What do you know about decimals?

What is 5.48 + 6.752?

What is 1.64 - 0.476?

What is 3.62 x 10?

What is 6.035 x 100?

What is 5.02 ÷ 10?

What is 32.4 ÷ 100?

What is 0.7 x 15?

Round 7.48 to the nearest integer.

Round 4.849 to 1 d.p.

How many 0.01's would make 5.2 in total?

60 000 fans watch Arsenal lose their opening game



… but the official attendance was 60 233.

New cruise liner holds 6500 passengers



… but the official capacity is 6360.

1 Why do newspapers, magazines and TV shows round numbers?

Rounding

How many people watched Premier League matches in London?

Team	Attendance
Arsenal	62 033
Chelsea	41 564
Crystal Palace	23 997
West Ham	56 977

How could we represent this data on a simple pictogram?



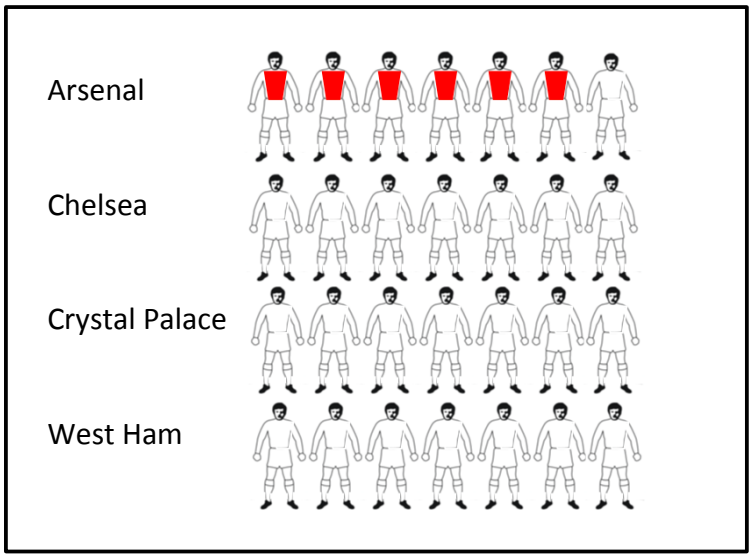
Matthew

The numbers are too detailed to use a pictogram.

Expressing approximate numbers

Marcus decided to represent the attendance data in a pictogram shown here.

Let's think about how to approximate exact numbers.





Jon

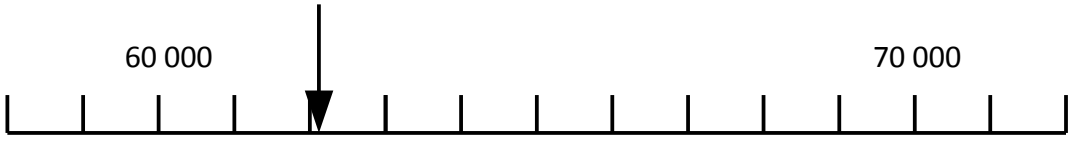
Why has he coloured in six players to represent Arsenal?

How many figures should he colour in for the other teams?

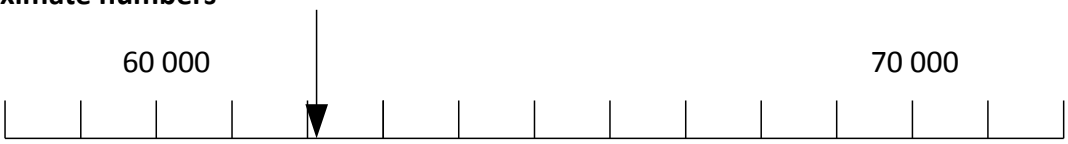


Sarah

Matthew decided to express the attendance for Arsenal, 62033 as 60000.
Think about why he did this by looking at the number line below.



Approximate numbers



Since the number 62 033 is near to 60 000, we say that it is about 60 000.
We also say that it is **approximately** 60 000. Numbers that express 'about how much' are called approximate numbers.

