What do you know about rounding?

Round 15.346 to 2dp.

Round 0.499 to 1dp.

What is 47523 to the nearest hundred?

What is 6027 to the nearest 10?

Roughly what is 28.58 x 39.6

Roughly what is 1821 ÷ 99?

What is 2.352 to the nearest hundredth?

A whole number rounds to 850 to the nearest fifty and 830 to the nearest ten.

What could the number be?

A ball of string is sold as 80m to the nearest metre.
Explain exactly what that means about the length of string.

95971 people watch Rihanna's concert at Twickenham.



1 Roughly how many people would you say were at the show.

Rounding numbers, especially large numbers is often helpful in estimating the answers to calculations. You already know about rounding decimals. In this section we look at another way of rounding numbers.

Significant figures

With integers all the digits matter and so are **significant**. The size of the number is influenced most by the first digit. For example, with the number 4689 the 4 is the most significant digit as it tells us the number is four thousand and something. It follows that the six in the next most significant figure and so on.

When we round to one significant figures we need only look at the first two digits and decide whether to round up or down. Your final answer will have a non zero digit first followed by the correct number of zeros to make sense of the answer.

For example, if you are asked to round 4689 to 1 significant figure, this really means round 4689 to the nearest thousand, and so the answer is 5000.

If you are asked to round 4689 to 2 significant figure, you are being asked to round it the nearest hundred, and so the answer would be 4700.

2 95971 people watch Rihanna's concert at Twickenham. Round in as many different ways as you can.

95 971 people watch Rihanna's concert at Twickenham.



Jon

Well I think that is 100 000 to 1 significant figure.

And 96 000 to 2 significant figures



Alice



Amber

Or even 96 000 to 3 significant figures. Now that's odd!

3

Do you agree with their answers?

What do you think Amber means by, “Now that's odd.”

Fitness check

Round these numbers as indicated

1. 284 (1 sf)
2. 12723 (2 sf)
3. 512654 (3 sf)
4. 26452 (1 sf)
5. 449 (1 sf)



Estimate the answers to these calculations by rounding each number to 1 sf.

1. 532 x 29
2. 3127 x 186
3. 422.3 x 18.4
4. 93578 x 527
5. 845.3 ÷ 19.2
6. 98.542 ÷ 8.6
7. 62874 ÷ 365.2
8. 6499 ÷ 24.5

9. A classroom measures 7.7m x 8.6m x 3.3m.
   Estimate the volume of the classroom.

10. A petrol tanker delivers 36875L of fuel to a garage. A family car holds roughly 48L of fuel. Estimate how many cars can be filled up from this delivery.

"X Factor ratings hit a 10-year low as only 6.8m viewers tune in."



Sarah

Presumably they have rounded the number of viewers to 2 s.f.
So how many viewers might have watched the show?

4 Assuming Sarah is correct, what is the lowest possible number of viewers?
What is the highest possible number of viewers.

Although rounding numbers can be very helpful especially in calculations, you must always remember that you are only estimating. It is often helpful to keep in mind the range of possible values involved.

5 What can you say about the length of a piece of wood that is 15 cm to the nearest centimetre?



Jon

Well is must be at least 14.5 cm long.



Alice

And no more than 15.4 cm.



Amber

That's not right Alice.
What about 15.45 cm?
That would round to 15 cm.

6 If our piece of wood rounds to 15cm, what can we actually be sure it is less than?

Upper and lower bounds

If the length of our piece of word is 15 cm to the nearest centimetre then it most be at least 14.5 cm long to round up to 15 cm as Jon said. This is called the **lower bound**.
It must also be less than 15.5 cm to round down to 15 cm. This is called the **upper bound**.
If the length of the wood is L cm, we can write this as an inequality

$$14.5 \leq L < 15.5$$



Matthew

So what are the bounds for the X factor ratings?


Sarah

Ok, I think I've got that. My mother is 40 to 1 significant figure. How old could she be?


Jon

She must be at least 35, that is the lower bound.


Alice

And less than 45, that is the upper bound.

7 The width of a pencil is 0.9 cm (to 1 d.p.). What are the lower and upper bounds of the width of the pencil?


Matthew

Well it must be more than 0.85, so that's the lower bound and less than 0.95, so that's the upper bound. It works exactly the same way as significant figures.

