What do you know about fractions, decimals and percentages?

Which is bigger

$\frac{2}{3}$ or $\frac{3}{4}$?

Write down three fractions that are equivalent to

$\frac{2}{5}$

What are the decimal equivalents to:

$\frac{2}{5}$ $\frac{3}{4}$ $\frac{2}{3}$ $\frac{7}{10}$ $\frac{5}{8}$ $\frac{41}{100}$ $\frac{7}{100}$

Calculate:

$\frac{1}{2}+\frac{3}{8}$

$\frac{5}{7}-\frac{2}{3}$

$\frac{3}{4}\times\frac{2}{5}$

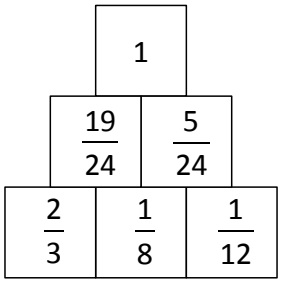
$\frac{2}{3}\div 4$

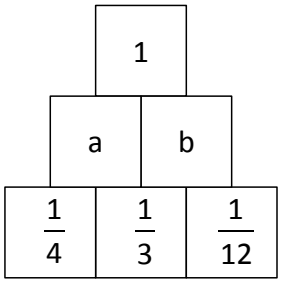
Write down a fraction that is between

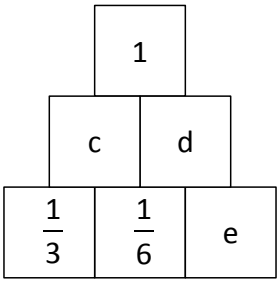
$\frac{2}{5}$ and $\frac{3}{5}$

Fraction triangles

Look carefully at these fraction triangles.







Find the values of a, b, c, d and e.



Question creation - Have fun creating problems for your friends to solve. Using this bar of chocolate to help you, create your own fraction triangle. Your fractions can not have the same denominator.



More chocolate bar questions

1. Find four fractions that add up to $\frac{23}{24}$ with none of the denominators being 24.

2. Find two different pairs of fractions that add up to $\frac{19}{20}$ with neither of the denominators being 20.



3. Find a pair of fractions that add up to $\frac{20}{21}$ with neither of the denominators being 21.



4. Find four fractions that add up to $\frac{35}{36}$ with none of the denominators being 36.

   Can you find more than one answer?

Adding and subtracting fractions

Adding and subtracting fractions

Work out 1. $\frac{1}{3}+\frac{3}{7}$ 2. $\frac{5}{6}-\frac{3}{4}$

Evie's working for Q1.



Evie

$\frac{1}{3} = \frac{7}{21}$ $\frac{3}{7} = \frac{9}{21}$

$\frac{1}{3} + \frac{3}{7} = \frac{7}{21} + \frac{9}{21} = \frac{16}{21}$

Jon's working for Q2.

$\frac{5}{6} = \frac{10}{12}$ $\frac{3}{4} = \frac{9}{12}$

$\frac{5}{6} - \frac{3}{4} = \frac{10}{12} - \frac{9}{12} = \frac{1}{12}$



Jon

Complete these addition grids

+	$\frac{5}{8}$	$\frac{3}{5}$
$\frac{1}{3}$	$\frac{23}{24}$	
$\frac{5}{12}$		

+		$\frac{1}{4}$
$\frac{3}{8}$	$\frac{9}{16}$	
		$\frac{11}{16}$

Working with mixed numbers

1. Write $1\frac{5}{6}$ as a top- heavy fraction or improper fraction.
2. Write $\frac{21}{4}$ as a mixed number.
3. Work out $1\frac{5}{6}+2\frac{1}{4}$.

Now complete these addition grids:

+	$1\frac{5}{6}$	$3\frac{2}{5}$
$2\frac{1}{4}$	$4\frac{1}{12}$	
$4\frac{7}{8}$		$8\frac{11}{40}$

+		$1\frac{3}{4}$
	$2\frac{7}{8}$	$3\frac{1}{4}$
$1\frac{5}{8}$		

+	$1\frac{2}{3}$	
	$3\frac{4}{5}$	$4\frac{7}{15}$
$2\frac{1}{5}$		

Multiplying fractions

Calculate $\frac{4}{9} \times \frac{3}{8}$.

Jon's working



Jon

$\frac{4}{9} \times \frac{3}{8} = \frac{\cancel{12}^{1}}{\cancel{72}_{6}} = \frac{1}{6}$

Alice's working

$\frac{\cancel{4}^{1}}{\cancel{9}_{3}} \times \frac{\cancel{3}^{1}}{\cancel{8}_{2}} = \frac{1}{6}$



Alice

Look at this calculation $\frac{5}{14} \times \frac{7}{9} \times \frac{18}{25}$.

