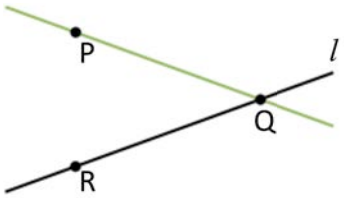


1. Let Q be any point on L and construct line PQ.

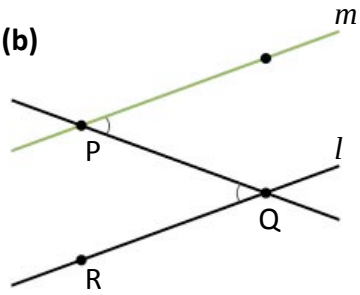
2. Let R be another point on l.

Copy angle PQR using construction 2 so that the vertex is at P, one side is on line QP and the other is on line m.

(a)



(b)



Classic Constructions

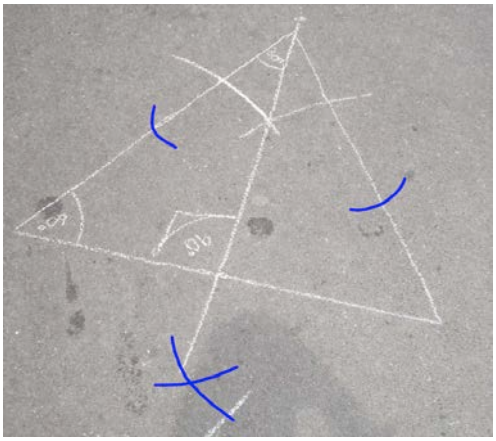
The Great Construction Challenge

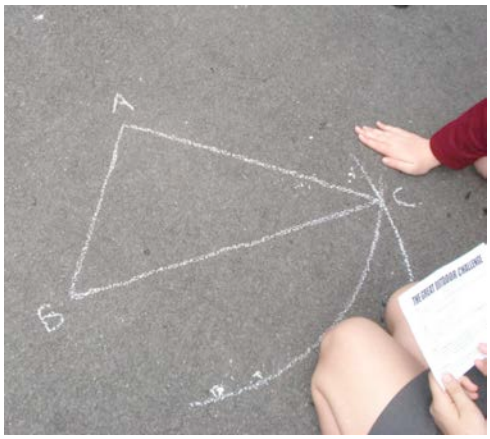
Using only a pencil, straight edge and a compass, can you construct the following?

1. A sixty degree angle
2. A thirty degree angle
3. A right angle
4. A square
5. A rectangle with one side twice the length of another
6. A triangle with sides 3 units, 4 units and 5 units.
7. An isosceles trapezium
8. A parallelogram without a right angle.

As an alternative, this can be presented as “The Great Outdoor Challenge”.

Using only a piece of chalk and a length of string, can you construct the list above?





Geometry 4a: Mastery Questions

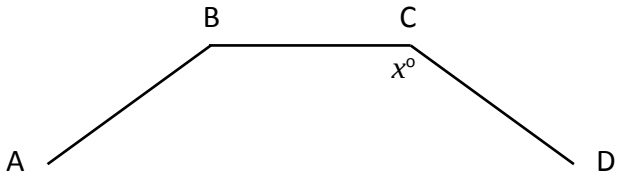
1. What is the size of the the exterior angle in a regular octagon?

A	45°	B	140°	C	135°	D	40°

2. What is the total of all the interior angles in a regular pentagon?

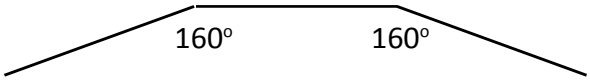
A	360	B	540	C	720	D	900

3. ABCD are 4 vertices of a 10-sided regular polygon. What is the size of the missing angle?



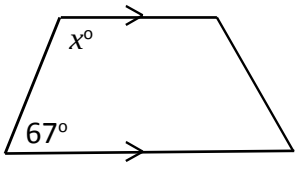
A	36°	B	78°	C	108°	D	144°

4. This diagram shows 3 sides of a regular polygon.How many sides does it have?



A	16	B	18	C	20	D	24

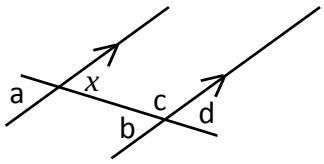
5. What is the size of the missing angle?