Fitness check

All of these numbers have been rounded as shown. Write down the lower and upper bounds for each of them.

1. 300 (1 s.f.)
2. 120 (2 s.f.)
3. 900 (2 s.f.)
4. 8 (1 s.f.)
5. 6.5 (1 d.p.)
6. 16.62 (2 d.p.)
7. 0.05 (2 d.p.)
8. 0.99 (2 d.p.)



9. There were 1000 supporters at the Rocks last match. If this has been rounded to 2 s.f., how many people could have been at the match?

10. Davy says the sum of the ages of his 5 brothers is 120 to 2 significant figures but if you include his age, the sum is 140 to 2 significant figures. How old could Davy be?


Amber

I think I might need some help with Q10.


Matthew

Just think about lower and upper bounds.

How big is the Postage Stamp at Troon?



The eighth hole at the Royal Troon Golf Club affectionately known as "The Postage Stamp" is one of the most difficult par 3 holes in championship golf.
It is not only subjected to strong sea breezes but it is also protected by many green side bunkers.
Added to that, the green is quite small.



Sarah

I looked it up on a golf map and it is quite rectangular.
According to my calculations it is 22.6m by 11.4m.
So the area is 22.6 x 11.4 = 257. 64m^2



Jon

I don't think you can be that accurate as you have already made lots of assumptions.



Alice

If we work to one significant figure we should get a reasonable answer.

8 Using Alice's idea, estimate the area of the green at the "Postage Stamp".

It is often helpful to round numbers to one significant figure to estimate answers.

For example suppose you need to estimate the value of $(2.6 \times 21.3)^2$



Amber

Amber's Idea

2.6 is 2 to 1 s.f.
21.3 is 20 to 1 s.f.
So
2.6 x 21.3 is roughly
3 x 20 = 60
60^2 = 3600 or
4000 to 1s.f.

Estimating

By rounding each number to one significant figure, estimate the values of

1. 38.3 x 51.7
2. 98.4 x 106.8
3. $784.8 \div 42.7$
4. $27.4 \div 64.6$
5. $\dfrac{82.4 \text{ x } 14.4}{38.4}$
6. $\dfrac{44.9 \text{ x } 324}{3.6^2}$
7. $\dfrac{32^2 + 144}{13^2 \text{ x } 4}$
8. Sophie has to deliver some equipment from the docks to a laboratory in Wales. Her SAT NAV says the distance is 346 miles. Her on board computer says she uses one litre of fuel to go 7.8 miles.
   Fuel costs £1.12 per litre.
   Estimate the cost of fuel for the journey.
9. The population of the UK in 2012 was 63.2 million. The cost of delivery the 2012 Olympics in London was £8.92 billion.
   Estimate the cost of the games per person in the UK.

What do you remember about factors?

Write down the first five multiples of 12.

Write down the first five multiples of 16.

What is the lowest common multiple of 12 and 16?

Write down all of the factors of 20.

Write down all the factors of 24.

What is the highest common factor of 20 and 24?

Explain what is meant by a prime number.

Write down the first ten prime numbers.

Why isn't 1 a prime number?

Write down the first five composite numbers.

Find two different numbers that have an odd number of factors.

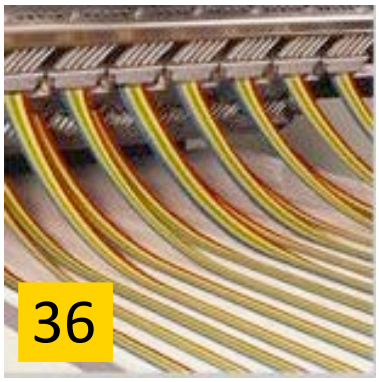
What type of numbers have an odd number of factors?

