Let's think about the size of divided parts



Miss R

Do you ever have to divide something into equal parts?



Alice

My birthday cake

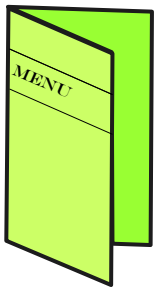


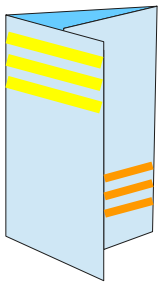
Pizza on Fridays



Evie









Jon

Making a menu card in Food Technology

My graphics project



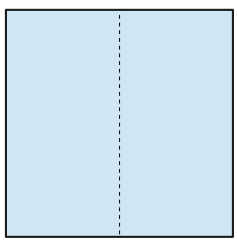
Matthew

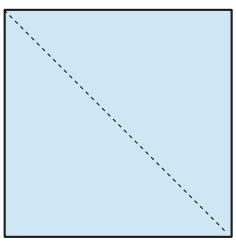


Miss R

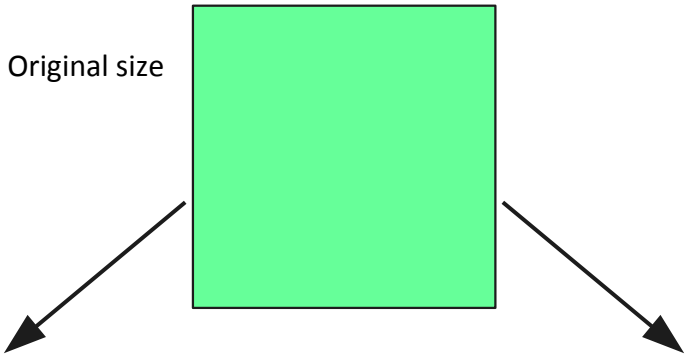
When something is divided into two equal parts, the size of one part is called one half of the original size and is written as $\frac{1}{2}$.

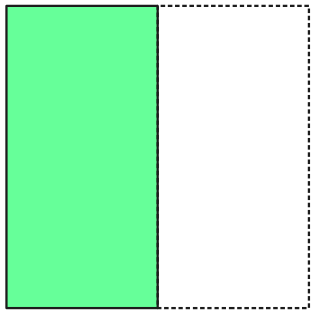
For example:



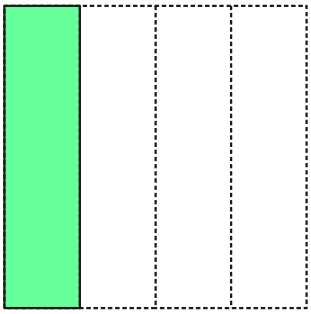


Let's think about fractions ...

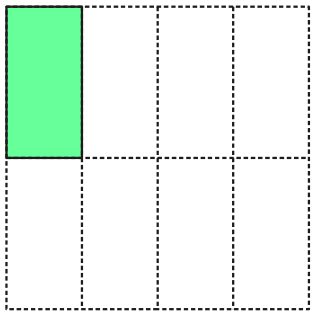




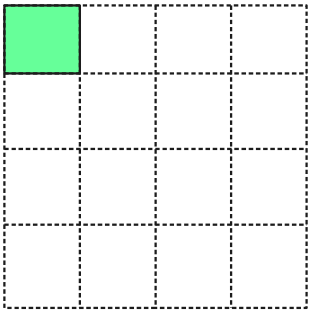
One half $\frac{1}{2}$



One ____ $\frac{1}{\square}$



One ____ $\frac{1}{\square}$



One ____ $\frac{1}{\square}$

Use your diagrams to:

1. Find the missing numbers in these equivalent fractions:

a) $\frac{1}{2} = \frac{?}{4} = \frac{?}{8} = \frac{?}{16}$ b) $\frac{1}{4} = \frac{?}{8} = \frac{?}{16}$ c) $\frac{3}{4} = \frac{?}{8} = \frac{?}{16}$

2. Place the fractions in order, smallest first:

a) $\frac{5}{8}, \frac{5}{16}, \frac{1}{4}, \frac{9}{16}, \frac{3}{8}$ b) $\frac{11}{16}, \frac{7}{8}, \frac{1}{2}, \frac{3}{4}, \frac{13}{16}$

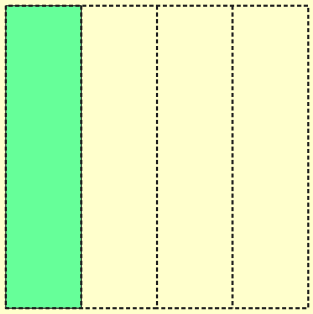
Sarah's working for 2(a)



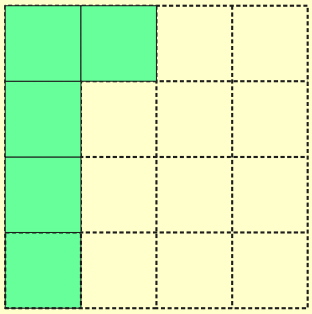
Sarah

It's easy if you use the drawings.

