What do you know about fractions, decimals and percentages?

Calculate

$1\frac{3}{5}+2\frac{3}{4}$

$3\frac{2}{3}-1\frac{3}{4}$

Calculate

$6\times\frac{2}{3}$ $\quad\frac{3}{4}\times16$

$6\div\frac{2}{3}$ $\quad\frac{3}{7}\div4$

Write down a fraction that is between

$\frac{2}{3}$ and $\frac{3}{4}$

Which is bigger

75 % of 120

or

60% of 160?

A TV normally sells for £650 but on the sale there is a 20% discount.
What is the sale price of the TV?

Making comparisons



The combined population of Wales and Scotland is approximately 8.1 million people.

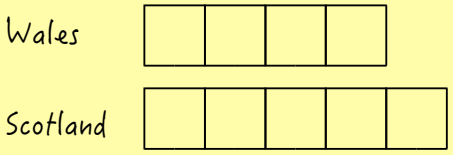
If the population of Scotland is approximately 25% higher than the population of Wales, what is the population of each country?

1 Can you draw a diagram that might help you with this question?

Sarah's working



Sarah



8.1 ÷ 9 = 0.9
Population of Wales = 4 x 0.9 = 3.6 million
Population of Scotland = 5 x 0.9 = 4.5 million.

2 Can you explain Sarah's diagram?



There are actually 55 seats on this bus but in fact it is carrying 140% more than its capacity.

How many people are on the bus?



Evie

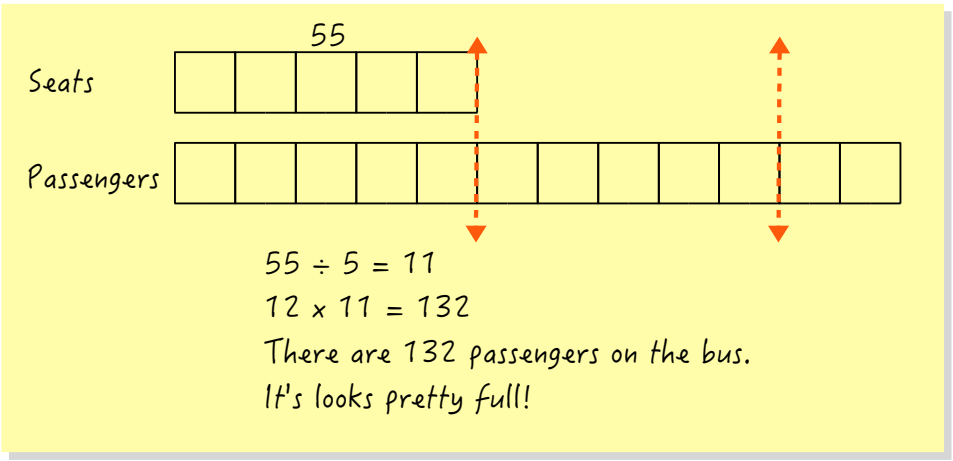
I think a diagram might help!

Making comparisons

Matthew's working



Matthew

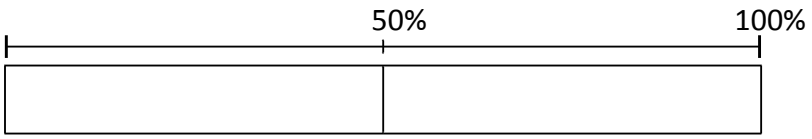


3 Can you explain Matthew's diagram?

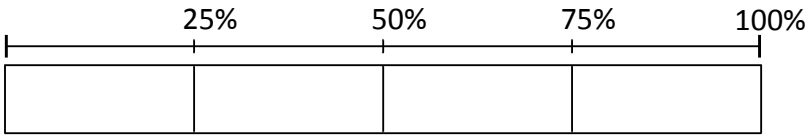
Helpful diagrams

Percentages are another way to work out parts of a quantity. Per cent actually means 'out of 100', so 50% means 50 parts out of 100. In other words, finding 50% of a quantity is the same as finding $\frac{1}{2}$ of a quantity. Finding 25% of a quantity is the same as finding $\frac{1}{4}$ of a a quantity and so.