1

Use either Jon's or Alice's approach for this calculation.
Which method do you think will be easier? Explain your thinking.

Evie's working



Evie

I am going to use Alice's approach. It makes the multiplication stage much easier.

$\frac{\cancel{5}^{1}}{\cancel{14}_{\cancel{2}_{1}}} \times \frac{\cancel{7}^{1}}{\cancel{9}_{1}} \times \frac{\cancel{18}^{\cancel{2}^{1}}}{\cancel{25}_{5}} = \frac{1}{5}$

Fitness check

Work out:

1. $\frac{7}{22} \times \frac{11}{33}$
2. $\frac{5}{6} \times \frac{8}{15} \times \frac{3}{16}$
3. $\frac{4}{9} \times \frac{18}{35} \times \frac{25}{24}$
4. $\frac{4}{7} \times \frac{4}{5} \times \frac{5}{4}$
5. $\frac{7}{11} \times \frac{9}{8} \times \frac{8}{9}$
6. $1\frac{2}{3} \times 1\frac{4}{5}$
7. $3\frac{3}{7} \times 1\frac{5}{16}$
8. $2\frac{2}{9} \times 1\frac{4}{11} \times 1\frac{11}{20}$
9. $1\frac{7}{9} \times 1\frac{3}{5} \times \frac{5}{8}$
10. $4\frac{4}{5} \times 2\frac{4}{7} \times \frac{7}{18}$

Sarah's work on Q5



Sarah

$$\frac{7}{11} \times \frac{\cancel{9}^{1}}{\cancel{8}_{1}} \times \frac{\cancel{8}^{1}}{\cancel{9}_{1}} = \frac{7}{11}$$

That's the same as I started with.

Reciprocals

In the question, Sarah realised she had got back to the number she started with.

This is because $\frac{8}{9}$ is the **reciprocal** (or **multiplicative inverse**) of $\frac{9}{8}$.

It may help to remember that "reciprocal" comes from the Latin *reciprocus* meaning *returning*.

Multiplying a number by its reciprocal always gives 1, and so their combined effect leaves everything unchanged.

For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ because $\frac{2}{3} \times \frac{3}{2} = 1$.

This also works the other way round, so the reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$.

To find the inverse of a fraction, you simply flip the fraction.

So the reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$.

2 What is the reciprocal of $\frac{1}{4}$?

Jon's working



Jon

The reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$ but $\frac{4}{1}$ is the same as 4, so the reciprocal of $\frac{1}{4}$ is 4.

Fitness check

Write down the reciprocals of :

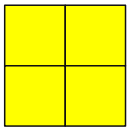
a) $\frac{5}{8}$ b) $\frac{7}{2}$ c) $2\frac{3}{4}$ d) $\frac{1}{9}$ e) 3 f) 6

Just say what you see

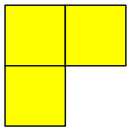


Sarah

If this shape represent 1



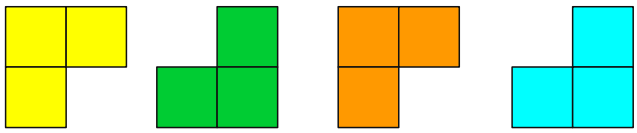
Then this shape represents ...

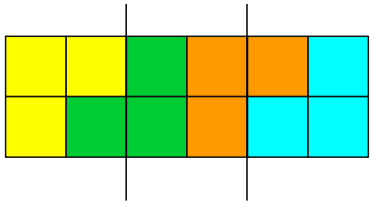




Amber

Look carefully at my diagrams. They show that ...





