One for you, two for me

Sometimes things in life simply don't seem fair.

Amber has been picking strawberries and agrees to share them with Matthew.





Amber

As I did all of the work, I am going to share my strawberries out so that it is one for you and two to me. Altogether I picked 81 strawberries.



Matthew

Ok, that sounds fair but how many will I get?

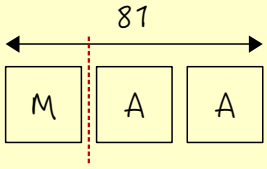
1 Can you draw a diagram that might help Matthew to answer his own question?



Evie

Evie's idea

You could have 3 boxes, one for Matthew and two for Amber and then share the strawberries out equally between the boxes.

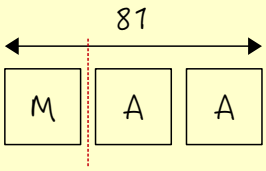


2 Can you use Evie's idea to solve the problem?



Evie

Evie's idea continued



81 ÷ 3 = 27
So Matthew has 1 x 27 = 27 strawberries
And Amber has 2 x 27 = 54 strawberries.

3 How many strawberries would Matthew receive if Amber had actually picked 144 strawberries?

Two for you, three for me



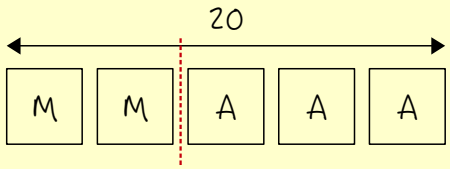
4

Can you split this money so that Matthew gets £2 for every £3 that Amber gets?



Taro

Taro's solution



20 ÷ 5 = 4
So Matthew has 2 x 4 = £8
And Amber has 3 x 4 = £12

Some useful terminology

A **ratio** is a way to compare amounts of something. In our first example, Matthew received 1 strawberry for every 2 strawberries that Amber received. We can express this mathematically by saying the ratio of Matthew's part to Amber's part is 1 to 2. We write this ratio as 1:2. In the second case, Matthew received £2 for every £3 that Amber received and we could say we split the money in the ratio of 2:3.

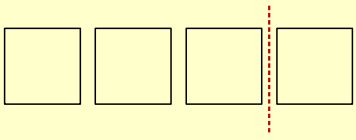
5

Draw a diagram to show how you would split a quantity in the ratio 3:1.

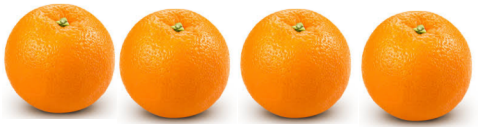


Jon

Jon's diagram



1. In a basket of fruit there are 4 oranges, 6 bananas and 8 apples.





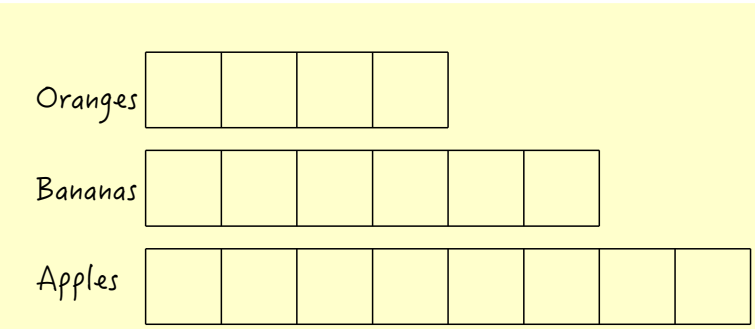


6 Draw a simple diagram to represent this information



Sarah

Sarah's diagram



a) How could we write the ratio of the number of oranges to the number of bananas?

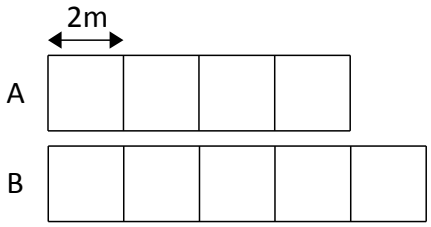
b) How could we write the ratio of the number of oranges to the number of apples?

c) How could we write the ratio of the number of bananas to the number of apples?

d) How could we write the ratio of the number of apples to the number of bananas?

e) How could we write the ratio of the number of oranges to the number of bananas to the number of apples?