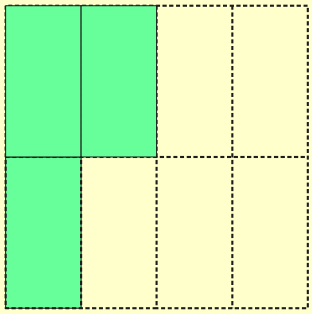
$\frac{1}{4}$



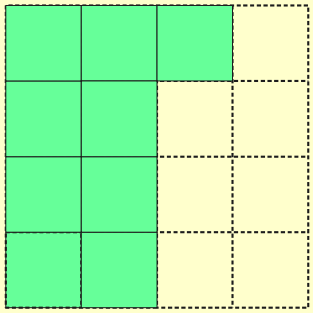
$\frac{5}{16}$



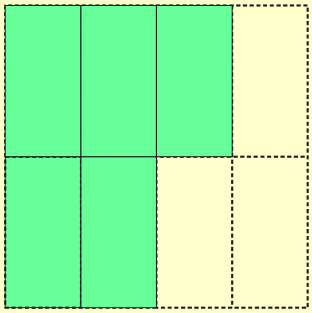
$\frac{3}{8}$



$\frac{9}{16}$



$\frac{5}{8}$



So the correct order is

$\frac{1}{4}, \frac{5}{16}, \frac{3}{8}, \frac{9}{16}, \frac{5}{8}$



Matthew

It's easy if you work in small squares.

1 Can you explain what Matthew means?

Alice's working for 2 (b)



Alice

$\frac{11}{16}, \frac{7}{8} = \frac{14}{16}, \frac{1}{2} = \frac{8}{16}, \frac{3}{4} = \frac{12}{16}, \frac{13}{16}$

So, in order smallest first

$\frac{1}{2}, \frac{11}{16}, \frac{3}{4}, \frac{13}{16}, \frac{7}{8}$

Fractions

A fraction is generally thought of as part of a whole but it Is probably better described as a collection of equal parts.

For example in the fraction, $\frac{3}{4}$, the denominator (that's the number on the bottom) tells you we have divided a whole into 4 equal parts. The numerator (the number on the top) tells you have many parts we are counting – in this case 3.

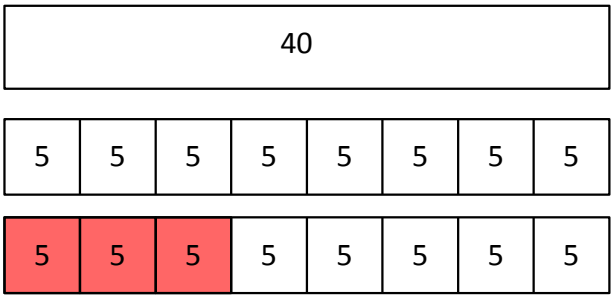
Amber's question



Amber

I have 40 sweets left in my sweet box. $\frac{3}{8}$ of them are red.

How many red sweets do I have?



Divide into 8 equal parts.

Three boxes will give you $\frac{3}{8}$.

So $\frac{3}{8}$ of 40 = 3 x 5 = 15

There are 15 red sweets.

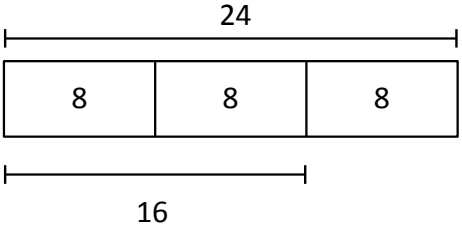
Evie's question



Evie

I bought 24 flowers for my mum. $\frac{2}{3}$ of them where white.

How many white flowers were there?



Divide into 3 equal parts.

Two boxes will give you $\frac{2}{3}$.

So $\frac{2}{3}$ of 24 = 2 x 8 = 16

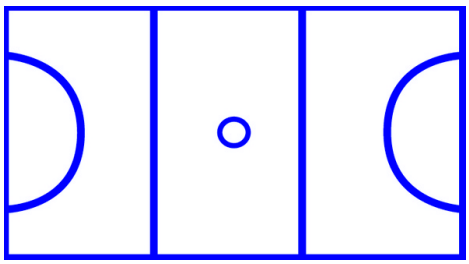
There are 16 white flowers.

Fitness test

1. There are 75 pupils in the Junior choir. $\frac{3}{5}$ of them are girls. How many boys are in the Junior choir?
2. Alison bought 16 eggs and used $\frac{3}{4}$ of them. How many did she have left?
3. There are 40 books in the fantasy section of the school library. James has read $\frac{5}{8}$ of them. How many fantasy books hasn't he read yet?
4. Juliette has 60 counters. $\frac{1}{3}$ are red, $\frac{1}{4}$ are blue and the rest are green. How many green counters has she got?
5. Trevor spent $\frac{4}{7}$ of his savings on a new computer game. If he had £24 left, how much had he originally saved?