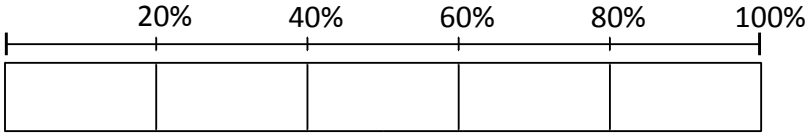
In each question a decision on the 'unit' of measure has to be made as illustrated below.



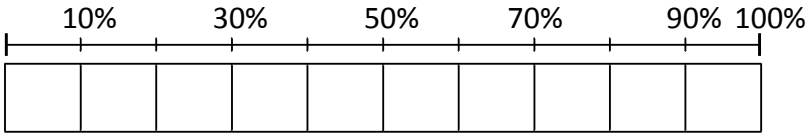
(2 units)



(4 units)



(5 units)



(10 units)

1% would mean drawing 100 boxes and probably isn't the most sensible approach!

Making comparisons



In the recent Arsenal v Manchester City game, the ground was only 90% full.

For a mid week testimonial match three days later it was 65% full.

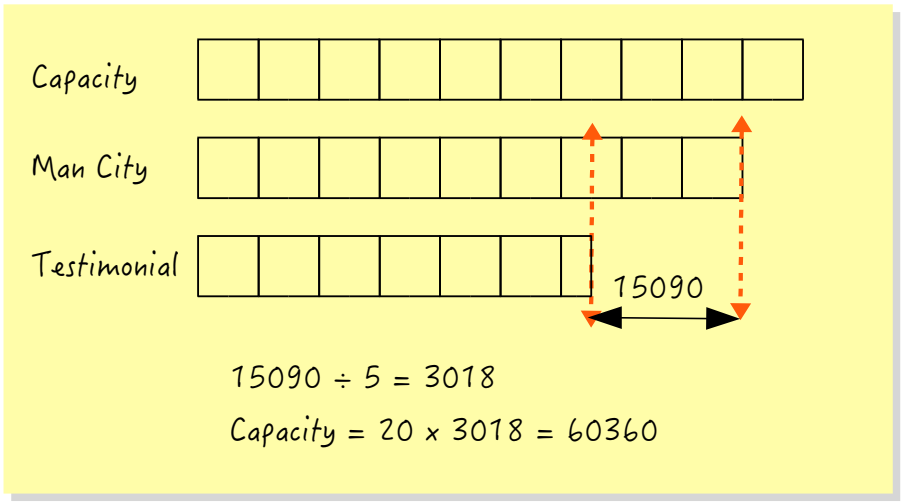
If there were 15090 more fans at the Man City match than the testimonial game, what is the capacity of the ground?