2. The lengths of two pipes, A and B, are represented using this model.



a) What is the the total length of each pipe?

b) What is the ratio of the length of pipe A to the length of pipe B?

c) What is the ratio of the length of pipe B to the length of pipe A?

d) What is the total length of both pipes?

e) What fraction of the total length is the length of pipe A?

f) What fraction of the total length is the length of pipe B?

g) What fraction of the length of B is the length of A?

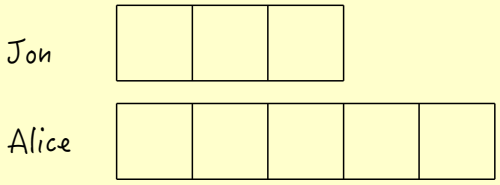
3. Jon and Alice both collect classic car magazines. The ratio of Jon's collection to Alice's collection is 3:5.

a) Represent this information on a simple diagram.



Evie

Evie's diagram



b) What is the ratio of Jon's collection to the total collection?

c) What is the ratio of Alice's collection to the total collection?

d) What fraction of the total collection is Jon's collection?

e) What fraction of the total collection is Alice's collection?

f) What fraction of Alice's collection is Jon's collection?

g) If they have 64 magazines altogether, how many does Jon have?

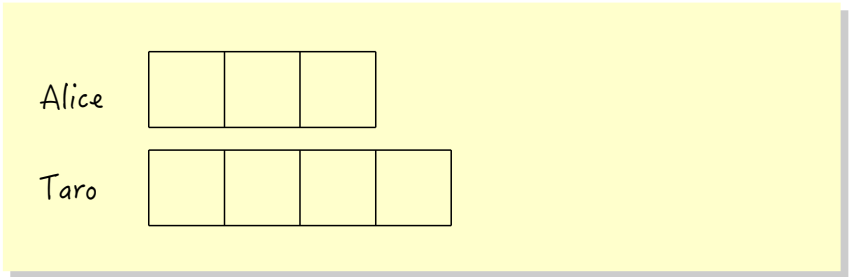
4. Alice has saved $\frac{3}{4}$ as much money as Taro.

a) Represent this information on a simple diagram.



Jon

Jon's diagram



b) What is the ratio of Alice's savings to Taro's savings?

c) What is the ratio of Alice's savings to their total savings?

d) What fraction of their total savings is Taro's savings?

e) If Taro has saved £128, how much as Alice saved?

f) If instead their total savings were £252, how much has Taro saved?

5. Matthew cut a 6m length of wood into two pieces in the ratio of 3:7

a) Represent this information on a simple diagram.

b) What is the length of each piece of wood?

6. Look at this chocolate bar. If we share it between two people in the ratio of 3:5, how many squares would each person receive?



7. If instead, the chocolate bar was shared out in the ratio of 1:5, how many squares would each person receive.

8. Look at this chocolate bar. If this is shared out in the ratio 3:4, how many squares would each person receive?



9. In Q8, if the chocolate bar was shared out so that Alice got 6 squares and Amber got 15 squares, what would be the ratio of their shares in its simplest form?

10. How many squares might there be in a chocolate bar that can be shared out in the ratio 2:3?

11. In the following table, you are given the ratio of one quantity to another. Complete the table.

Ratio statement	Fraction statements	
A:B = 2:5	A is $\frac{\square}{\square}$ of B	B is $\frac{\square}{\square}$ of A
C:D = 3:7	C is $\frac{\square}{\square}$ of D	D is $\frac{\square}{\square}$ of C
E:F = 5:9	E is $\frac{\square}{\square}$ of F	F is $\frac{\square}{\square}$ of E

12. In the following table, you are given the ratio of one quantity to another. Complete the table.

Ratio statement	Fraction statements	
A:B = 4:5	A is $\frac{\square}{\square}$ of the total	B is $\frac{\square}{\square}$ of the total
C:D = 5:7	C is $\frac{\square}{\square}$ of the total	D is $\frac{\square}{\square}$ of the total

13. A library classifies its books as fiction and non-fiction books.

    The number of fiction books is $\frac{4}{5}$ of the number of non-fiction books.

    a) Draw a suitable diagram to model this information.

    b) If there are 30 more non-fiction books that fiction books. How many books are in the library?

14. A box contains apples, oranges and pears. The ratio of the number of apples to the number of pears is 4:3.

    The number of oranges is $\frac{2}{3}$ of the number of pears.

    a) Draw a suitable diagram to model this information.