Measuring a line

7X have been asked to estimate the distance around the goal circle on their netball court in metres. They are not allowed to use a ruler or a tape measure.





Jon

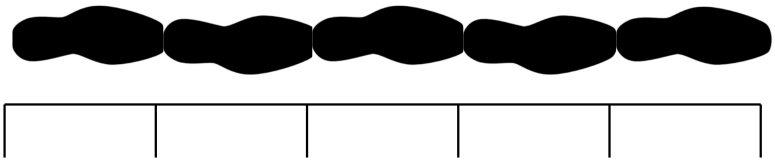
We could pace it out. My pace is about half a metre.

We could use our shoes as a measure. My shoe length is 25cm that's a quarter of a metre.



Alice

2 Using Alice's idea, what is the length of this line in metres?





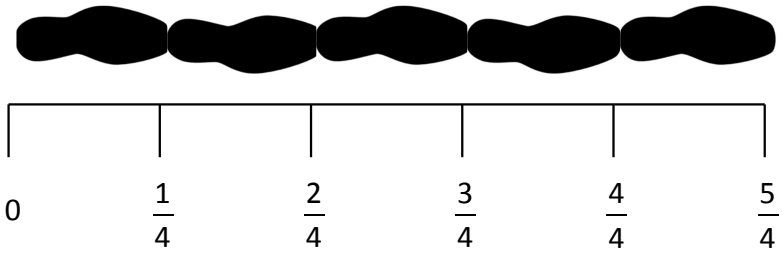
Sarah

The line is 5 shoe lengths long. So we could say its $\frac{5}{4}$ m long.



Sarah

The line is $\frac{5}{4}$ m long.





Evie

But that is more than one metre, so we could write it as ...

3 What do you think Evie was going to says?

Three types of fractions

As you can see, sometimes the numerator can be equal to or bigger than the denominator. These fractions are called **improper fractions**.

Fractions where the numerator is smaller than the denominator are called **proper fractions**.

As Evie noticed, you can write improper fractions in a different way.

For example $\frac{5}{4}$ can be written as $1\frac{1}{4}$. This is called a **mixed number**.

Fitness check

1. Write these improper fractions as mixed numbers:

   a) $\frac{7}{4}$ b) $\frac{5}{3}$ c) $\frac{5}{2}$ d) $\frac{9}{4}$ e) $\frac{11}{6}$ f) $\frac{13}{3}$

2. Write these mixed numbers as improper fractions:

   a) $1\frac{3}{5}$ b) $2\frac{1}{6}$ c) $3\frac{3}{8}$ d) $5\frac{4}{7}$ e) $1\frac{5}{9}$ f) $4\frac{2}{3}$

Addition and subtraction of fractions



Amber

I spent $1\frac{3}{4}$ hrs on my homework last night.

Wow, I only spent $\frac{3}{4}$ of an hour on my homework.



Matthew

4 Together, how long did Amber and Matthew spend on their homework?



Sarah

Sarah's solution

$$1\frac{3}{4} = \frac{7}{4}$$

$$\frac{7}{4} + \frac{3}{4} = \frac{10}{4}$$

$$\frac{10}{4} = 2\frac{2}{4} = 2\frac{1}{2}$$

They spent $2\frac{1}{2}$ hours altogether on their homework

Fitness test

Work out the following:

1. $2\frac{3}{5} + 3\frac{1}{5}$
2. $4\frac{3}{4} - 2\frac{1}{4}$
3. $2\frac{1}{3} - 1\frac{2}{3}$
4. $4\frac{5}{7} + 2\frac{4}{7}$
5. $1\frac{3}{8} - \frac{5}{8}$
6. $5\frac{4}{9} - 1\frac{5}{9}$

Jon's question



Jon

I ate $\frac{1}{2}$ of my chocolate bar today and gave $\frac{1}{3}$ to Alice.

What fraction of my chocolate bar is left?





Sarah

It would help if we knew how many squares of chocolate were in the bar.

5 How many squares of chocolate could there be in the bar?



Alice

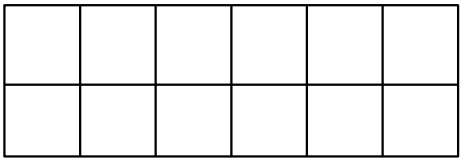
12 would work

So would 18



Matthew

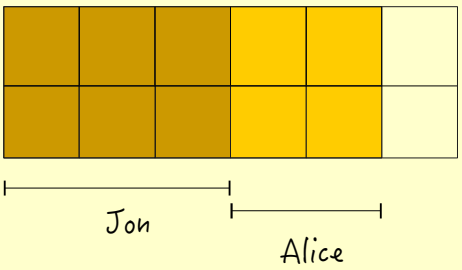
Using Alice's suggestion of 12 squares, try to answer Jon's question.