4 Can you draw a suitable diagram to help you with this question?

Alice's work



Alice



5 Can you explain Alice's diagram?

Fitness check

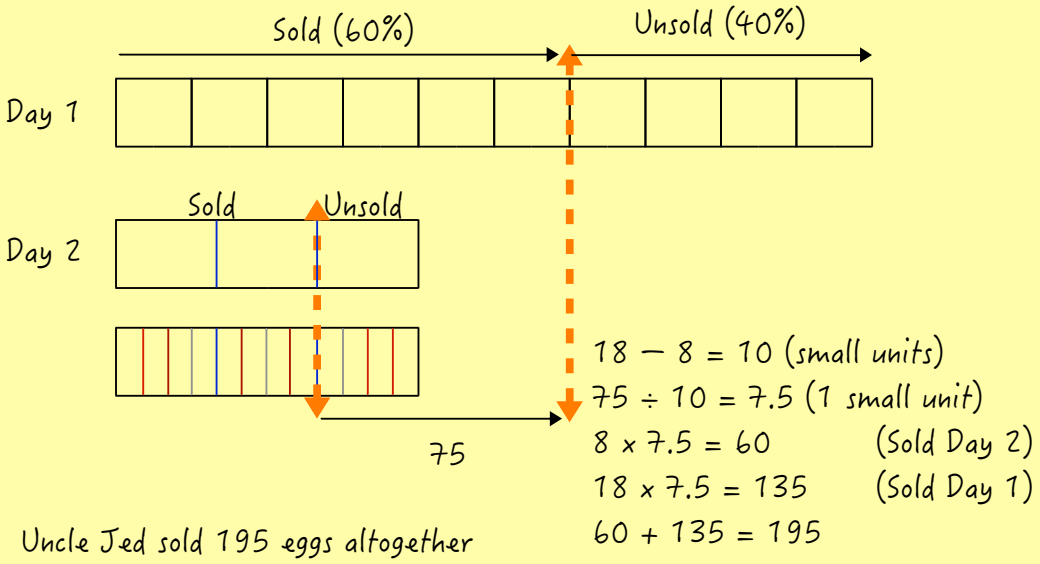


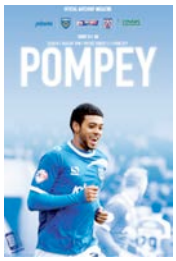
1. Mrs Charge poured 350ml of water into a jug and noticed it was only 40% full. How much water would the jug hold altogether?
2. 300 pupils took part in a fun run to raise money for charity. 25% of them took under an hour to complete the course. How many pupils took longer than an hour?
3. There were 196 children at a concert. This was 35% of the total audience. How many people were at the concert?
4. Alisha spent 30% of her money on a pair of shoes and 55% of the remainder on a dress.If she had £45 left, how much did she spend on the dress?
5. Uncle Jed keeps chickens and sells their eggs at the local market. He sold 60% of them on Thursday and $\frac{2}{3}$ of the remainder on Friday.
   He sold 75 more eggs on Thursday than on Friday. How many eggs did he sell altogether?

Jon's answer to Q5



Jon





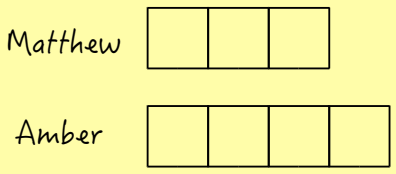
Amber and Matthew have been collecting soccer programmes for the last two years.
Amber has $\frac{1}{3}$ more programmes than Matthew.
Altogether they have 294 programmes. How many programmes does Amber have?

6 Can you draw a helpful diagram for this question?

Evie's work



Evie



294 ÷ 7 = 42
Matthew has 3 x 42 = 126 programmes.
Amber has 4 x 42 = 168 programmes.

Fitness check

1. Increase £1.70 by 20%
2. Decrease 180 by $\frac{2}{3}$
3. Decrease £4.48 by 25%
4. Increase 4102 by $\frac{2}{7}$

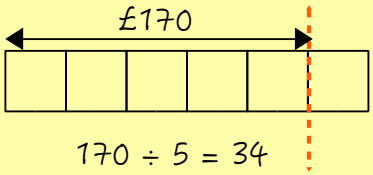


Increasing and decreasing quantities

Sarah's work on Q1



Sarah



170 ÷ 5 = 34

34 x 6 = 204



Matthew

I've thought of a quicker way. I am going to use a multiplier.

7 What do you think Matthew means by a multiplier?

Matthew's explanation



Matthew

Increasing by 20% is the same as increasing by 1/5

So I actually want to end up with 6/5 of what I started with.

So all I need to do is work out 170 x 6/5 = 204

I didn't need a diagram!

Increasing and decreasing quantities using multipliers

Matthew's suggestion that this type of calculation can be simplified using multipliers is a very useful skill to fully understand and develop.

For example



Amber

So if I want to increase something by 25%, that's a quarter more, so the multiplier is $\frac{5}{4}$.

… and if I want to decrease by 25%, that's a quarter less, so the multiplier is $\frac{3}{4}$. That is easy.



Jon

Increasing and decreasing quantities using multipliers

A company's profits increased by 12% in 2016 from the previous year.
If the profits were £4800 in 2015, what were the profits in 2016?

Evie's working



Evie

1% of £4800 = £48

So 12% is 48 x 12 = £576

So the profit in 2016 = £4800 + £576 = £5376



Matthew

Evie, you can do this much more quickly if you use multipliers.

8 What should the multiplier be to increase the value by 12%?

Alice's working



Alice

If I increase the profit by 12% then I will actually end up with 112%.

So the multiplier is 1.12

The profit in 2016 will be 4800 x 1.12 = £5376

Fitness check

1. What is the multiplier to to increase a quantity by:

a) 40%	d) 90%	g) 18%
b) 75%	e) 50%	h) 42%
c) 30%	f) 150%	i) 130%



2. What is the multiplier to decrease an amount by:

a) 60%	d) 10%	g) 24%
b) 50%	e) 25%	h) 32%
c) 75%	f) 90%	i) 67%

Increasing and decreasing quantities using multipliers

Fitness check



3. Increase
   a) £250 by 12%
   b) £4250 by 45%
   c) £940 by 62%
   d) £25 by 120%
   e) £250000 by 108%

4. Decrease
   a) £250 by 12%
   b) £4250 by 45%
   c) £940 by 65%
   d) £25 by 18%
   e) £250000 by 56%

5. The retail price of a laptop was £240. In the Christmas sale, the price was reduced by 40%. It was further reduced by 20% of the discounted price in the January sale. How much was the laptop sold for in the January sale?