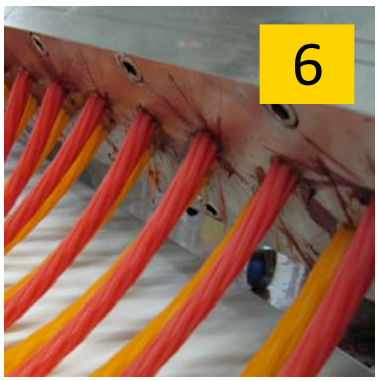
Prime factorisation



Matthew

My dad works at our local licorice factory. Customers can order their licorice in any length from one to 100. If a customer wants some length '36' licorice, they go to the '36' machine, feed in a unit length, and the machine stretches it out to length 36. Actually they do 10 at a time as you can see in my photos. It's great fun to watch.





Yesterday, they had a large order for length '6' licorice but the number '6' machine has broken down and can't be fixed today.

My dad knows I am good at Maths, so he asked me if I had any ideas about how he could solve the problem.

9 Matthew quickly solved the problem. What did he tell his dad to do?



Alice

Well I would put a unit piece into machine 2 which will stretch it to two units. I would then feed those pieces into machine 3 which will stretch to 6 units. Job done!

In that case they don't need a number '6' machine at all. In fact they could get rid of quite a few other machines.



Sarah



Amber

You definitely don't need machine 10 because you could use machine 2 and then machine 5.

10 Can you work out which machines are essential for the factory to fulfil all of their orders?

Prime factorisation



Jon

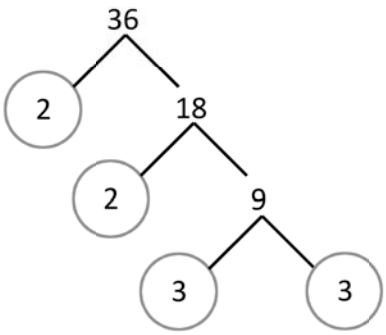
I am confident I know how to work this out. You just need to split the numbers into prime factors.

Splitting numbers into a product of prime factors.



Amber

I know how to do that. You use factor trees.



So $36 = 2 \times 2 \times 3 \times 3$

$= 2^2 \times 3^2$



Alice

I know a different way. It's called reducing to one.

2	36
2	18
3	9
3	3
	1

So $36 = 2 \times 2 \times 3 \times 3$

$= 2^2 \times 3^2$

11 So how would we make a length of 36 units using only the 2 and 3 machines?

Fitness check

Write the following numbers as the product of prime numbers.

1. 54
2. 126
3. 210
4. 360
5. 630
6. 792



12 What is the highest common factor (HCF) of 54 and 126?

Prime factorisation



Matthew

Highest common factor – that means the largest factor that 54 and 126 share.
So if I write down all of the factors of both numbers, it should be obvious.

Matthew's work

54 = 1 x 54 = 2 x 27 = 3 x 18 = 6 x 9.
So the factors of 54 are 1, 2, 3, 6, 9, 18, 27 and 54

126 = 1 x 126, 2 x 63, 3 x 42, 6 x 21, 7 x 18, 9 x 14
So the factors of 126 are 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63 and 126
The HCF of 54 and 126 is 18



Evie

Evie's work

54 = 2 x 3 x 3 x 3

126 = 2 x 3 x 3 x 7

The HCF of 54 and 126 is 2 x 3 x 3 = 18



Amber

Amber's work

$54 = 2 \times 3^3$

$126 = 2 \times 3^2 \times 7$

HCF of 54 and 126 is $2 \times 3^2 = 18$

13 Can you explain Amber's method?

Fitness check

Use your results from earlier to find the HCF of

1. 210 and 360
2. 126 and 630
3. 360 and 792



14 What is the lowest common multiple (LCM) of 36 and 54?

Lowest common multiple – that means the lowest multiple they share.
So if I write down all of the multiples of both numbers, it should be obvious.


Matthew

Matthew's work

Multiples of 36

36, 72, 108, 144, 180, ...

Multiples of 154

54, 108, 162, 216, 270, 324, ...

The LCM of 36 and 54 is 108.