2

Approximate the attendance at the other three clubs to the nearest 10 000.
Create your own pictogram to represent this data.

Two cruise ships left Southampton this morning. There were 2346 passengers aboard the Queen Elizabeth and 2620 aboard the Queen Mary 2.



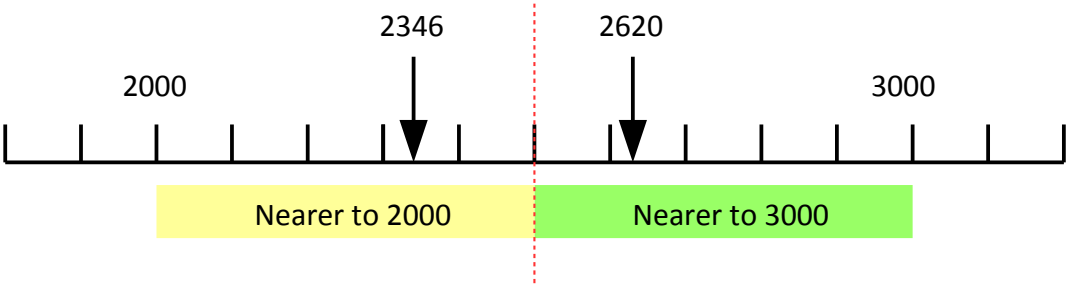
Queen Elizabeth



Queen Mary 2

3

Approximately how many thousands of people were on each ship?





Evie

If you draw a line exactly in the middle, all you have to decide is whether it is greater than or less than 2500.

So there are approximately 2000 aboard the Queen Elizabeth and approximately 3000 aboard the Queen Mary 2.



Amber



Matthew

But you can tell by just looking in the hundreds place.

4

Explain what Matthewmeans.

Rounding


Matthew

If you want to round 3457 to the nearest thousand, you only need look at the hundreds digit.
If the digit in the hundreds place is 0, 1, 2, 3, or 4, you round down.
So in this case 3457 is 3000 to the nearest thousand.

Fitness check

Round these numbers to the nearest thousand:

1. 7319 2. 6501 3. 4689 4. 2495 5. 9846



Round these numbers to the nearest 10000

1. 18321 2. 44962 3. 89160 4. 50983 5. 97452


Evie

What if I want to round 4689 to the nearest 100?


Matthew

It's the same idea, nearest hundred – so look in the tens column.
It's an 8, so round up. 4689 to the nearest hundred is 4700.

Fitness check



Round these numbers to the nearest hundred

1. 7319 2. 6501 3. 4689 4. 2495 5. 9846

Flying off for a holiday?



The maximum weight of hand luggage on a well-known international carrier is 23kg per person.
The passenger capacity for their long haul flights is 315.

5 Estimate the total weight of hand luggage that could be on board their flights.

Estimating answers

It is often useful to be able to estimate the answers to calculations.



Amber

To find the total weight, we need to estimate 315 x 23.
So my estimate is 300 x 20 = 6000kg.

Yes, but it could be more.



Matthew

6 Explain Matthew's thinking?

Fitness check

Estimate the answer to each of these calculations:

1. 257 x 28
2. 4187 x 67
3. 9842 x 387
4. 26.8 x 47.4
5. 3.512 x 4.365
6. 372 ÷ 38
7. 7256 ÷ 27
8. 8256 ÷ 216
9. 73.81 ÷ 5.89
10. 26.63 ÷ 3.56

Making deductions



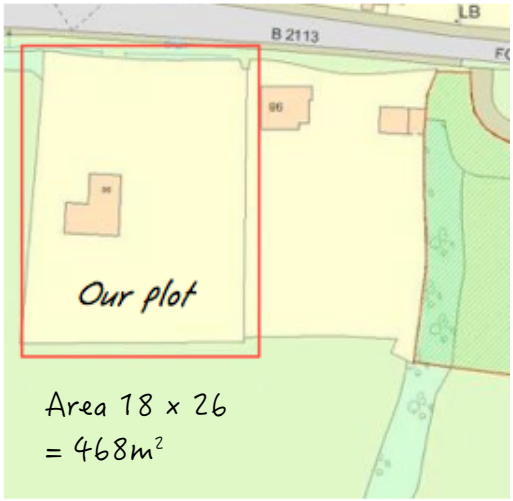
Matthew

We are building a new house. Our plot measures 18m by 26m with an area of 468m^2.