3 What do you think Amber was about to say?



Sarah

It shows that 4 lots of $\frac{3}{4}$ make 3

or $4 \times \frac{3}{4} = 3$

It also shows how many $\frac{3}{4}$ are in 3

So I would write $3 \div \frac{3}{4} = 4$

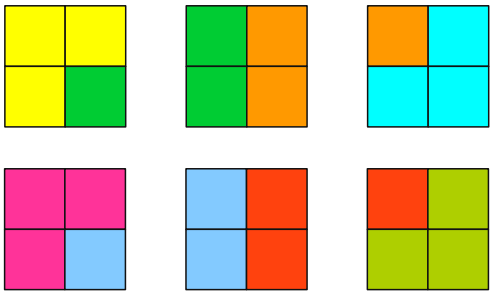


Matthew



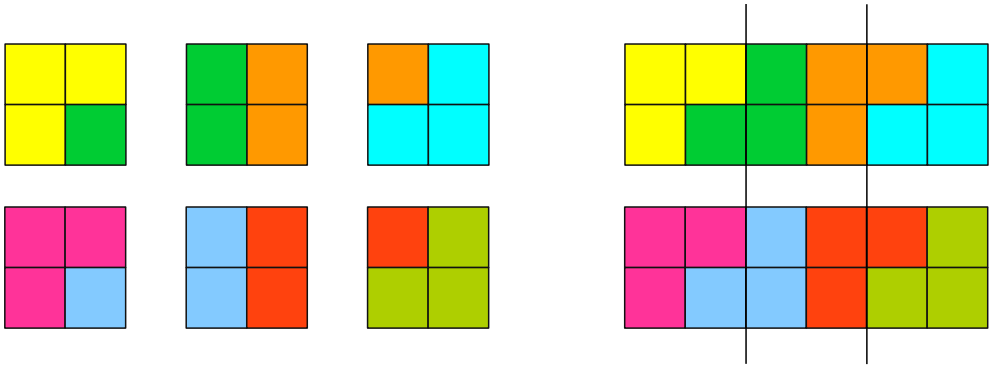
Matthew

In that case, this diagram shows that ...



4 What do you think Matthew was about to say?

Just say what you see





Amber

It shows that 8 lots of $\frac{3}{4}$ make 6

or $8 \times \frac{3}{4} = 6$

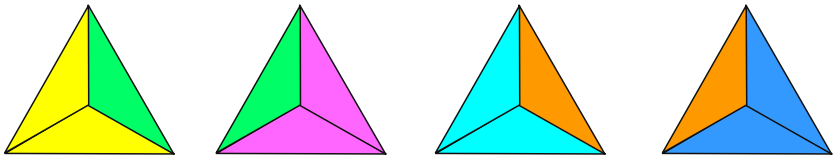
It also shows how many $\frac{3}{4}$ are in 6

So I would write $6 \div \frac{3}{4} = 8$



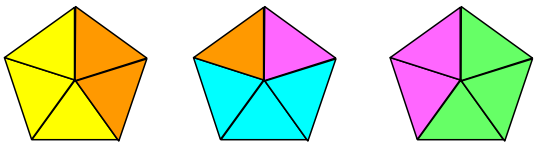
Jon

Look at this set of triangles.



5 Write down two maths statements that these diagrams illustrate.

Now, look at these pentagons.



6 Write down two maths statements that these diagrams illustrate.

Let's think about dividing by fractions

$3 \times 2 = 6$ means 3 lots of 2 are 6

$6 \div 2 = 3$ can mean "How many twos are in six?"

$2 \times \frac{1}{2} = 1$ means two lots of a half are 1.

$1 \div \frac{1}{2} = 2$ can mean "How many halves are in 1?"

$3 \times 5 = 15$ can be written in various different ways....



$5 \times 3 = 15$

Matthew

$15 \div 3 = 5$



Jon



$15 \div 5 = 3$

Sarah

1. Write each of these maths statements in 2 other ways...

   a) $4 \times 6 = 24$

   b) $21 \div 7 = 3$

   c) $5 \times 9 = 45$

   d) $32 \div 4 = 8$

   e) $10 \times \frac{1}{2} = 5$

   f) $6 \times \frac{2}{3} = 4$

   g) $10 \div \frac{1}{2} = 20$

Dividing by fractions

The calculation: $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$

Can be written as: $\frac{10}{21} \div \frac{5}{7} = \frac{2}{3}$

Or: $\frac{10}{21} \div \frac{2}{3} = \frac{5}{7}$

1. How does this help evaluate: $\frac{8}{15} \div \frac{2}{3}$?

Alice's working



Alice

$\frac{8}{15} \div \frac{2}{3} = \frac{8 \div 2}{15 \div 3} = \frac{4}{5}$

Fitness check



1. $\frac{12}{22} \div \frac{4}{11} =$

2. $\frac{8}{9} \div \frac{2}{3} =$

3. $\frac{63}{75} \div \frac{9}{25} =$

4. $\frac{2}{5} \div \frac{3}{4} =$



Evie

The last one doesn't work. There must be another way of doing this.

You can do it if you use equivalent fractions...



Sarah

6

Can you use Sarah's idea to work out $\frac{2}{5} \div \frac{3}{4}$?