    Altogether there are 30 apples less than the total number of oranges and pears.

    How many pears are in the box?

15. Taro spent $\frac{4}{7}$ much as Amber on Christmas presents last year.

    Altogether they spent £99 on presents. How much did Taro spend?

16. Mrs Sharma's salary is $\frac{7}{2}$ times her daughter's salary.

    a) Represent this information on a simple diagram.

    b) If their combined salary is £4500 per month, how much does Mrs Sharma's daughter earn each month?

Some more questions

17. The sides of a triangle are in the ratio 2:3:4. If the sum of the lengths of the smaller two sides is 13cm more than the length of the largest side, what is the perimeter of the triangle?

18. The ratio of boys to girls in class 8X is 5:4. How many girls might there be in 8X? Explain your thinking clearly.

19. Jon has some blue and some red marbles in a box. The ratio of blue marbles to red marbles is 3:5. If Jon gives away 12 blue marbles the ratio of blue marbles to red marbles becomes 1:2.

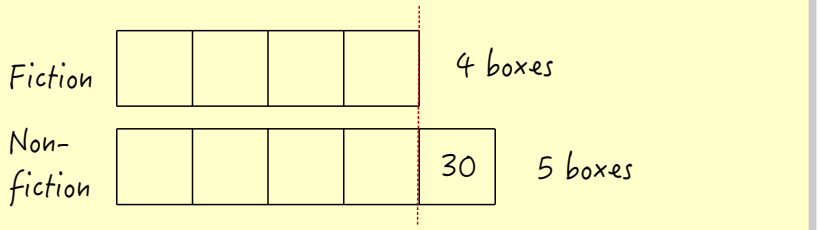
    How many red marbles does Jon have in his box?

20. Alice baked some cakes and shared them with her brother in the ratio of 4:1. She took some to school for the cake sale and took the rest to her youth club for the tuck shop in the ratio of 3:1. If Alice took 14 more cakes to school than she gave her brother, how many cakes did she make?

Taro's solution to Q13



Taro

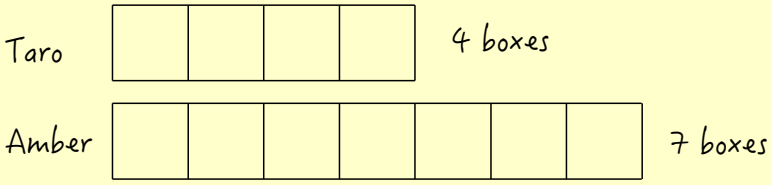


4 boxes for fiction books, so 5 boxes for non-fiction books.
So 30 books in one box.
So 9 x 30 = 270 books in the library.

Alice's solution to Q15



Alice

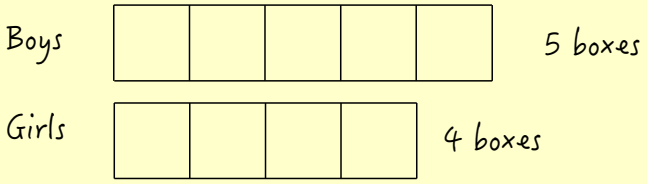


11 boxes altogether
£99 ÷ 11 = £9
Taro spent 4 x 9 = £36



Amber

Amber's solution to Q18

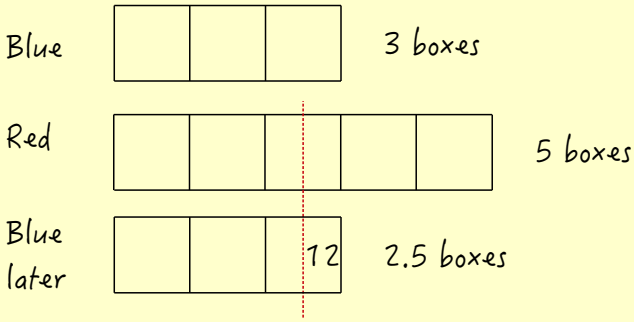


9 boxes altogether
There could be 18, 27 or 36 in a class
So each box could be 2, 3 or 4
So the number of girls could be 8, 12 or 16