I drew a one-tenth scale plan onour garage floor so we could decide where to put things.

So my drawing is 1.8m by 2.6m.

What is the area of my drawing?





Evie

Easy 18 x 26 = 468
So 1.8 x 2.6 = 46.8
The area of your drawing is 46.8m^2.

That doesn't sound right!



Sarah

7 Why does Sarah think this is wrong?

If we round the numbers to the nearest whole number, then 1.8 x 2.6 becomes 2 x 3 = 6.
So Evie is way out.



Alice



Jon

This is how I worked it out.
18 x 26 = 468
1.8 x 26 = 46.8 because it must be 10 times smaller.
1.8 x 2.6 = 4.68 because It must be 10 times smaller again.
So the area of Matthew's drawing is 4.68m^2.

Now that does look right!



Sarah

Fitness check

Using Jon's ideas, write down the answers to

1. 1.8 x 0.26
2. 1.8 x 260
3. 0.18 x 0.26
4. 180 x 0.026
5. 0.018 x 0.26
6. 1800 x 0.26



Making deductions



Evie

I've noticed something interesting about multiplying decimals.

Look

1.8 x 2.6 = 4.68

0.18 x 2.6 = 0.468

0.18 x 0.26 = 0.0468

8 What do you think Evie noticed?

Evie's explanation



Evie

Suppose you want to calculate 3.72 x 5.4, then calculate as if there are no decimals points.
Now add up the number of decimal places.
2 + 1 = 3
Your answer must also have 3 decimal places.
So count in 3 from the right and put the decimal point.

Evie's notes

3.72	2 d.p.
X5.4	1 d.p.
18600	
1488	
20.088	3 d.p.

Fitness check

Using Evie's method, write down the answers to these calculations:

1. 3.72 x 0.54
2. 0.372 x 0.54
3. 37.2 x 5.4
4. 0.372 x 54
5. 0.0372 x 5.4
6. 0.0372 x 0.54

In the following questions, estimate the product first and then use the multiplication algorithm to calculate

7. 2.83 x 6.7
8. 7.2 x 8.9
9. 34.9 x 0.7
10. 1.25 x 0.8



The top of a school desk measures 1.2m by 0.6m. The top is 0.018m thick.

11. Calculate the area of the desk top.
12. Calculate the volume of the desk top.

Dividing by decimals



But how much is that per litre?



Amber



Sarah

Sarah's solution

£13 for 2.5L, so that would be
£26 for 5L
26 ÷ 5 = 5.2
So £5.20 per litre



Matthew

Matthew's solution

£13 for 2.5L, so that would be
£52 for 10L
52 ÷ 10 = 5.2
So £5.20 per litre



Evie

Evie's solution

£13 ÷ 2.5 that's the same as
£130 ÷ 25 that's long division

$$25\overline{)130.^{5}0}\quad 5.2$$

So £5.20 per litre

In each of these methods, the calculation was changed so that the divisor was a whole number. This is the simplest way of dividing by decimals.

Where would you put the brackets?

Every week Amber earns £10 for helping her Mum in the shop on Saturdays and £5 for delivering her Grandma's shopping.
In February, she worked on four Saturdays.
She wrote down this sum to calculate her total earnings.

10 + 5 x 4 = 15 x 4 = £60


Jon

Mathematically that isn't right. The correct answer to your calculation is

10 + 5 x 4 = 10 + 20 = £30

Which, of course, is the wrong answer!

Neither Amber nor Jon actually did anything wrong in their calculations, but they ended up with different answers.

Amber added up 10 and 5 to show she earned £15 each Saturday.

As she worked for four Saturdays she then worked out 15 x 4 = £60, which was what she actually earned.