Sarah's explanation



Sarah

$\frac{2}{5} = \frac{24}{60}$ (×12 on both)

$\frac{2}{5} \div \frac{3}{4} = \frac{24}{60} \div \frac{3}{4} = \frac{8}{15}$

Fitness check



Use Sarah's method to work out:

1. $\frac{7}{11} \div \frac{2}{3}$

2. $\frac{3}{4} \div \frac{2}{7}$

3. $\frac{5}{9} \div \frac{4}{5}$

Amber's working



Amber

$\frac{7}{11} = \frac{42}{66}$ (×6 on both)

$\frac{7}{11} \div \frac{2}{3} = \frac{42}{66} \div \frac{2}{3} = \frac{21}{22}$

$\frac{3}{4} = \frac{42}{56}$ (×14 on both)

$\frac{3}{4} \div \frac{2}{7} = \frac{42}{56} \div \frac{2}{7} = \frac{21}{8}$

$\frac{5}{9} = \frac{100}{180}$ (×20 on both)

$\frac{5}{9} \div \frac{4}{5} = \frac{100}{180} \div \frac{4}{5} = \frac{25}{36}$



Matthew

I've noticed something very interesting about the answers.

7 Look at the four answers below. What do you think Matthew has noticed?

$$\frac{2}{5} \div \frac{3}{4} = \frac{8}{15}$$

$$\frac{7}{11} \div \frac{2}{3} = \frac{21}{22}$$

$$\frac{3}{4} \div \frac{2}{7} = \frac{21}{8}$$

$$\frac{5}{9} \div \frac{4}{5} = \frac{25}{36}$$

8 Can you now write down the answer to $\frac{4}{7} \div \frac{5}{9}$

Alice's explanation



Alice

$\frac{2}{5} \div \frac{3}{4} = \frac{8}{15}$ $\quad 2 \times 4 = 8$ $\quad 5 \times 3 = 15$

You just multiply the diagonals to get the answer!

$\frac{4}{7} \div \frac{5}{9} = \frac{36}{35}$ $\quad 4 \times 9 = 36$ $\quad 7 \times 5 = 35$

Matthew's explanation



Matthew

Or you could just say that dividing by a fraction is the same as multiplying by its reciprocal.

So $\frac{4}{7} \div \frac{5}{9}$ is the same as $\frac{4}{7} \times \frac{9}{5} = \frac{36}{35}$

Dividing by fractions

Fitness check



1. $\frac{3}{5} \div \frac{4}{7} =$

2. $\frac{6}{13} \div \frac{5}{11} =$

3. $\frac{15}{21} \div \frac{3}{7} =$

4. $2\frac{1}{2} \div 1\frac{1}{3} =$

5. $3\frac{2}{3} \times 4\frac{1}{4} =$

6. $3\frac{1}{2} \div 4\frac{1}{3} =$

7. $2\frac{1}{5} \times 2\frac{1}{3} =$

8. $2\frac{1}{5} \div 2\frac{1}{3} =$

9. There are some marbles in a bag. Aimee took half of the marbles out of the bag and then put one back into the bag. She repeated this 5 times. There were then 3 marbles left in the bag. How many marbles were in the bag to start with?

10. PhoneFast sold 20 more than half its mobile phone stock in January. It sold 15 more than half the remaining number of mobile phones in February. It had 75 mobile phones left in March before any more were sold. How many mobile phones did PhoneFast have in stock initially?

11. Several commuters were on a bus when it left the bus station. At the first stop, half of the commuters got off. At the second stop, half of the remaining commuters got off. This was repeated at each stop until after the fifth bus stop, there were left only 4 commuters left on the bus. How many commuters boarded the bus at the bus station?

12. I have some beads. When I divide all the beads into 4 equal groups, there is one bead left over. When I divide one of these groups into 4 equal groups, there is also one left over. When I split one of these groups into 4, there is still one left over. What is the minimum number of beads I had?

13. Steph is playing with her younger sister's bubble wand. Her bubble wand ejects 100 bubbles at a time. Half of the bubbles burst within one minute. 25% of the original number of bubbles survive for at least 2 minutes. Only 2% of the original number of bubbles survive for more than 3 minutes. All the bubbles have burst at the end of the fourth minute. Steph blew the bubble wand every minute; how many bubbles were there altogether after 10 minutes?

