Let's think about ratio

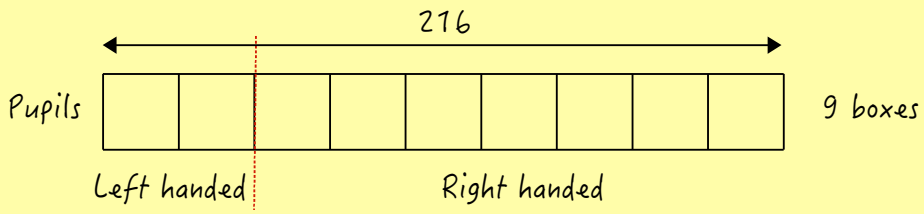
1. The ratio of left handed pupils to right handed pupils is 2:7 in Year 9. If there are 216 pupils in Year 9, how many pupils are right handed?

1 Can you draw a diagram that might help to answer this question?



Alice

Alice's idea



9 boxes, so 216 ÷ 9 = 24 in each box.
8 boxes for right handed pupils
So 8 x 24 = 192 right handed pupils

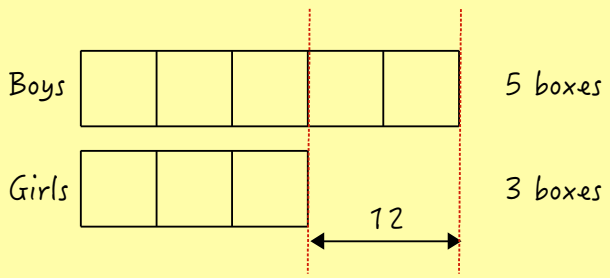
2. The ratio of the number of boys to the number of girls in a sports lesson is 5 :3. There are 12 more boys than girls. How many pupils are there in the class?

2 Can you draw a diagram that might help to answer this question?



Taro

Taro's idea



2 extra boxes for the boys, so each box represents 6 pupils.
5 x 6 = 30 boys and 3 x 6 = 18 girls
So 30 + 18 = 48 pupils altogether.

Some more questions

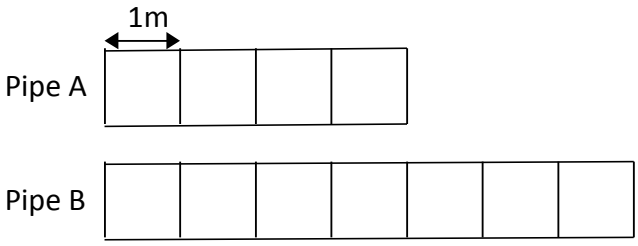
1. If 6 large donuts cost £2.10, how much should 4 similar donuts cost?

2. Avril cut a length of ribbon into three parts so that the ratio of the lengths was 1 : 2 : 4 The total length of the ribbon was 21m. What was the length of the longest part after cutting?

3. Write down 3 more ratios that are equivalent to 1 : 3 : 5

4. If 12 : 18 : 24 = 48 : a : b, what are the values of a and b?

5. Look at this diagram. It represents two lengths of pipe.



   a) What is the the total length of each pipe?
   b) What is the ratio of the length of pipe A to the length of pipe B?
   c) What is the ratio of the length of pipe B to the length of pipe A?
   d) What is the total length of both pipes?
   e) What fraction of the total length is the length of pipe A?
   f) What fraction of the total length is the length of pipe B?
   g) What fraction of the length of B is the length of A?

6. In the following table, you are given the ratio of one quantity to another. Complete the table.

Ratio statement	Fraction statements	
C:D = 2 : 5	C is $\frac{\square}{\square}$ of D	D is $\frac{\square}{\square}$ of C
E:F = 3 : 8	E is $\frac{\square}{\square}$ of F	F is $\frac{\square}{\square}$ of E

7. In the following table, you are given the ratio of one quantity to another. Complete the table.

Ratio statement	Fraction statements	
A : B = 2:9	A is $\frac{\square}{\square}$ of the total	B is $\frac{\square}{\square}$ of the total
C : D = 4 : 11	C is $\frac{\square}{\square}$ of the total	D is $\frac{\square}{\square}$ of the total

8. As part of a recycling project Amber has been collecting plastic milk bottle tops.
   $\frac{2}{5}$ of the tops are red and the rest are green.
   If there are 18 more green tops than red tops, how many tops had Amber collected?

9. Ms Parker spent $\frac{1}{4}$ of her salary on transport. Mr Ashton spent $\frac{1}{3}$ of his salary on transport. If they both spent the same amount of money on transport. What was the ratio of Ms Parker's salary to Mr Ashton's salary?

10. Taro's young brother has a large collection of marbles.
    $\frac{5}{7}$ of the marbles were red and the remainder were blue.
    Taro then swapped $\frac{2}{5}$ of the red marbles for extra blue marbles.
    What was the final ratio of red marbles to blue marbles?