Jon chose to work out the multiplication first and the added the 10.

As you can see, the order in which you carry out a calculation can greatly change your answer.
To avoid this problem, everyone in the world follows the same convention, although they use different acronyms to help them remember!
In the UK, we tend to use **BIDMAS**.

The order of operations should be carried out as follows:

- **B**rackets (which means work out the inside of the bracket first)
- **I**ndices (powers, such as squares, cubes, square roots)
- **D**ivision
- **M**ultiplication
- **A**ddition
- **S**ubtraction


Amber

So I should have written down (10 + 5) x 4 to avoid any confusion.

Fitness check



Some of these calculations need brackets to make them correct.
Copy them out and put in brackets if they are needed:

1. 2 x 5 + 6 = 16
2. 17 – 5 x 3 = 2
3. 17 – 5 x 3 = 36
4. 20 ÷ 10 ÷ 5 = 10
5. 9 x 8 ÷ 4 = 18
6. 6 x 4 + 14 ÷ 7 = 26

Where would you put the brackets?

Now look at this calculation:

24 ÷ 4 + 4 x 3

Where would you put the brackets to get the following answers?

1. 18
2. 9
3. 1
4. 30



Matthew

Without any bracket
24 ÷ 4 + 4 x 3
= 6 + 12
= 18



Evie

And 24 ÷ (4 + 4) x 3
= 24 ÷ 8 x 3
= 3 x 3 = 9



Amber

Or (24 ÷ 4 + 4) x 3
= (6 + 4) x 3
= 10 x 3
= 30



Sarah

Look what happens if you use brackets inside brackets
24 ÷ ((4 + 4) x 3)
= 24 ÷ (8 x 3)
= 24 ÷ 24
= 1

Fitness check



Some of these calculations need brackets to make them correct.
Copy them out and put in brackets if required.

1. 2^2 x 5 + 6 = 26
2. 1 + 5 + 3^2 = 65
3. 3 + 3^2 ÷ 2 = 6
4. 8^2 ÷ 4^2 ÷ 2^2 = 16
5. 3^2 x 2^3 ÷ 4 = 18
6. 3^2 + 4^2 ÷ 5 = 5

Multiple puzzles

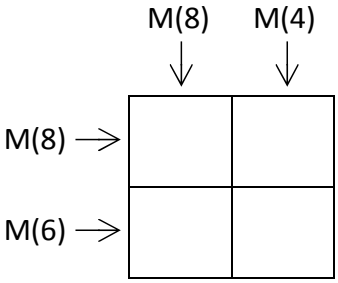
In these puzzles, M(8) means a number that is a multiple of 8.



Amber

So it could be 8, 16, 24, 32 and so on.

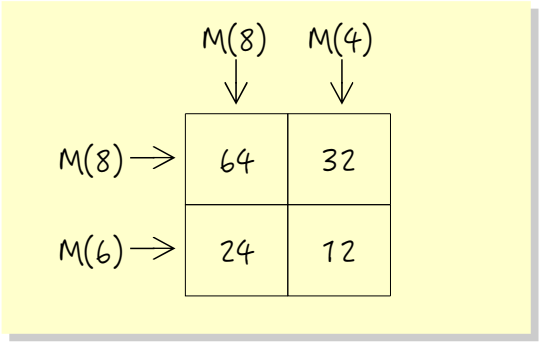
In this puzzle, find four digits (all different) to go one in each of the boxes to make 2-digit multiples of the required number across and down.