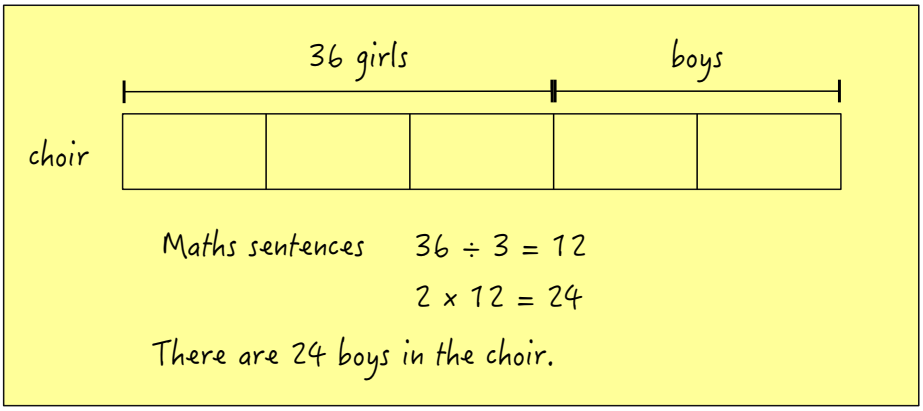
Puzzle corner

1. $\frac{3}{5}$ of the choir are girls and the rest are boys. If there are 36 girls, how many boys are there in the choir?

Alice's working for Q1



Alice

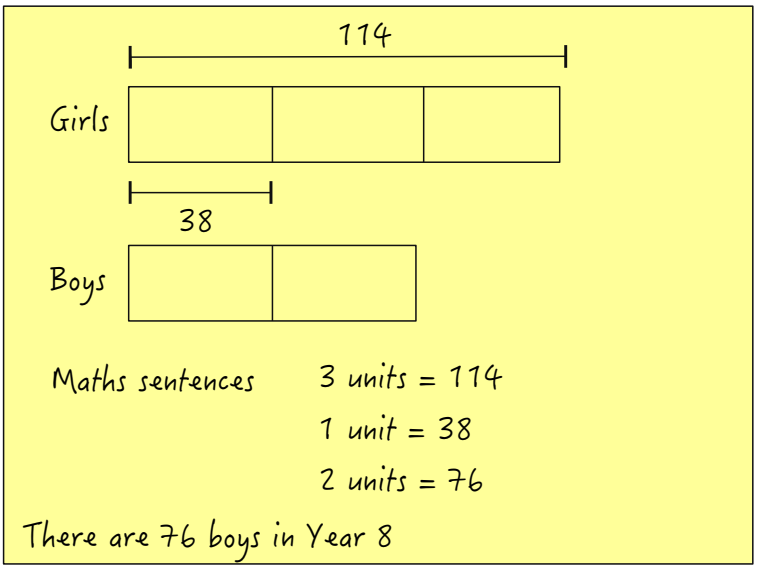


2. Sarah has bought 28 flowers. $\frac{3}{4}$ of them are white. How many white flowers did Sarah buy?
3. $\frac{4}{7}$ of the stamps on a page are red. The rest are blue or green. There are twice as many blue stamps as green stamps. There are 24 more red stamps than blue stamps. How many of each colour are there?
4. Year 7 brought in 360 cakes for their cake sale. They sold $\frac{5}{6}$ of them. They gave $\frac{2}{3}$ of the remainder to their teachers. How many cakes did they have left?
5. There are $\frac{2}{3}$ as many boys in Year 8 as there are girls. If there are 114 girls, how many boys are there?

Jon's working for Q5



Jon



6. Ross and his sister Rosie have been saving for their holidays. Ross has saved $\frac{3}{4}$ as much as Rosie. Altogether they have saved £84. How much has each of them saved?

7. There are 48 books in the fantasy section of the school library. James has read $\frac{3}{8}$ of them. Alison has read $\frac{2}{3}$ of the 48 books. How many more books has Alison read than James?

8. In a pile of counters, $\frac{1}{4}$ are red, $\frac{1}{3}$ are blue, and the rest are yellow. If there are 36 counters altogether, how many of each colour are there and what fraction are yellow?

9. Maddie spent an equal amount of money each day. After 4 days, she had $\frac{3}{4}$ of her money left. After another 9 days, she had £12 left. How much money did she have to start with?