Evie

Evie's work

$54 = 2 \times 3 \times 3 \times 3$

$126 = 2 \times 3 \times 3 \times 7$

The LCM of 36 and 54 and $2 \times 3 \times 3 \times 3 \times 7 = 108$


Amber

Amber's work

$54 = 2 \times 3^3$

$126 = 2 \times 3^2 \times 7$

LCM of 36 and 54 $= 2 \times 3^3 \times 7 = 108$

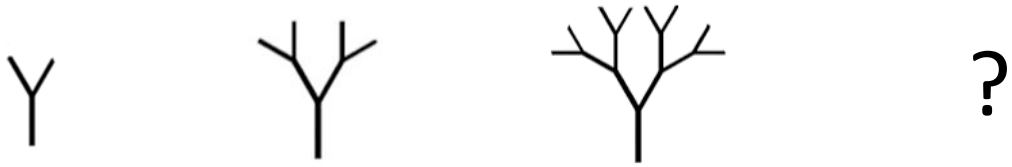
15 Can you explain Amber's method?

Fitness check

Use your results from earlier to find the LCM of the following:

1. 210 and 360
2. 126 and 630
3. 360 and 792





16 Describe how the number of branches on tree changes.
How many branches would there be in tenth picture?


Sarah

Easy, they double every go, so
2, 4, 8, 16, 32, 64, 128, 256, 512, 1024.
There will be 1024 in the tenth picture.

2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 = 1024

Or you could just write 2^{10}. It is a lot neater.


Jon

Repeated multiplication can be written using **index notation** just as Jon suggested.

So 3 x 3 x 3 x 3 = 3^4 = 81

3 is called the **base** and 4 is called the **power** or **index**.

Numbers written like this can easily by multiplied.

$3^4 \times 3^2 = (3 \times 3 \times 3 \times 3) \times (3 \times 3) = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

17 What happens to the powers when you are multiplying numbers written in index form with the same base?


Alice

You just add the powers. That makes things pretty easy.
In that case, I think I know what happens when you divide numbers written in this form.

18 Can you explain Alice's thinking?
How can you be sure she is right?

Alice's work

Suppose you want to work out $5^6 \div 5^2$, then if you write the numbers as a fraction, you can cancel

$$\frac{5 \times 5 \times 5 \times 5 \times \cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5}}$$

So $5^6 \div 5^2 = 5^4$ You just subtract the powers

Fitness check

Write these calculations in index notation.



1. 4 x 4 x 4 x 4 x 4
2. 7 x 7 x 7
3. 3 x 3 x 3 x 3 x 3 x 3 x 3
4. What is bigger 7^3 or 3^7?

Write these calculations as a single power of the base.

5. $2^3 \times 2^4$
6. $3^5 \times 3^2$
7. $6^2 \times 6^2$
8. $(5^3)^2$
9. $4^5 \div 4^2$
10. $9^7 \div 9^6$
11. $8^{11} \div 8^7$
12. $8^{11} \div 8^7$

19 How would you describe the rule for questions such as $(5^3)^2$?

Evie's explanation

$$(5^3)^2 = 5^3 \times 5^3$$
$$= 5^6$$

So with power to a power, you just multiply the powers!

Fitness check

Write as a single power of the base

1. $(6^2)^4$
2. $(2^4)^3$
3. $(10^2)^5$



20 Which is bigger 4^3 or 2^5?



Jon

$4^3 = 4 \times 4 \times 4 = 64$

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

So $4^3 > 2^5$



Amber

Or, you could write 4^3 as $(2^2)3 = 2^6$

So $4^3 > 2^5$

And I didn't have to work either of them out!

Fitness check

Complete the following



1. $25^2 = 5^?$
2. $9^3 = 3^?$
3. $2^8 = 4^?$
4. $2^8 = 16^?$
5. Which is bigger 11^4 or 121^2?



1. Find an integer that has exactly 8 factors including 6 and 15.
2. What is the smallest possible integer that is odd, not prime and the next largest odd number is not prime?
3. What is the smallest prime number that is the sum of three different primes?
4. Find the smallest prime number which is greater than 23 and which is the sum of three primes.
5. Find the smallest prime number which is the sum of five different prime numbers.Find a prime number that cannot be written as $4k \pm 1$ for any positive integer k.
6. Which of these numbers are prime 1, 12, 123, 1234, 1235, 123456?
7. Find a prime number that cannot be written as $6m \pm 1$ for any positive integer m.
8. Find a prime number that cannot be written as $8n \pm 1$ for any positive integer n.
9. Is it possible to find a prime number than be expressed as $4k$, $4k + 1$, $4k + 2$, as $4k + 3$ for any positive integer k?
10. Which of the following numbers is not a square number

    A 16 B 25 C 34 D 43 E 52

11. What is the remainder when $2^2 \times 3^3 \times 5^5 \times 7^7$ is divided by 8?
12. What is the remainder when $2^2 \times 3^3 \times 5^5 \times 7^7 \times 11^{11}$ is divided by 8?
13. What is the remainder when $2^2 \times 3^3 \times 5^5 \times 7^7$ is divided by 16?
14. Which of the following numbers are divisible by all of the integers from 1 to 10 inclusive?