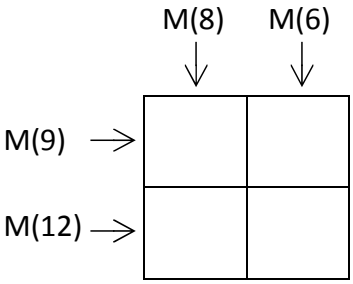
Alice

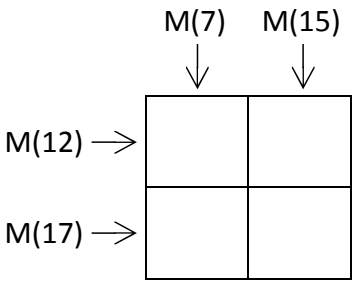
Alice's solution

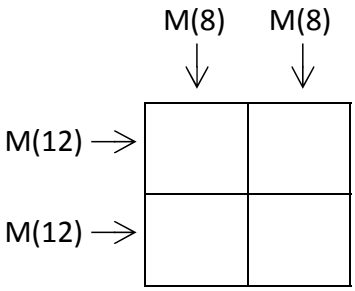


9 Is Alice's solution the only answer?

Three more multiple puzzles for you to try.







Which is bigger: -1 or -3?

Extending our number system

The set of positive whole numbers are called **natural numbers** or **counting numbers**. They are the first numbers you came across when you were younger.

Whole numbers are useful for solving many problems and applications in the real world. However, there are many situations where **negative numbers** are useful.

10 Can you think of any practical examples of using negative numbers?



Jon

Describing temperature when it is below zero.

Elevation when it is below sea level.



Sarah



Alice

Losses in the stock market.

In golf, when you are “under par”.



Matthew



Evie

Overdrawn at the bank.

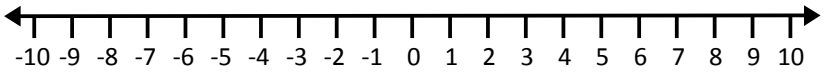
Integers

To help us to solve problems with positive, zero or negative whole number answers, we introduce a new set of numbers, the **integers**.
The integers include all positive and negative whole numbers as well as zero.
The integers are often written as

…, -3, -2, -1, 0, 1, 2, 3, …

Where the … means "and so on.”
1, 2, 3, … are called **positive integers.**
-1, -2, -3, … are called **negative integers**.
Zero is neither positive nor negative.

It is often helpful to think of this as extending the number line to the left and referring to as the **Integer Number Line** as shown below.



11 Which is bigger: -3 or -1?



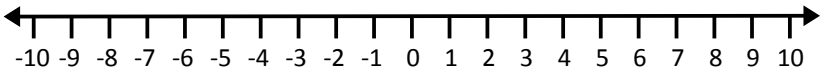
Alice

You could use the number line...

Positive numbers get bigger as you move to the right. So ...



Matthew



Notation "greater than" and "less than".

To show that 6 is greater or bigger than 4, we write 6 > 4.
To show that 2 is less or smaller than 4, we write 2 < 4.
We use the same notation for all integers as well as fractions and decimals.
So we could write -1 > -3 or we could write -3 < -1.
These mean exactly the same thing, that -3 is further to the left than -1 on our integer number line.
These new signs are called **inequality signs**.

Fitness check



Copy these questions into your exercise book and insert the correct inequality sign in each.

1. 3 ☐ 5
2. -3 ☐ -5
3. -3 ☐ 5
4. 7 ☐ 0
5. -4 ☐ -6
6. -2 ☐ 0

Copy these questions into your exercise book and insert a possible integer solution in each box.

7. 2 < ☐
8. -3 > ☐
9. -5 < ☐
10. 3 < ☐ < 5
11. -1 < ☐ < 1
12. -4 < ☐ < -1

Put these integers into ascending order from left to right.

13. 5, -2, 3, 7, -10, 0, -4, 6

What's the difference?



12 Why are some of the numbers on this outdoor thermometer marked in red?



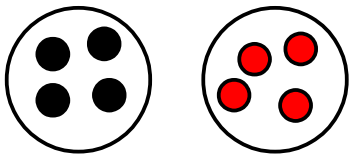
Alice

Red numbers must mean they are negative integers.
So the temperature is about -6°C.

Opposites

The integer number line is clearly symmetrical about zero. This symmetry leads to a useful idea about positive and negative integers. We can think about -4 being the **opposite** of 4 and 4 being the **opposite** of -4.
In this section, we will represent positive integers with black discs and negative integers with red discs.