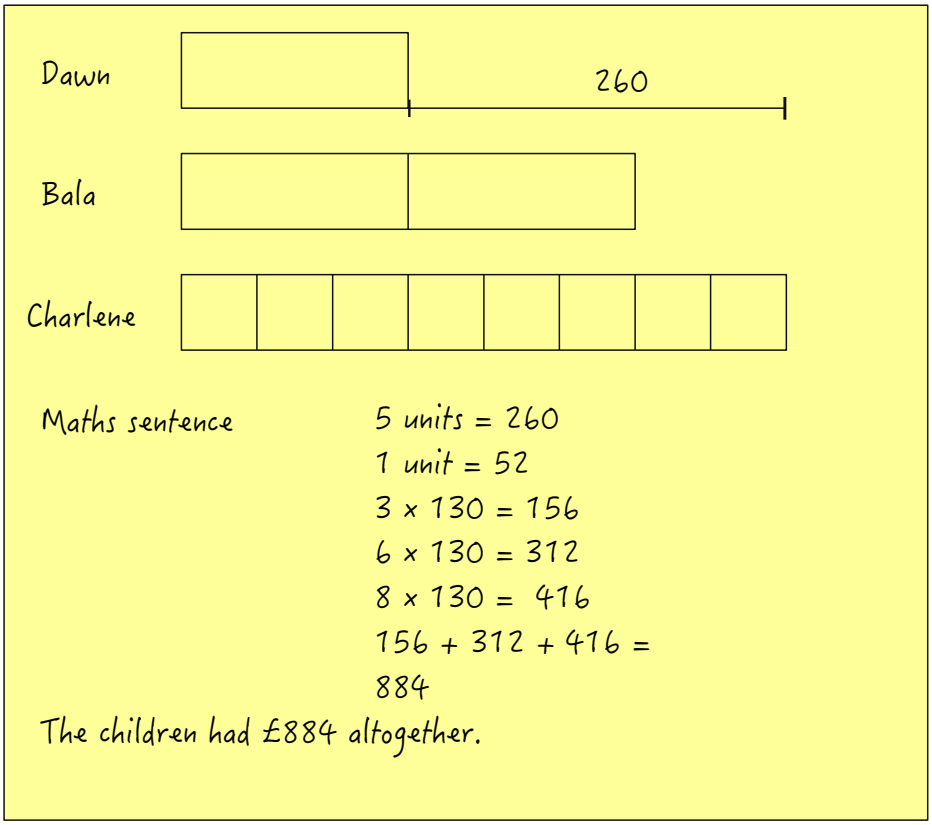
10. Mr Harris gives $\frac{2}{5}$ of his salary to Mrs Harris. Mrs Harris spends $\frac{3}{4}$ of this on groceries and saves the rest. What fraction of Mr Harris' salary did she save?

11. Bala had $\frac{3}{4}$ as much money as Charlene. Dawn had $\frac{1}{2}$ as much as Bala. If Dawn had £260 less than Charlene, find the sum of money the three children had altogether.

Sarah's working on Q11



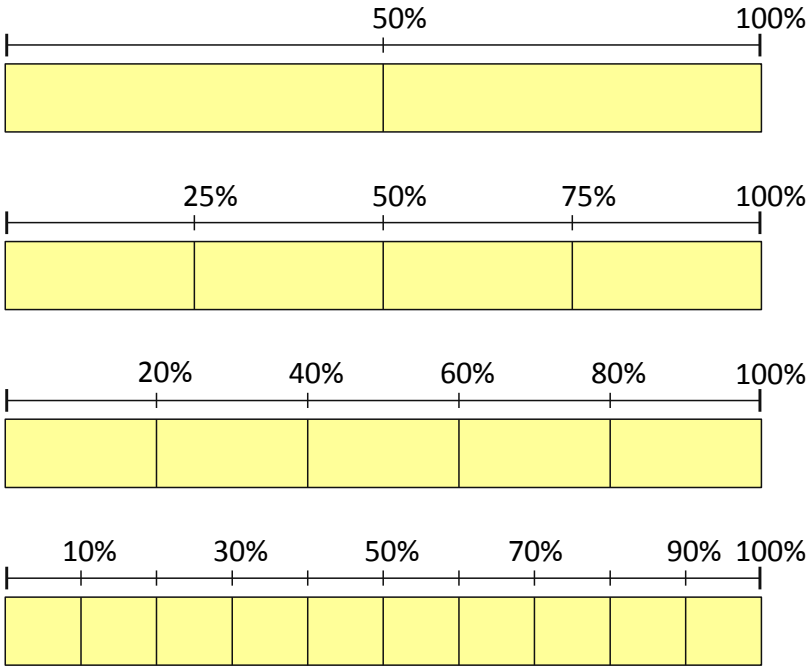
Sarah



Working with percentages

Percentages are another way to work out parts of a quantity. Per cent actually means 'out of 100', so 50% means 50 parts out of 100. In other words, finding 50% of a quantity is the same as finding $\frac{1}{2}$ of a quantity. Finding 25% of a quantity is the same as finding $\frac{1}{4}$ of a quantity and so on.