Amber

Amber's working



So 2 squares left, that is $\frac{2}{12}$

Or $\frac{1}{6}$

6 What difference would it have made if you had used Matthew's idea?

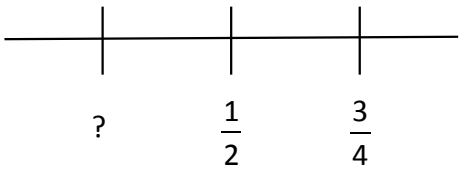
What size chocolate bar would help you to work out ...

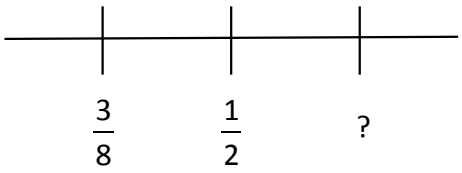
1. $\frac{3}{4} + \frac{5}{8}$
2. $\frac{1}{4} + \frac{7}{12}$
3. $\frac{2}{3} + \frac{2}{5}$
4. $\frac{7}{8} - \frac{1}{2}$
5. $\frac{4}{5} - \frac{3}{10}$
6. $\frac{5}{6} - \frac{7}{12}$

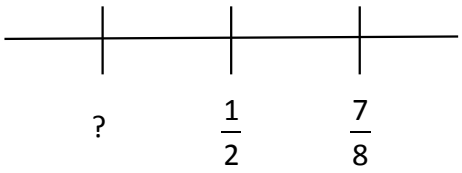
7 Use your ideas to complete the calculations above

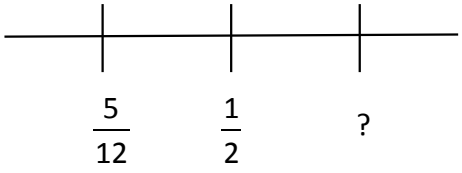
Half is in the middle

In each of these diagrams $\frac{1}{2}$ is in the middle. Find the missing numbers:









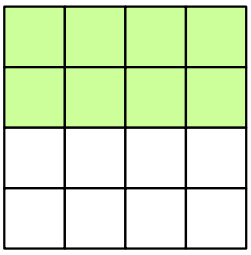
8 What do you notice about your results?

Let's investigate fractions of equal size

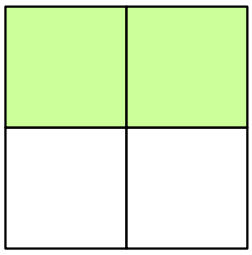


Evie

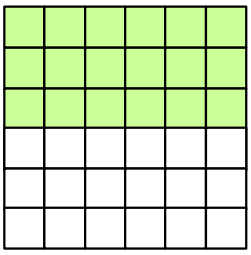
Equivalent fractions are different ways of representing fractions of the same size.
For example the shaded areas ...



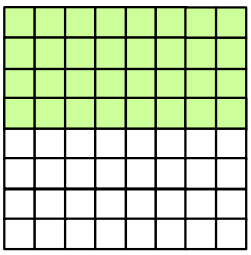
$\frac{8}{16}$



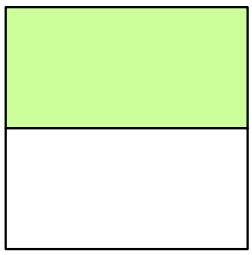
$\frac{2}{4}$



$\frac{18}{36}$



$\frac{32}{64}$



$\frac{1}{2}$

... pictured here are different ways of representing the same fraction.

So they are equivalent fractions.



Evie

9

Draw three more fractions that belong to this family

Look at this list of fractions.

$\frac{1}{4}, \frac{4}{16}, \frac{3}{10}, \frac{9}{36}, \frac{1}{40}, \frac{15}{60}$

a) Which of the these fractions are equivalent to $\frac{1}{4}$?

b) How did you decide which were equivalent?

c) Write down two more examples for fractions equivalent to $\frac{1}{4}$.

Generating equivalent fractions



Matthew

Is there an easy way of generating equivalent fractions without actually drawing them?

10

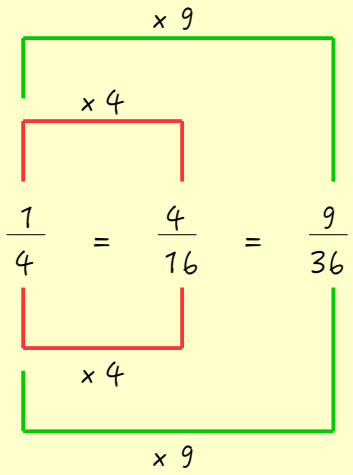
How would you answer Matthew's question?