Matthew

Matthew's solution to Q19

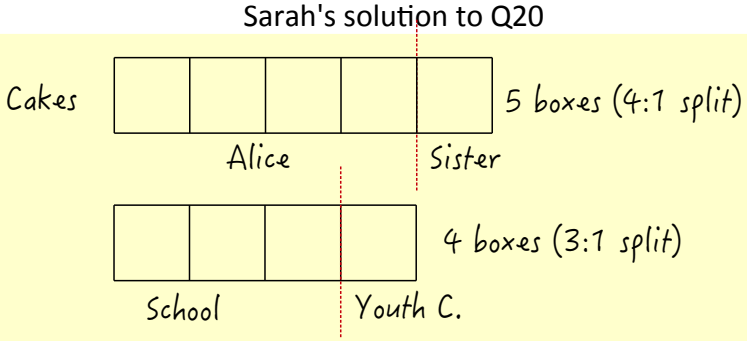


0.5 box = 12
So one box = 24
The number of red marbles= 5 x 24 = 120
There are 120 red marbles in the box.



Sarah

Sarah's solution to Q20



Alice took 3 boxes to school. Her brother had one box.
Alice took 14 more cakes to school than she gave her brother.
So there 14 cakes in 2 box, so 7 in each box.
Altogether she made 5 x 7 = 35 cakes.

Which is stronger?

8Z have been asked to run the refreshment stand at sports day.
They have decided to sell orange squash by the cupful for 20p.



Alice

Some people like their squash strong but some people like it weak. I think we should sell 2 versions – strong or weak.

7 Can you explain what Alice means by strong or weak orange squash?

Which is stronger, or are they the same?

In each of these 5 cases, decide whether A is stronger than B, B is stronger than A or they are both the same strength,

A	B
1 orange, 1 water	1 orange, 3 water
1 orange, 2 water	2 orange, 1 water
1 orange, 1 water	2 orange, 2 water
1 orange, 2 water	2 orange, 3 water
1 orange, 2 water	2 orange, 4 water



Sarah

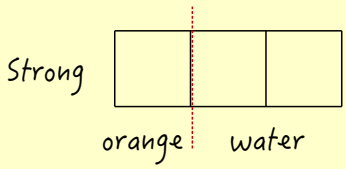
Eventually we decided that strong squash would be 1 parts orange juice to two parts water. So the ratio of orange to water is 1:2.
And for weak squash we would have a ratio 1:3.

8 8Z decided to make 60 litres of both strong squash and weak squash. How many litres of orange juice will they need?

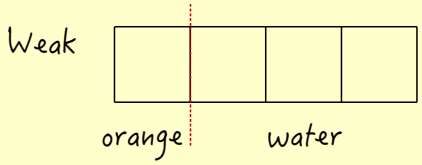


Evie

Evie's solution



3 boxes = 60 litres
So 20 litres per box



4 boxes = 60 litres
So 15 litres per box

8Z will need 1 x 20 = 20 litres for the strong squash
And 1 x 15 = 15 litres for the weak squash,
Altogether they will need 35 litres of orange juice

Different ways of writing the same thing

Earlier we said the ratio of boys to girls in 8Z was 5:4.
Now suppose this ratio is true for all the pupils in Year 8.
How many boys could be in Year 8?



Taro

Taro's working

We don't know how many pupils are in Year 8 but we do know the ratio of boys to girls is 5:4.
So I have made a table of some possible answers.

Boys	5	50	80	120	135
Girls	4	40	64	96	108
Total	9	90	144	216	243

The number of boys must be a multiple of 5
The number of girls must be a multiple of 4
The total number of pupils must be a multiple of 9

9 Can you explain Taro's working? And can you find three more possible solutions?

Different ways of writing the thing

1. Look at this table. Can you fill in the blanks assuming the ratio of boys to girls remains constant?

Boys	3	30	?	?
Girls	5	?	70	?
Total	8	?	?	216

2. Look at this table. Can you fill in the blanks assuming the ratio of boys to girls remains constant?

Boys	18	45	?	?
Girls	20	?	100	?
Total	38	?	?	209

10

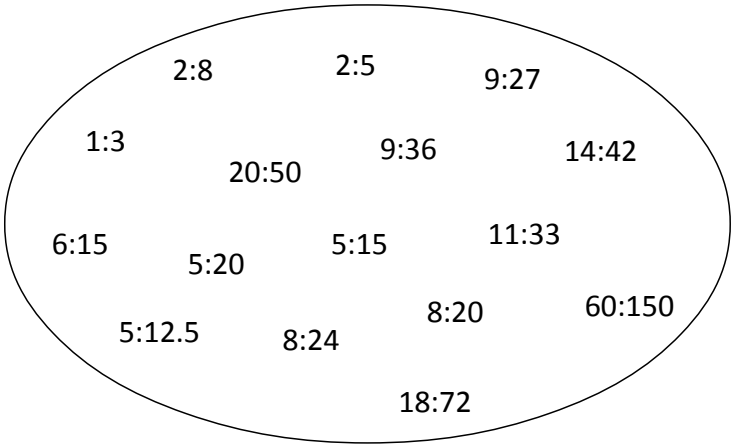
In Q2 above, what is the ratio of boys to girls in its simplest form?