In each question, a decision on the 'unit' of measure has to be made as illustrated below.



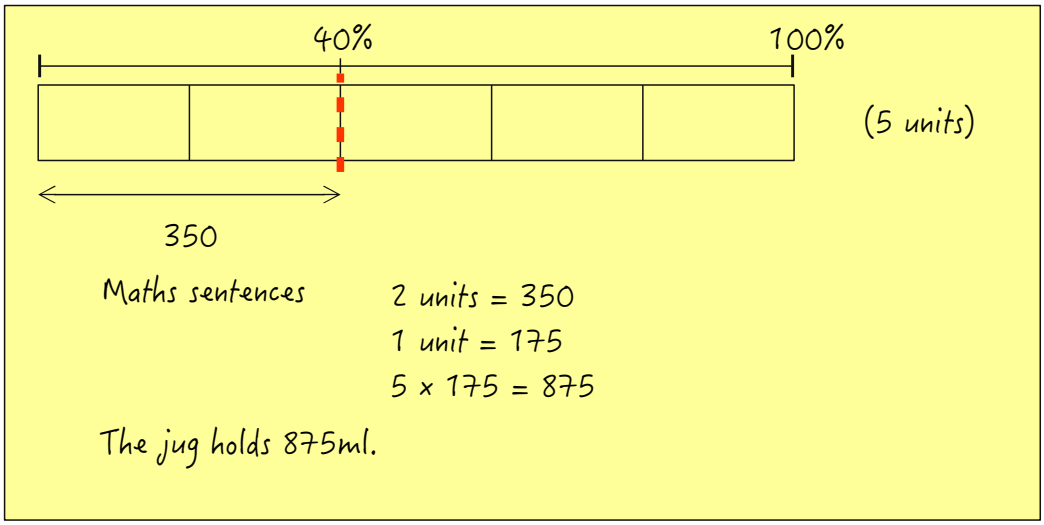
1% would mean drawing 100 boxes and probably isn't the most sensible approach!

Puzzle Corner - Percentages

1. Mrs Charge poured 350ml of water into a jug and noticed it was only 40% full. How much water would the jug hold altogether?

Matthew's working on Q1





2. 300 pupils took part in a fun run to raise money for charity. 25% of them took under an hour to complete the course. How many pupils took longer than an hour?

3. There were 500 people at a concert. 30% of them were children. How many children were at the concert?

4. 1400 pupils helped make the world's biggest smiley face last year, but only 40% of these were female. How many females took part?

5. 65 000 fans watched Arsenal's last home game, of which 90% supported Arsenal. How many fans did not support Arsenal?

6. 198 children went on a school trip to the Cathedral. This was 10% more than last year. How many went last year?

7. There has been a 15% rise in the number of pupils having school lunch this year. If 400 pupils had lunch each day last year, how many extra lunches should the kitchen now make per day?

8. The estimated number of people travelling by train to London each day from our region is 3600. Next year the train operators are predicting a 4% rise in numbers. If their predictions are correct, how many people will be travelling to London each day next year?

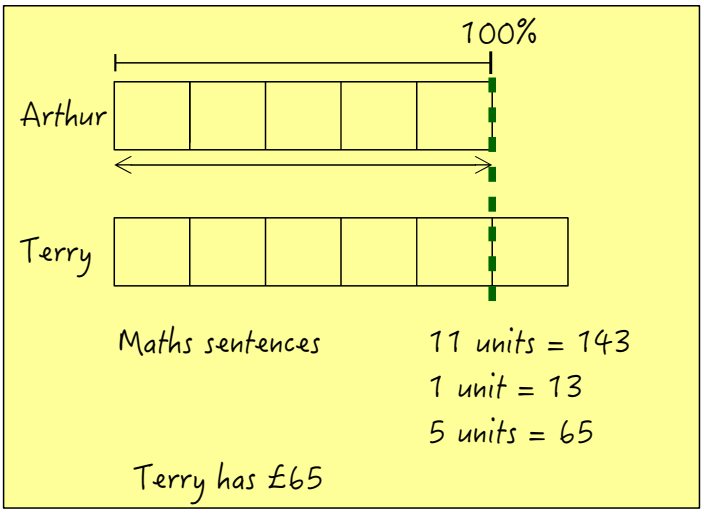
9. Brian's parcel weighs 5kg. Jenny's parcel is 15% lighter. Find the mass of Jenny's parcel.