4 and -4 are
opposites of each other



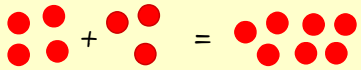
One black disc and one red disc always cancel each other out or "make a zero". They are called a **zero pair**.
Likewise, 4 and -4 will be a zero pair as they cancel each other out.

Of course, 4 and -4 are equally distant from zero. The negative sign tells you that -4 is to the left of zero and $^{+}4$ or just 4 tells you it is to the right of zero.
The **absolute value** of both numbers is 4, which specifies how far the integer is from zero but not in which direction.

13 What is (-4) + (-3)?


Evie

Evie's solution



(-4) + (-3) = (-7)


Jon

So adding negative numbers is exactly the same as adding positive numbers. You just have to remember that the answer is negative. I think I'll call that the **same sign rule**.

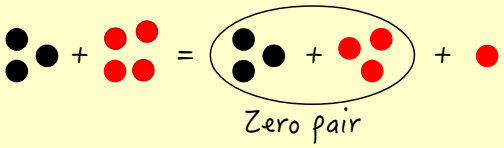

Alice

But what happens if one is positive and the other is negative?

For example 3 + (-4)?


Matthew

Matthew's solution



3 + (-4) = (-1)

14 Can you think of a rule for this kind of calculation?


Jon

So to find the sum of two numbers with different signs. Subtract the smaller absolute value from the larger and then decide if the answer should be negative or positive. I think I will call that the **mixed sign rule**.

For example,

(-5) + 3 = -(5 – 3) = -2

Fitness check

Using Matthew's method or Jon's rules, work out the following calculations:



1. (-2) + (-3)
2. (-4) + 2
3. (-1) + 3 + (-5)
4. 7 + (-6) + (-1)
5. (-3) + (-4) + (-2)
6. 6 + (-4) + (-3)



Sarah

What about subtraction? What is 4 - (-3)?

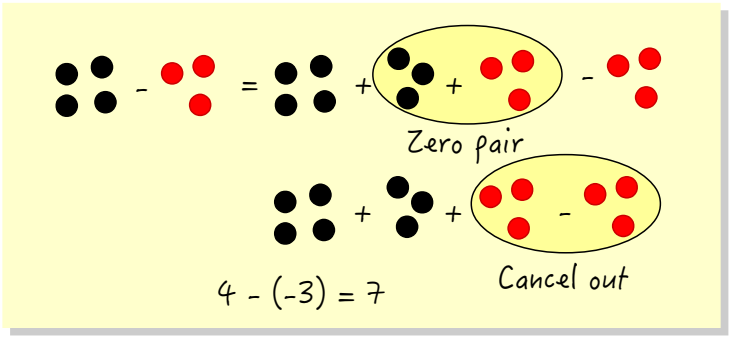
15

Can you think of a way of using the discs to solve Sarah's question?



Amber

Amber's solution



Fitness check

Using Amber's method, work out the following calculations:



1. 7 - (-2)
2. 5 - (-3)
3. 4 - (-5)
4. (-2) - 2
5. (-3) - (-4)
6. (-6) - (-4) - (-3)



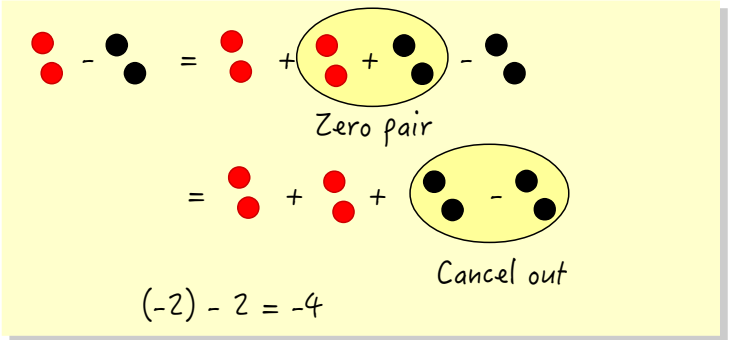
Alice

Some of these are very confusing!
What is (-2) – 2?
And how do I work out (-3) - (-4)