    A 23×24 B 34×45 C 45×56 D 56×67 E 67×78

15. Which is the smallest positive integer that 45×56 is ***not*** divisible by?
16. Find a positive integer n such that neither 12 nor 30 is divisible by n, but the product 12×30 is divisible by n. How many positive integers, n, with this property are there?

Find an integer that has exactly 8 factors including 6 and 15

If …. then …

In mathematics we often use what is already known to work out new facts. This is known as deductive reasoning. A good example of this technique is to use "if … then …" questions.

21 If $40^2 = 1600$, then what is 41^2?



Sarah

Sarah's solution

$40^2 = 40 \times 40 = 1600$
$41 \times 40 = 1640$
$40 \times 41 = 1640$
$41 \times 41 = 1681$

Fitness check

1. If $52^2 = 2704$, then what is 53^2?
2. If $60^2 = 3600$, then what is 59^2?
3. If $10^3 = 1000$, then what is 11^3?



22 If $36 \times 47 = 1692$, then what is 37×48?



Matthew

Matthew's solution

$36 \times 47 = 1692$
$37 \times 47 = 1739$
$47 \times 37 = 1739$
$48 \times 37 = 1776$

Fitness check

1. If $16 \times 18 = 288$, then what is 17×19?
2. If $34 \times 63 = 2142$, then what is 35×64?
3. If $29 \times 72 = 2088$, then what is 29×73?
4. If $13 \times 14 = 182$ and $12 \times 13 \times 14 = 2184$, then what is $13 \times 14 \times 15$?



If …. then ...

Another handy use of deductive reasoning is being able to interchange between multiplication and division facts.

For example, if 24 x 53 = 1272, then 1272 ÷ 24 = 53 and 1272 ÷ 53 = 24

Or if you are told that 2664 ÷ 72 = 37, then 2664 = 72 x 37 and 2664 ÷ 37 = 72

Fitness check

In each of these questions, complete the sentence by writing down two other related results.

1. If 43 x 57 = 2451, then ...
2. If 7.8 x 8.5 = 66.3, then ...
3. If 1886 ÷ 23 = 82, then …
4. If 4.56 ÷ 1.9 = 2.4, then ...



23 If 37 x 46 = 1702, then what is 17.02 ÷ 4.6



Jon

Jon's solution

37 x 46 = 1702
3.7 x 4.6 = 17.02
17.02 ÷ 4.6 = 3.7

Fitness check

In each of these questions, complete the sentence by writing down two other related results.

1. If 83 x 37 = 3071, then what is 30.71 ÷ 37?
2. If 17 x 59 = 1003, then what is 1003 ÷ 5.9?
3. If 2352 ÷ 98 = 24, then what is 23.52 ÷ 2.4?
4. If 168.64 ÷ 13.6 = 12.4, then what is 1.36 x 1.24?





Question creation - create your own question and solution using "if … then ...."
Have fun creating problems for your friends to solve.
Who can make the trickiest question?



1. What is (999 – 99 + 9) ÷ 9
2. What is the difference between the smallest 4-digit number and the largest 3-digit number?
3. Albert, Brenda, Carl and Daniel have their 12^{th}, 14^{th}, 15^{th} and 15^{th} birthdays today. In how many years time will their combined age add up to 200?
4. Which of these numbers are divisible by 3

   A 123 B 234 C 345 D 456 E 567
5. Find the smallest 2 digit integer that has the same remainder when divided by 2 and 3.
6. Find the smallest positive integer that has a remainder of 1 when divided by 3 and 5.
7. Find the smallest positive integer that has a remainder of 3 when divided by 5 and 7
8. Show that every positive integer which gives the same remainder when divided by 2 as when divided by 3 is either a multiple of 6 or one more than a multiple of 6.
9. If m is an even integer and n is an odd integer, which of these is an odd integer?