10. Arthur and Terry have £143 between them. If Terry has 20% more than Arthur, how much money does Arthur have?



Evie

Evie's working on Q10



11. Jazzy and Megan have 828 stickers altogether. Jazzy has 40% more stickers than Megan. How many more stickers does Jazzy have than Megan?

12. 84 000 supporters watched Chelsea's last 2 games. If there were 25% fewer at the second game, how many watched the first game?

13. Darren saved 50% more each month than in the previous month. After 3 months, he had saved £95. How much did he save in the second month?

Number 3b: Mastery questions

1. What is $1\frac{7}{9}+2\frac{1}{3}$?

A	$3\frac{8}{9}$	B	$4\frac{1}{9}$	C	$3\frac{10}{9}$	D	$3\frac{8}{12}$

2. What is $3\frac{2}{3}-1\frac{3}{4}$?

A	$1\frac{11}{12}$	B	$2\frac{1}{12}$	C	$2\frac{1}{7}$	D	$1\frac{6}{7}$

3. What is $\frac{5}{3}\times 1\frac{3}{4}$?

A	$\frac{35}{12}$	B	$\frac{15}{12}$	C	$\frac{20}{21}$	D	$2\frac{3}{4}$

4. What is $2\div\frac{1}{4}$?

A	$\frac{2}{4}$	B	$\frac{1}{8}$	C	8	D	4

5. What is $\frac{9}{20}\div\frac{3}{4}$?

A	$\frac{3}{5}$	B	$1\frac{4}{5}$	C	$\frac{27}{80}$	D	$\frac{27}{5}$

6. What is $2\frac{1}{4}\div\frac{3}{8}$?

A	$\frac{27}{32}$	B	$1\frac{5}{32}$	C	$1\frac{2}{3}$	D	6

7. What is $1\frac{7}{9}\div 2\frac{1}{3}$?

A	$\frac{48}{63}$	B	$\frac{16}{21}$	C	$\frac{9}{27}$	D	$1\frac{1}{3}$

8. Put these numbers into order, smallest first. $\frac{7}{8}$, 0.25, 0.8, $\frac{9}{16}$, 0.625

A	0.8, 0.25, $\frac{9}{16}$, 0.625, $\frac{7}{8}$
B	0.25, 0.625, 0.8, $\frac{7}{8}$, $\frac{9}{16}$
C	0.25, $\frac{9}{16}$, 0.625, 0.8, $\frac{7}{8}$
D	0.8, 0.25, 0.625, $\frac{9}{16}$, $\frac{7}{8}$

9. Which of the sets of numbers is in order smallest to largest?

A	0.33, $\frac{3}{8}$, 3%, 0.303
B	$\frac{1}{3}$, 30%, $\frac{10}{30}$, 0.35
C	0.15, $\frac{1}{5}$, 25%, $\frac{10}{15}$
D	80%, $\frac{6}{8}$, 0.68, 0.86

10. Which of the sets of numbers is in order smallest to largest?

A	3.5, $\frac{9}{4}$, 300%, 3.25
B	$\frac{9}{4}$, 300%, 3.25, 3.5
C	$\frac{9}{4}$, 3.25, 3.5,300%
D	80%, $\frac{6}{8}$, 0.68, 0.86

11. If $\frac{2}{3}$ of an amount is 14, what is the original amount?

A	9	B	14	C	18	D	21

12. Which is the smallest amount?

A	75% of 600	B	$\frac{5}{8}$ of 640
C	60% of 700	D	0.375 x 1200

Number 3b: Mastery questions

13. Which is the largest amount?

A	$33\frac{1}{3}\%$ of 960	B	$\frac{7}{8}$ of 360
C	40% of 900	D	0.6 x 610

14. What is 80 as a percentage of 120?

A	80%	B	75%	C	$66\frac{2}{3}\%$	D	150%

15. What is 120 as a percentage of 80?

A	80%	B	75%	C	$66\frac{2}{3}\%$	D	150%

16. Write 35% as a fraction in its simplest terms.

A	$\frac{35}{100}$	B	$\frac{7}{20}$	C	0.35	D	$\frac{350}{100}$

17. Write $\frac{1}{2}$ % as a decimal.

A	0.5	B	0.05	C	0.005	D	0.2

18. A local supermarket is offering 20% off Easter eggs this week. If the original price was £3.20, what is the reduced price?

A	£3.00	B	£2.56	C	£2.60	D	£2.80

19. A laptop originally cost £484. But the price has now been increased by 25%. What is the new price?

A	£363	B	£509	C	£605	D	£534

20. A painting was valued at £300 two years ago. It is now valued at £420. What is the percentage rise in its value?

A	120%	B	140%	C	30%	D	40%