6. A painting was valued at £4000 in 2010. It was revalued in 2015 to be £5260.
   What percentage rise is that?
   If the painting continues to appreciate at the same rate, what will be its value in 2020?

7. A retailer buys in a particular brand of mobile phone for £340. His normal mark up is 35%.
   a) What is the retail price of the mobile phone?
   b) One month later he has a sale and offers a discount of 20%. What is the sale price of the mobile phone?
   c) What percentage profit does he make on the mobile phones in the sale?

9

Look at Q5. Can you explain why the retailer did not make 15% profit.



Jon

Jon's explanation

To increase £340 by 35% the multiplier is 1.35

So 340 x 1.35 = £459. That is the retail price.

You have added 35% of £340

But in the sale price you reduce £459 by 20%

So the multiplier is 0.8

And the sale price is £459 x 0.8 = £367.20

You have deducted 20% of £459

The retailer's actual profit is £27.20

So his percentage profit is $\frac{27.20}{340} \times 100 = 8\%$

10 What happens if you increase the price of an item by 50% and then reduce it by 50%? Does it matter how much you start with?



Sarah

Sarah's explanation

Suppose I start with £100, then after a price rise of 50%, the price would be £150.

If the price is then reduced by 50%, it will go down by £75.

So the final price is £75

This is 25% less than the starting price,



Evie

Evie's working

Suppose the original price is £n, then after a 50% rise the price would by £1.5 x n = £1.5n

And then if it is reduced by 50%, the price would be

£0.5 x 1.5n = £0.75n

So it always goes down by 25%

Do you use twitter to get the news?

Apparently lots of people do as the recently published figures show the number of twitter users in the UK has risen from 14.8 million in 2015 to 15.8 million in 2016.

11 If this rate of increase continues, how many people will be using twitter in the UK by 2017?

Alice's working



Alice

Increase in users from 2015 to 2016 is 1 million.
So the rate of increase is (1 ÷ 14.8) x 100 = 6.75%
So in 2016 the number will be 15.8 x 1.0675 = 16.87 million
So in 2017 the number will be 16.87 x 1.0675 = 18.0 million.



Jon

I used multipliers to find the rate of increase

Jon's idea

Rate of increase = 15.8 ÷ 14.8 = 1.0675
So in 2 years time the number will be
15.8 x 1.0675 x 1.0675
So in 2017 the number will be 18.0 million

12 If this rate of increase continues, how many people will be using twitter in the UK by 2020?



Amber

How do you find the percentage change from a multiplier?

13 How would you answer Amber's question?

Matthew's explanation



Matthew

To increase an amount by 25%, the multiplier is 1.25
If you started with 100%, then you now have 125% or 25% more.
To decrease an amount by 20%, the multiplier is 0.80 or simply 0.8
If you started with 100%, then you now have 80% or 20% less.

Fitness check

1. Match the multiplier with the correct percentage change

1.35	30% increase
0.3	30% decrease
1.3	35% increase
1.7	35% decrease
0.35	70% increase
1. 65	70% decrease
0.65	65% increase
0.7	65% decrease



2. Using Alice's approach find the percentage change if the amount changes from:
   a) 90 eggs to 70 eggs
   b) £520 to £590
   c) 20p to 55p

3. Using Jon's approach find the percentage change if the amount changes from:
   a) 320 km to 440 km
   b) 5p to 16p
   c) 760 g to 520 g.