   A $3m + 4n$ B $5mn$ C $(m + 3n)^2$ D m^3n^3 E $5m + 6n$
10. In a Maths competition there are 15 questions. 2 points are awarded for every correct answer and 1 deducted for every wrong or incomplete answer. How many questions did Alex get right is she scored 21 in total?
11. There are 30 questions in a Science quiz. There are 5 points fare awarded for each question answered correctly and 3 points deducted for wrong or incomplete answers. Harry scored 126 points. How many questions did he get right.
12. A farmer's wife sells eggs at the local but, when asked by her husband how many eggs she sold, she teases him by answering riddles. On Monday her reply was, “Taken in groups of 11, 5 would be left over but taken in groups of 23, 3 are left over. How many eggs might she have sold on Monday?
13. On Tuesday she answered, “Taken in groups of 2, 3, 4, 5, 6 and 7 there remain 1, 2, 3, 4, 5 and no eggs respectively. How many eggs did she sell on Tuesday?

Number 4a Mastery questions

In questions 1 – 12, four possible answers are given. Only one of them is the correct. Record the letter which you think indicates the correct answer. You must be able to give reasons for your choice.

1. Round 54462 to 2 significant figures.

A	54000	B	55000	C	54	D	55

2. Round 154.965 to 2 significant figures.

A	15	B	154.97	C	150	D	160

3. Which of these is the most sensible estimate for 98.6 ÷ 495

A	2	B	0.2	C	0.5	D	5

4. A plane is flying at 32000 feet. The height has been rounded to 3 significant figures. Which of these inequalities shows the correct upper and lower bounds for the height h?

A	$32000 \leq h < 32100$	B	$31950 \leq h < 32050$
C	$31500 \leq h < 32500$	D	$31190 \leq h < 32049$

5. The radius of a circle, r is 3.2 cm rounded to 1 decimal place. What is lower bound of r?

A	3.20	B	3.25	C	3.00	D	3.15

6. Aaron is 1.68m tall. His height is given to 2 decimal places. Which of these inequalities shows the correct upper and lower bounds for his height, h m?

A	$1.67 \leq h < 1.69$	B	$1.65 \leq h < 1.75$
C	$1.675 \leq h < 1.685$	D	$1.675 < h < 1.685$

7. By rounding each number to one significant figure, estimate the values of 5.45 x 6.54 x 4.49

A	120	B	175	C	210	D	140

8. Express 144 as the product of its prime factors

A	12 x 12	B	$2^4 \times 3^2$	C	$6^2 \times 6^2$	D	$4^2 \times 3^2$

Number 4a: Mastery questions

In questions 1 – 12, four possible answers are given. Only one of them is the correct. Record the letter which you think indicates the correct answer. You must be able to give reasons for your choice.

9. Two numbers are written as the product of their prime factors as shown.

$2^2 \times 3 \times 5$ and $2 \times 3 \times 7$

Which of these is the lowest common multiple (LCM) of the two numbers?

A	12	B	420	C	210	D	55

10. Three numbers are written as the product of prime factors as shown.

$2^3 \times 3^2 \times 5^2 \times 7$ and $2^4 \times 3^3 \times 5^2 \times 7^2$ and $2^3 \times 3^3 \times 5 \times 7$

Which of these is the highest common factor (HCF) of the three numbers?

A	2520	B	210	C	504	D	1680

11. What is $6^2 \times 6^2$?

A	6^4	B	2×6^2	C	6^2	D	36

12. What is $3^4 \times 3^2$?

A	9^6	B	9^8	C	3^8	D	3^6

In questions 13 – 15, you must show all of your working.

13. If $42 \times 53 = 2226$, then work out 43×54, without using long multiplication or a calculator.

14. Estimate the answer to this calculation by rounding each number to 1 significant figure.

$$\frac{387 \times 409}{196 \times 94}$$

15. A packet of screws contains 100 screws to the nearest 10.
Bryan buys 10 packets of screws and puts them in an empty tin.
Write down an inequality which shows how many screws he might actually have in his tin.