3. Evie says that 1:2, 2:4, 5:10 and 15:30 are all **equivalent ratios**. What do you think Evie means by equivalent ratios?

4. Write three equivalent ratios to each of the following ratios.

   a) 3:5 b) 2:7 c) 3:2

5. Look at this collection of ratios. Can you split them in to three different groups so that each group contains **equivalent ratios**? Explain your reasoning.



Properties of equivalent ratios

Earlier Evie said that the ratios 1:2, 2:4, 5:10 and 15:30 were all equivalent. You were asked to explain what this might mean.


Taro

Taro's explanation

If we mixed 1L of orange juice with 2L of water TWICE we would have 2L of orange juice and 4L of water but the strength of the squash would be unchanged.
If we did this another three times we would have 5L of orange juice and 10L of water but the strength of the squash would still be the same.
We can just keep doing this until we get to 15L of orange juice and 30L of water.


Amber

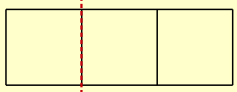
Amber's explanation

Ratios work just like fractions. We can think of 1:2 as the first quantity being $\frac{1}{2}$ of the second quantity.
In each of the ratios this is true.
So any fraction equivalent to a $\frac{1}{2}$ will work.
For example, since $\frac{45}{90} = \frac{1}{2}$
The ration 45:90 is also equivalent to 1:2

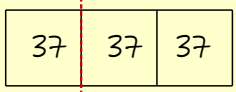

Jon

Jon's explanation

If you want to find ratios that are equivalent to 1:2, you simply draw a suitable diagram.


3 boxes

You can then put whatever quantity you want in the first and that fixes what goes in the other two boxes.
For example,



3 boxes

So 37:74 is also equivalent to 1:2

Properties of equivalent ratios

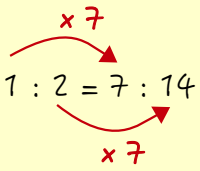


Evie

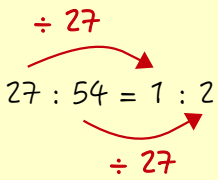
Evie's explanation

You can multiply or divide each part of a ratio by the same number the value of the ratio is unchanged.

For example,



And



A ratio is said to be in its **simplified form** when it is given in terms of the smallest possible integers for that ratio.

For example 12:18 = 4:6 (dividing by 3) but this is not in its simplest form because 12:18 = 2:3 (dividing by 6).

1. Write down three equivalent ratios for each of these examples. Identity the simplest form of the ratio in each case.
   a) 10:50
   b) 14:22
   c) 3:4

2. Write these ratios in their simplest form.
   a) 24:108
   b) 32:56
   c) 81:117

3. Kevin wants to paint the outside walls of his house orange by mixing yellow paint with red paint in the ratio of 3:5. He can buy paint in 2L cans and 5L cans.
   Can you suggest what he should buy to make sure he mixes the correct shade of orange?

4. A small box of chocolates contains 12 milk chocolates and 16 dark chocolates. Assuming the ratio of milk chocolates to dark chocolates remains the same, how many milk chocolates would be in
   a) a medium size box containing 42 chocolates altogether?
   b) a large size box containing 98 chocolates altogether?

Some more questions

1. The ratio of boys to girls in the Yr 8 athletics squad is 3 : 5. If there are 42 boys in the squad, how many girls are there?

2. Mrs Knight cuts a piece of ribbon 40m long into three pieces in the ratio 3 : 2 : 5. What is the length of the longest piece?

3. The ratio of blue counters to red counters is 5 : 7. If there are 35 blue counters, how many red counters are there?

4. Angus and Angie have both been collecting newts. For every 3 newts Angus manages to collect, Angie collects 4 newts. Altogether they have 63 newts. How many newts did Angie collect?