Jon

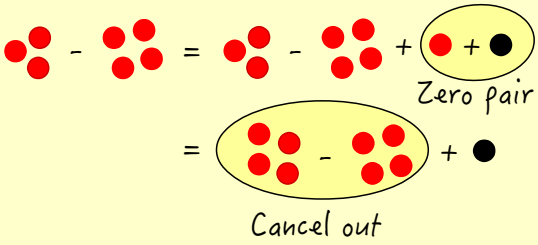
Jon's solution to (-2) - 2





Matthew

Matthew's solution to (-3) -(-4)





Jon

I've made up my own rule to make life easier.
For example, if you want to subtract a positive or negative number, it is the same as changing the sign and adding it!

16 Can you explain Jon's idea with examples?



Amber

5 - (-2) = 5 + 2 = 7
Easy – I can use the same sign rule!



Sarah

6 – 4 = 6 + (-4) = 2
But I knew that already!



Matthew

-3 - (-2) = -3 + 2 = -1
Simple – I can used the mixed sign rule!

Fitness check



Calculate the following:

1. (-6) - (-1)
2. 7 - (-9)
3. 0 - (-6)
4. -5 - (-5)
5. (-5) - 0
6. 6 - (-3)

Subtraction revisited

When you first learnt about subtraction, you were always asked to subtract the smaller number from the larger number.
Now you learned more about the integers, positive and negative, you can now do the following calculation with confidence.

5 – 8 = 5 + (-8)
= -3 (Using Jon's rule above)

Calculations with a mixture of addition and subtraction

17 Can you now work out -6 + 3 – 2 + 8 efficiently?



Sarah

Sarah's solution

$$-6 + 3 - 2 + 8 = -3 - 2 + 8$$
$$= -5 + 8$$
$$= 3$$



Matthew

Matthew's solution

$$-6 + 3 - 2 + 8 = -(6 + 2) + (3 + 8)$$
$$= -8 + 11$$
$$= 3$$

18 Can you explain the difference between these two methods?

Fitness check

Calculate the following:

1. -4 + 12 - (-3)
2. 6 + (-5) - 2
3. -7 + (-6) - (-4)
4. -6 + (-12) + 9
5. -3 – 7 + 2 - 8
6. 32 – 48 + (-15) + 26

Number 3a: Mastery questions

In questions 1 – 12, four possible answers are given. Only one of them is the correct. Record the letter which you think indicates the correct answer. You must be able to give reasons for your choice.

1. What is 2685 rounded to the nearest 10?

A	2680	B	2700	C	2690	D	90

2. Which of these is the most sensible estimate for 39 x 47?

A	1786	B	1600	C	1200	D	2000

3. Which of these is the most sensible estimate for 19.6 x 3.1?

A	60	B	57	C	62	D	60.76

4. Which of these is the most sensible estimate for 3558 ÷ 63?

A	60	B	50	C	53	D	55

5. Which of these is the most sensible estimate for 216.6 ÷ 18.7?

A	20	B	11.6	C	10	D	15

6. Given that 28 x 52.6 = 1472.8, what is 0.28 x 526?

A	1472.8	B	147.28	C	14.728	D	1.4728

7. What is 1.2^2?

A	14.4	B	0.144	C	1.4	D	1.44

8. Calculate $3 + 7^2 - 5 \times 2$

A	90	B	42	C	94	D	91

9. Calculate $(2 + 3)^2 - 3 \times 2^2$

A	88	B	1936	C	13	D	44

10. What is -6 + (-9)?

A	5	B	15	C	-15	D	-5

11. Put these numbers in order starting with the smallest.

0.5 -0.4 -0.35 0.25 -0.346

A	-0.346 -0.35 -0.4 0.5 0.25
B	-0.346 -0.35 -0.4 0.25 0.5
C	-0.4 -0.35 -0.346 0.25 0.5
D	-0.4 -0.346 -0.35 0.25 0.5

12. What is 7 - (-4)?

A	3	B	-11	C	3	D	11