Sarah

Sarah's explanation

The diagram below shows how to generate equivalent fractions.



You just multiply the numerator and the denominator by the same number each time

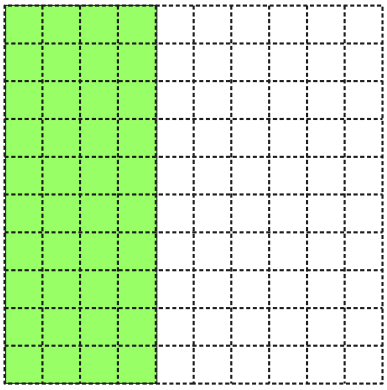
Use Sarah's idea to find 3 more equivalent fractions in each of these cases:

a) $\frac{1}{3}$, $\frac{2}{6}$, $\frac{5}{15}$, $\frac{25}{75}$

b) $\frac{2}{5}$, $\frac{8}{20}$, $\frac{4}{10}$, $\frac{40}{100}$

c) $\frac{7}{10}$, $\frac{14}{20}$, $\frac{35}{50}$, $\frac{70}{100}$

Look at this picture



What fraction of the grid is shaded?

How many different ways could you express this?


Alice $\frac{40}{100}$


Jon $\frac{20}{50}$


Amber $\frac{4}{10}$


Matthew $\frac{2}{5}$

11 Which would you say is the simplest way of writing the fraction?

Writing fractions in their simplest form

As you can see fractions can be written in different ways.
We can **simplify** a fraction by **cancelling** any common factors.


Sarah

Sarah's explanation

$\frac{\cancel{40}^{20}}{\cancel{100}_{50}} = \frac{20}{50}$ Divide top and bottom by 2

$\frac{\cancel{20}^{4}}{\cancel{50}_{10}} = \frac{4}{10}$ Divide top and bottom by 5

$\frac{\cancel{4}^{2}}{\cancel{10}_{5}} = \frac{2}{5}$ Divide top and bottom by 2

$\frac{2}{5}$ is the simplest form as 2 and 5 have no common factors

Let's think about multiplying fractions by whole numbers


Amber

Amber and Matthew are delivering leaflets to publicise their Mother's new business venture. They have 300 leaflets to deliver and estimate they can deliver 60 leaflets each hour.


Matthew

1. What fraction will they deliver in one hour?
2. What fraction will they deliver in 2 hours?
3. How long will it take them to deliver all 300 leaflets?


Evie

Evie's working

They will deliver 120 leaflets per hour between them.
As a fraction that is

$$\frac{120}{300} = \frac{12}{30} = \frac{2}{5}$$

In 2 hours they will deliver twice as many, so that is

$$\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$$

They will then have delivered 240 leaflets, so they have 60 left.
That will take another half an hour.
So it will take then 2 and half hours altogether


Jon

So multiplying fractions by a whole number is easy.

Just multiply the numerator by the whole number and leave the denominator alone.

For example

$$2 \times \frac{2}{5} = \frac{2 \times 2}{5} = \frac{4}{5}$$


Sarah

So $5 \times \frac{3}{4} = \frac{5 \times 3}{4} = \frac{15}{4} = 3\frac{3}{4}$

Fitness check



1. Write these fractions in their simplest terms.

   $(a)\ \frac{8}{10}\quad (b)\ \frac{12}{18}\quad (c)\ \frac{20}{24}\quad (d)\ \frac{60}{100}\quad (e)\ \frac{55}{100}$

2. Write these as equivalent fractions with a denominator of 100

   $(a)\ \frac{3}{4}\quad (b)\ \frac{3}{5}\quad (c)\ \frac{9}{10}\quad (d)\ \frac{17}{20}\quad (e)\ \frac{22}{40}$

3. For each pair of fractions

   a) Write down the fractions with a common denominator

   b) Put the fractions in order, smallest first

   c) Write down another fraction that fits between in size order

   $(i)\ \frac{1}{4}\ ,\ \frac{1}{2}\quad (ii)\ \frac{3}{4}\ ,\ \frac{1}{2}\quad (iii)\ \frac{1}{2}\ ,\ \frac{7}{10}\quad (iv)\ \frac{2}{5}\ ,\ \frac{3}{10}\quad (v)\ \frac{1}{4}\ ,\ \frac{3}{10}$

4. By using equivalent fractions where necessary, calculate

   $(a)\ \frac{3}{10}+\frac{1}{2}\quad (b)\ \frac{3}{5}+\frac{1}{10}\quad (c)\ \frac{3}{8}+\frac{1}{2}\quad (d)\ \frac{3}{10}+\frac{7}{20}$

5. Calculate the following. Write your answer in the simplest form.

   $(a)\ 5\times\frac{1}{4}\quad (b)\ 7\times\frac{1}{3}\quad (c)\ 2\times\frac{3}{4}\quad (d)\ 3\times\frac{2}{5}\quad (e)\ 6\times\frac{4}{5}$

6. Calculate the following. Write your answer in the simplest form.

   $(a)\ 3\times 1\frac{1}{4}\quad (b)\ 4\times 2\frac{2}{3}\quad (c)\ 5\times\frac{5}{6}\quad (d)\ 6\times 4\frac{2}{5}$

Comparing results ...