5. Mrs Williams has 3 children, Grace, David and Fiona. Their ages are 3, 5 and 7. Mrs Williams has been saving some money and decides to divide it in the ratio of their ages. If Fiona got £42, how much money had Mrs Williams saved and how much did the other children receive?

6. Three friends, John, Paul and George started up a part-time business. John invested £450, Paul £350 and George £200.

   a) What was the ratio of the three friends investments in its simplest form?

   b) In the first three months, the business made £200 profit. How much should each of the three friends receive?

7. Over the season the Year 8 soccer team had a surprising number of penalty kicks awarded. Their coach, Mr T, noted the ratio of scoring them to missing them was 3:2. He also noticed that of the penalty kicks missed the ratio of saves by the goalkeeper to shooting wide was 1:3. If only 4 penalties were saved by the goalkeeper, how many penalties were awarded altogether?

Ratio – Mastery questions

In questions 1 – 12, four possible answers are given. Only one of them correct. Record the letter which you think indicates the correct answer. You must be able to give reasons for your choice.

1. Express 20 : 28 in its simplest form.

A	10 : 14	B	5 : 7	C	1 : 1.4	D	0.71: 1

2. Write 0.6m to 1.8m in its simplest form.

A	0.6 : 1.8	B	6 : 18	C	2 : 6	D	1 : 3

3. Leah and Oscar share out their chocolates in the ratio 3 : 4. If Leah gets 24 chocolates, how many does Oscar receive?

A	18	B	28	C	32	D	56

4. What is the ratio of 45 minutes to 2 hours in its simplest form?

A	45 : 120	B	45 : 2	C	1 : 2.67	D	3 : 8

5. The ratio of girls to boys in 8X is 2 : 3. Which of these numbers can not be the number of pupils in 8X?

A	20	B	24	C	25	D	30

6. Mark and Petrina share a bag of sweets. Petrina gets three times as many as Mark. What fraction of the total number of sweets does Petrina receive?

A	$\frac{1}{4}$	B	$\frac{1}{3}$	C	$\frac{2}{3}$	D	$\frac{3}{4}$

7. Farmer Coombe harvests pears. He splits his crop into Premium, Budget and Reject pears in the ratio 4 : 7 : 2. Calculate the number of pears he picked if he had 16 rejects.

A	104	B	88	C	55	D	56

8. Molly is making a fruit drink using orange juice, pineapple juice and blackcurrant juice in the ratio 5 : 3 : 2. She has lots of orange juice and blackcurrant juice but only 90ml of pineapple juice. How much fruit drink can she make?

A	180ml	B	300ml	C	450ml	D	900ml

9. The lengths of two pieces of wood are in the ratio 4 : 5. What fraction of the total length is the shorter piece?

A	$\frac{4}{5}$	B	$\frac{5}{4}$	C	$\frac{4}{9}$	D	$\frac{5}{9}$

10. The lengths of two pieces of wood are in the ratio 4 : 5. What fraction of the shorter piece is the longer piece?

A	$\frac{4}{5}$	B	$\frac{5}{4}$	C	$\frac{4}{9}$	D	$\frac{5}{9}$

11. A small box of screws contains 24 flat head screws and 36 cross head screws. How many flat head would be in a similar box if there are 140 screws altogether in the box?

A	56	B	70	C	72	D	84

12. Sarah has been collecting stamps for a number of years. She splits here stamps into European, Asian and American in the ratio 7 : 2 : 4. If she has 546 non-European stamps, how many Asian stamps does she have?

A	84	B	156	C	182	D	273

In Q13 – 15, you must show all of your working.

13. The ratio of blue soccer shirts to red soccer shirts sold in a sports shop is 4 : 7. If the shop sold 54 more red shirts than blue shirts, how many blue shirts did they sell?

14. Petra has a large collection of action figures. The ratio of male to female is 4 : 3. For her birthday, her brother gave her 5 more female action figures. The ratio of male to female figures was then 8 : 7. How many female action figures has Petra now got in her collection?

15. Three enterprising students in Year 8 decided to set up their own sweet shop in the playground at break time and lunch time selling chocolate bars. Initially James put £25 into the kitty, Sara put £15 and Adrianne put in £10. They agreed to split the profits in the ratio of their initial contributions. At the end of the first month, Sara's share was £16 less than James' share. How much was Adrianne's share of the profits?