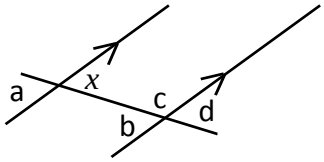
A	67°	B	113°	C	134°	D	56°

Geometry 4a: Mastery Questions

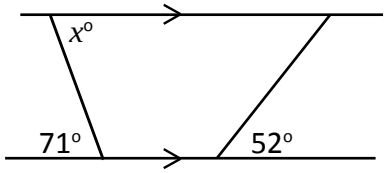
6. Which is the corresponding angle to x?



7. Which is the alternative angle to x?

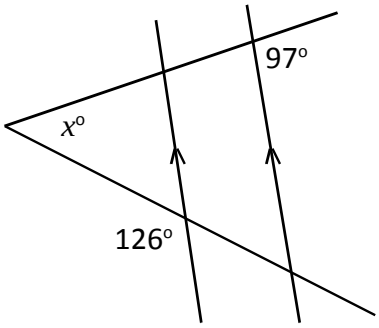


8. What is the size of the the marked angle?



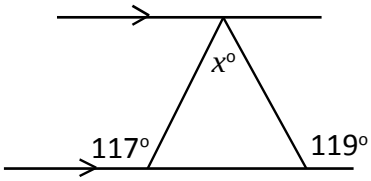
A	52°
B	71°
C	123°
D	57°

9. What is the size of the missing angle?



A	97°
B	54°
C	83°
D	43°

10. What is the size of the missing angle?



A	63°
B	61°
C	56°
D	46°

Geometry 4a: Mastery Questions

11. Which word means cut into 2 equal parts?

A	Intersect	B	Parallel
C	Bisect	D	Perpendicular

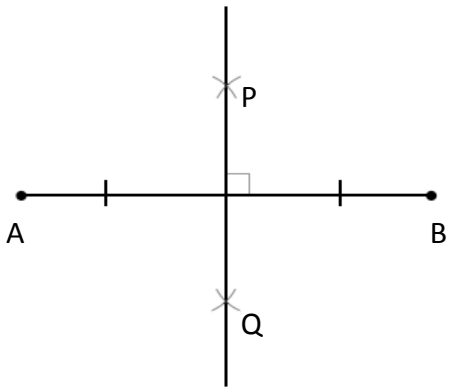
12. What would you do first to construct an angle of 45° using a pair of compasses?

A	Angle bisector	B	Construct an equilateral triangle
C	Use a protractor	D	Perpendicular bisector

13. Which of the following sets of 3 lengths could not be used to construct a triangle?

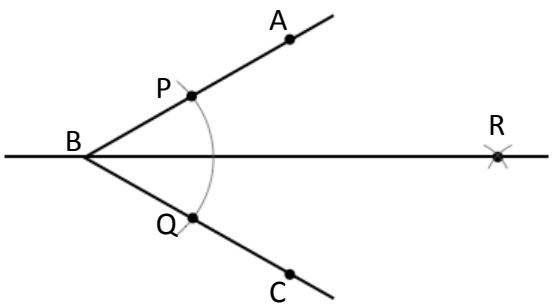
A	3cm, 3cm, 3cm	B	3cm, 6cm, 9cm
C	3cm, 12cm, 12cm	D	3cm, 4cm, 5cm

14. What construction is this an illustration of?



A	Angle bisector
B	Shortest distance from a point to a line
C	Perpendicular bisector
D	Centre of a circle

15. What construction is this an illustration of?



A	Angle bisector
B	Shortest distance from a point to a line
C	Perpendicular bisector
D	Centre of a circle