Alice

Alice is determined to get into the netball team this year.
She has been regularly practising her shooting and every Monday she takes a number of shots and records her results.
Her records for last month are shown below.



Week	⊕ goal scored ⊗ missed										Goals	Attempts
1	⊕	⊗	⊕	⊕	⊗						3	5
2	⊗	⊕	⊕	⊕	⊗	⊕	⊕	⊗			5	8
3	⊗	⊕	⊕	⊕	⊗	⊕	⊕	⊕	⊕	⊗	7	10
4	⊕	⊗	⊕	⊕							3	4

12 On this evidence, would you say Alice is improving?



Sarah

Alice scored the most in week 3

… but she also missed the most in week 3.



Matthew



Amber

Perhaps we could use fractions.



Jon

The fraction of goals scored out of the number of shots was

$$\frac{3}{5}, \frac{5}{8}, \frac{7}{10}, \frac{3}{4}$$

13 Does this help you to decide?

Out of 100 ...



Evie

We need to be able to compare these fractions

$$\frac{3}{5}\ ,\ \frac{5}{8}\ ,\ \frac{7}{10}\ ,\ \frac{3}{4}$$



Matthew

We could use percentages. That means "out of 100".



Amber

Amber's working

Week 1 $\frac{3}{5} = \frac{60}{100} = 60\%$ (× 20 top, × 20 bottom)

Week 2 $\frac{5}{8} = \frac{25}{40} = \frac{62.5}{100} = 62.5\%$ (× 5, × 2.5 top; × 5, × 2.5 bottom)

Week 3 $\frac{7}{10} = \frac{70}{100} = 70\%$ (× 10 top, × 10 bottom)

Week 4 $\frac{3}{4} = \frac{75}{100} = 75\%$ (× 25 top, × 25 bottom)

So Alice is getting better!

Fitness check



1. A jumper is made from 65% acrylic. The rest is wool. What percentage is wool?
2. 48% of the pupils in 7X were absent at least once this year. What percentage of pupils were not absent.
3. In an opinion poll for the election, 42% said they would vote for the Blue party, 37% for the Red party and the remainder for the Yellow party. What percentage said they would vote for the Yellow party.
4. Convert these fractions into percentages

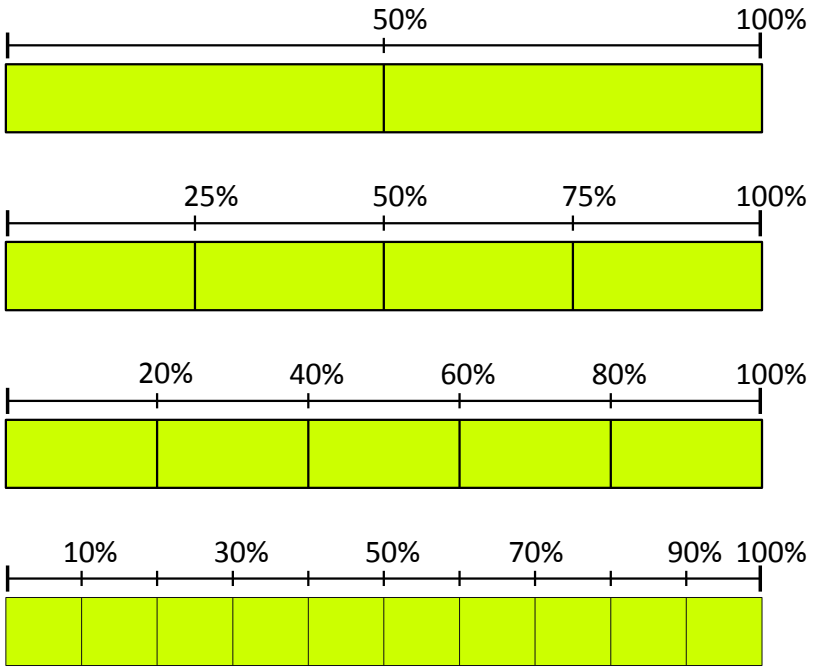
   $(a)\ \frac{1}{4}\quad (b)\ \frac{4}{5}\quad (c)\ \frac{6}{10}\quad (d)\ \frac{13}{20}\quad (e)\ \frac{18}{40}$

Finding percentages of ...

Working with percentages

Percentages are another way to work out parts of a quantity. Per cent actually means 'out of 100', so 50% means 50 parts out of 100. In other words, finding 50% of a quantity is the same as finding $\frac{1}{2}$ of a quantity. Finding 25% of a quantity is the same as finding $\frac{1}{4}$ of a quantity and so on.