4. Use either method to find the percentage change if the amount changes from:
   a) £75 to £80
   b) $365 to $315
   c) 67 kg to 52 kg
   d) 42000 to 4600

Fractions, decimals and percentages 4b

Evie's working for Q2



Evie

1 (a) Decrease = 90 − 70 = 20 eggs

Percentage decrease = (20 ÷ 90) x 100 = 22.2%

1(b) Increase = 590 − 520 = £70

Percentage increase = (70 ÷ 520) x 100 = 13.5%

1 (c) Increase = 55 − 20 = 35p

Percentage increase = (35 ÷ 20) x 100 = 175%

Sarah's working for Q3



Sarah

2 (a) The multiplier is 440 ÷ 320 = 1.375

Percentage increase = 37.5%

2 (b) The multiplier is 16 ÷ 5 = 3.2 or 3.20

Percentage increase = 220%

2 (c) The multiplier is 0.68

Percentage decrease is 32%

Here is a snap shot of the average prices for a litre of unleaded fuel in the United Kingdom in January and July of each year since 2007.

	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Jan	87.4	104.0	87.2	111.4	127.8	133.4	132.9	130.1	108.2	102.3
July	96.7	118.7	103.0	117.2	135.1	132.3	135.8	131.3	115.9	109.3

1. What is the percentage rise in fuel between Jan 2007 and 2016?
2. What is the percentage rise in price between January 2007 and January 2012?
3. What is the percentage fall in price between January 2012 and January 2016?
4. In what way might your answers to these questions be confusing?
5. In which year is the greatest percentage increase between the January and July prices?
6. Between which two consecutive years is the greatest percentage rise in price?
7. Between which two consecutive years is the greatest percentage fall in price?
8. A newspaper claims that, “Petrol prices always go up before the summer holidays”. Do these figures support this claim. Explain your reasoning.

Chapter Mastery Questions A

1. What 15% of 120?

A	30	B	18	C	24	D	35

2. What is £50 as a percentage of £200?

A	$\frac{1}{4}$	B	50%	C	25%	D	400%

3. What is 144 as a percentage of 72?

A	$\frac{1}{2}$	B	50%	C	2%	D	200%

4. Which of these is largest?

A	70% of 300	B	150% of 120
C	90% of 200	D	25% of 500

5. Alice earns £32000 per year. She is given a 3% increase as a bonus. How much does she now earn per year?

A	£32960	B	£32320	C	£9600	D	£960

6. Mark bought a new car for £15600. The car depreciates in value by 20% in the first year. How much is his car worth one year later?

A	£3120	B	£18720	C	£14040	D	£12480

7. A mobile phone that originally sold for £400 has had its price increased by 20%. What is the new price of the phone?

A	£80	B	£480	C	£420	D	£440

8. Jon restores vintage toys. He bought a coach for £40 and sold it for £100. What was his percentage profit?

A	50%	B	100%	C	150%	D	250%

9. Annie buys and sells antique jewellery. She recently bought a ring for £80 and plans to make a 30% profit. What is the selling price of the ring?

A	£88	B	£96	C	£104	D	£240

10. Gerald collects old comics. Over the last year his collection has increased from 480 comics to 648. What is the percentage increase?

A	0.35	B	135%	C	35%	D	168

11. What is the correct multiplier to increase a quantity by 30%?

A	0.3	B	0.7	C	1.3	D	3

12. What is the correct multiplier to decrease a quantity by 15%?

A	0.85	B	0.15	C	1.15	D	15

13. What calculation decreases £60 by 8%?

A	60 x 1.8	B	60 x 1.08
C	60 x 0.08	D	60 x 0.92

14. What calculation increases £60 by 8%?

A	60 x 1.8	B	60 x 1.08
C	60 x 0.08	D	60 x 0.92

15. The length of a rectangle increases by 10%. The width of the rectangle decreases by 10%. What is the effect on the area of the rectangle.

A	No change	B	1% increase
C	1% decrease	D	21% increase

16. What is the multiplier to increase a quantity by $\frac{1}{3}$?

A	$\frac{2}{3}$	B	$\frac{1}{3}$	C	$\frac{4}{3}$	D	$\frac{7}{3}$

17. What is the multiplier to decrease a quantity by $\frac{2}{5}$?

A	$\frac{2}{5}$	B	$\frac{3}{5}$	C	$\frac{7}{5}$	D	$\frac{8}{5}$

18. An amount is increased by 150%. What fraction of the original amount do I now have?

A	$1\frac{1}{2}$	B	$2\frac{1}{2}$	C	2.5	D	1.5

19. A number increases by 125%. Which of these statements is true?

A	It's one and a quarter times its original size.
B	It's 25% bigger than it was.
C	It's a quarter as big again.
D	It's more than double its original size.