In each question a decision on the 'unit' of measure has to be made as illustrated below.



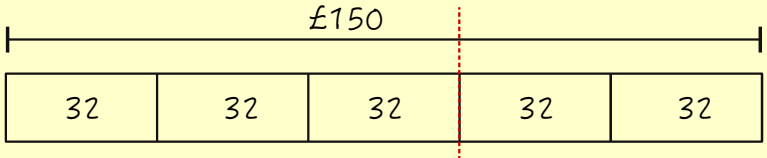
1% would mean drawing 100 boxes and probably isn't the most sensible approach!

14 Explain how you can use these ideas to find 60% of £160.



Jon

Jon's working



60% - that's 3 boxes out of 5

160 ÷ 5 = 32

3 x 32 = 96

So 60% of £160 = £96

Fitness check



1. Will collects stamps. Altogether he has 540 stamps. 30% are European. How many European stamps does he have?
2. Bryony earns £1200 per month. She saves 20% of her money each month. How much does she save in 6 months?
3. Arsenal's stadium holds 60000 people. For the final match of the season it was only 75% full. How many people were in the stadium?
4. Which is greater: 40% of £400 or 60% of £300?
5. Alice scored 45 out of 60 in her latest Science assignment. What is this as a percentage?
6. Matthew scored 48 in the same assignment. What is this as a percentage?
7. Amber scored 54 in the same assignment. What is this as a percentage?
8. All cars over 3 years old have to have an MOT test once a year. When analysing the test results of 3 garages in a local town the Ministry of Transport reported the results in the table below.

Garage	Number of tests	Passed	Failed
A	400	320	80
B	450	315	135
C	550	330	220

What percentage of cars failed at each garage?

9. Of the cars that failed the test, the Ministry of Transport reported the reasons for failure were as follows:

Garage	Failed	Brakes	Lights	Emission
A	80	40	30	20
B	135	81	54	27
C	220	165	88	44

a) Find the percentage of the cars that failed for a) brakes b) lights and c) emission for each garage.

b) Why does the total reasons for failure not equal the number of failures at each garage?

Let's investigate the connection between fractions, decimals and percentages

The term "percent" is simply another name for **hundredths**.

For example, think about the number $\frac{3}{4}$.

How can we write this as an equivalent fraction with the denominator of 100?



Evie

Evie's working

$\frac{3}{4} \overset{\times 25}{=} \frac{75}{100} = 75\%$

But $\frac{75}{100}$ can be written as 0.75 as a decimal

So 75% and 0.75 are just different ways of writing the same number.

Fractions, decimals, percentages

1. Convert these fractions to percentages and then into decimals.

a)	$\frac{1}{2}$	e)	$\frac{7}{10}$	i)	$\frac{9}{25}$
b)	$\frac{1}{4}$	f)	$\frac{9}{10}$	j)	$\frac{21}{25}$
c)	$\frac{1}{10}$	g)	$\frac{1}{5}$	k)	$\frac{37}{50}$
d)	$\frac{3}{10}$	h)	$\frac{3}{5}$	l)	$\frac{47}{50}$

2. Convert these percentages into fractions in their simplest form and into decimals.

a)	40%	d)	35%	d)	18%
b)	80%	e)	55%	e)	46%
c)	15%	f)	95%	f)	91%

3. Convert these decimals into percentages and fractions in their simplest form.

a)	0.2	d)	0.85	d)	0.125
b)	0.45	e)	0.54	e)	0.385
c)	0.65	f)	0.86	f)	0.875



Amber

Amber's working

0.2 means two tenths, so that is $\frac{2}{10} = \frac{1}{5}$

0.45 means 4 tenths and 5 hundredths or 45 hundredths, so that is $\frac{45}{100} = \frac{9}{20}$

0.125 means 1 tenth, 2 hundredths and 5 thousandths or 125 thousandths,

so that is $\frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$



Alice

Alice's working

0.2 means two tenths, so that is $\frac{2}{10} = \frac{20}{100} = 20\%$

0.45 means 4 tenths and 5 hundredths or 45 hundredths, so that is $\frac{45}{100} = 45\%$

0.125 means 1 tenth, 2 hundredths and 5 thousandths or 125 thousandths,

so that is $\frac{125}{1000} = \frac{12.5}{100} = 12.5\%$

Number 2b: Mastery questions

1. In a survey 30 pupils out of 45 felt that they should have extra PE lessons. Write this as a fraction in its simplest form.

A	B	C	D
$\frac{30}{45}$	$\frac{3}{4}$	$\frac{6}{9}$	$\frac{2}{3}$

2. Find $\frac{3}{4}$ of £36

A	B	C	D
£9	£18	£27	£48

3. Work out 4 x $\frac{3}{5}$

A	B	C	D
$\frac{12}{20}$	$\frac{3}{20}$	$\frac{12}{5}$	$\frac{23}{5}$

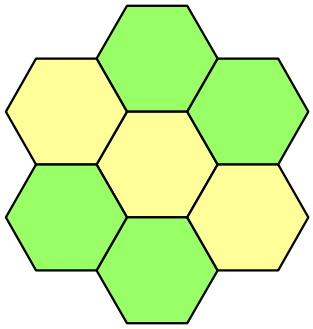
4. Work out $\frac{1}{2} + \frac{1}{8}$

A	B	C	D
$\frac{2}{8}$	$\frac{2}{10}$	$\frac{5}{16}$	$\frac{5}{8}$

5. Write $\frac{13}{6}$ as a mixed number in its simplest form.

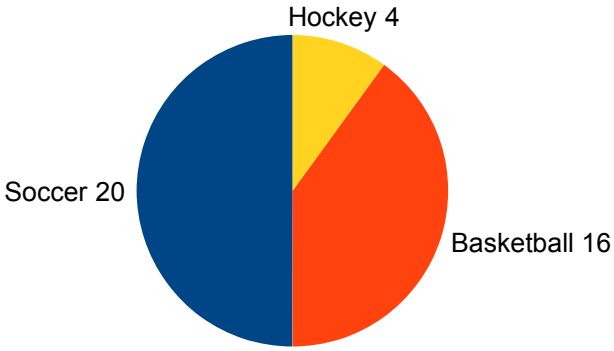
A	B	C	D
$1\frac{3}{6}$	$1\frac{1}{2}$	$2\frac{1}{6}$	$3\frac{1}{6}$

6. What fraction of this picture is yellow?



A	B	C	D
$\frac{3}{7}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{4}{3}$

7. 40 Year 7 pupils were given the choice of playing soccer, basketball or hockey. The results are shown in this pie chart. What fraction of the pupils chose basketball? Write your answer in its simplest form.



A	B	C	D
$\frac{16}{24}$	$\frac{16}{40}$	$\frac{2}{5}$	$\frac{4}{10}$

8. Put these fractions into ascending order (smallest first).

$$\frac{3}{2}\ ,\ \frac{4}{5}\ ,\ \frac{5}{4}\ ,\ \frac{6}{8}$$

A	$\frac{3}{2}\ ,\ \frac{4}{5}\ ,\ \frac{5}{4}\ ,\ \frac{6}{8}$	B	$\frac{6}{8}\ ,\ \frac{4}{5}\ ,\ \frac{5}{4}\ ,\ \frac{3}{2}$
C	$\frac{6}{8}\ ,\ \frac{5}{4}\ ,\ \frac{4}{5}\ ,\ \frac{3}{2}$	D	$\frac{4}{5}\ ,\ \frac{6}{8}\ ,\ \frac{3}{2}\ ,\ \frac{5}{4}$

9. What is $\frac{1}{4} + \frac{1}{8}$?

A	B	C	D
$\frac{2}{8}$	$\frac{3}{8}$	$\frac{1}{12}$	$\frac{2}{12}$

10. Which of these fractions is not equivalent to $\frac{9}{12}$?

A	B	C	D
$\frac{16}{24}$	$\frac{3}{4}$	$\frac{60}{80}$	$\frac{27}{36}$

11. What is $3\frac{1}{4} - 1\frac{5}{8}$?

A	B	C	D
$2\frac{1}{2}$	$2\frac{3}{8}$	$1\frac{3}{8}$	$1\frac{5}{8}$

12. What is 60% of £60?

A	B	C	D
£40	£36	£60	£66

13. Write 2% as a decimal.

A	B	C	D
0.02	0.2	2.0	20.0

14. Write 35% as a decimal.

A	B	C	D
3.5	0.35	0.7	0.07

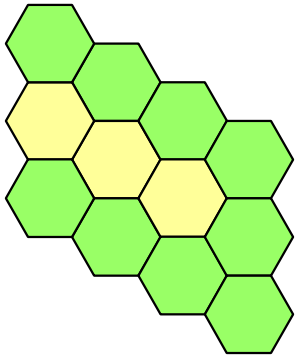
15. What fraction is exactly half between $\frac{3}{5}$ and 0.68? Write the answer in its simplest form.

A	B	C	D
$\frac{64}{100}$	$\frac{32}{50}$	$\frac{16}{25}$	0.64

16. What is $\frac{3}{20}$ written as a percentage?

A	B	C	D
3%	6%	15%	60%

17. Look at this picture. What percentage is yellow?



A	B	C	D
3%	12%	25%	30%

18. Work out the difference between $\frac{1}{4}$ and 30%.

A	B	C	D
2.5%	5%	7.5%	10%

19. What is 0.465 as a percentage?

A	B	C	D
4.65%	46%	46.5%	465%

20. What is 15% of £80?

A	B	C	D
£8	£12	£15	£20