11. The ratio of the number of red beans to that of green beans is 4 : 5 . The ratio of the number of green beans to that of black beans is 3 : 5 . There are 50 black beans.
    a) How many red beans are there?
    b) How many beans are there altogether?

12. 9Z were selling sweets to raise money for charity. Initially the ratio of packets of sweets to tins of sweets was 5 : 6. They decided to sell only the tins of sweets first until they had sold half of them. They then had 104 packets and tins left to sell. How many packets of sweets were left to sell?

13. There are 5 boiled sweets to every 2 toffees in a bag of sweets. If there are 15 more boiled sweets than chocolates, how many chocolates were in the box?

14. On a farm the ratio of cows to sheep is 2 : 9. If there are 441 more sheep than cows, how many cows are on the farm?

15. In a group of 72 children, the ratio of boys to girls is 3 : 5. How many boys would need to join the group so the ratio of boys to girls became 5 : 3 ?

16. Mike cuts a piece of wood into 4 parts in the ratio 2 : 3 : 4 : 5. If the combined length of the smallest 3 pieces is 32cm more than the length of the largest piece, what is the length of the smallest piece of wood?

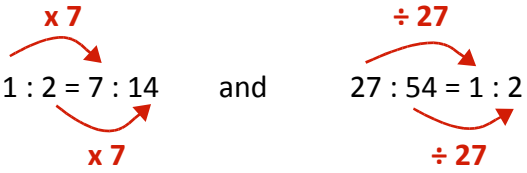
17. Last month Danni and Rohan saved some money in a ratio of 3 : 5. This month they saved an additional £176 together, and Danni now has three times as much money as she had last month, while Rohan has twice as much as he had last month. How much money did they save last month?

Equivalent ratios

In Ratio 3, we discovered that multiplying or dividing each part of a ratio by the same number does not change the ratio. Such ratios are called equivalent ratios.

For example,

1 : 2 = 7 : 14 and 27 : 54 = 1 : 2 as shown below



Ratios are generally given in terms of their smallest possible integers and this is known as the **simplest form**.

For example,

12 : 30 = 6 : 15 (dividing by 2) is not yet in its simplest form because if we can divided again by 3 and so 12 : 30 = 6 : 15 = 2 : 5. This is now in its simplest form.

Of course we could have initially divided by 6!

Fitness check

Write these ratios in their simplest form

1. 18 : 45
2. 25 : 75
3. 64 : 16
4. 48 : 78
5. 1.5 : 2
6. 1 : 1.2
7. 3.2 : 4.8
8. 2.6 : 15.6
9. 8 : 12 : 20
10. 0.6 : 0.9 : 1.5
11. $\frac{1}{2} : \frac{1}{3}$
12. $\frac{2}{5} : \frac{3}{4}$

Sometimes it can be useful to write simple ratios in an equivalent form.

For example, if $a : b = 3 : 4$ and $a : c = 2 : 3$, what is $b : c$?

3

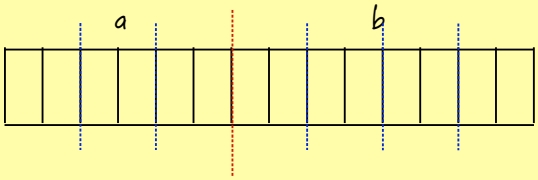
Can you think of a way to solve this question? Using equivalent fractions might help!



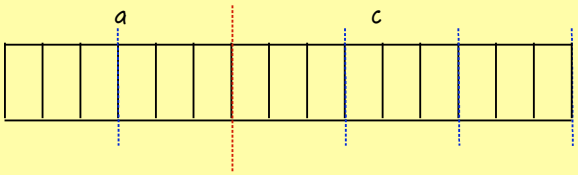
Matthew

Matthew's solution

Because the ratio of a : b = 3 : 4 and that suggests I use 3 boxes for a and 4 boxes for b.
But because the ratio of a : c = 2 : 3 that suggests I should use 2 boxes for a and 3 boxes for c.
So I really need the number of boxes for a to be a multiple of 6, so that I can split into 3 or 2 parts as necessary.
So my diagram looks like this.



Split in the ratio
6 : 8 = 3 : 4



Split in the ratio
6 : 9 = 2 : 3

So the ratio of b : c = 8 : 9



Evie

Evie's solution

The ratio of a : b is 3 : 4 or 6 : 8
The ratio of a : c is 2 : 3 or 6: 9
So the ratio of b : c is 8 : 9
Easy!

I just made "a" the same in both ratios

Both Matthew and Evie made use of equivalent ratios. Matthew used a diagram to show his thinking, Evie just used words. Both solutions are equally valid.

Fitness check

1. If $a : b = 3 : 4$ and the ratio of $a : c = 2 : 5$, what is $b : c$?

2. If $a : b = 2 : 7$ and the ratio of b : $c = 2 : 9$, what is $a : c$?

3. If $a : b = 3 : 11$ and the ratio of $a : c = 4 : 5$, what is $b : c$?

Some more questions

1. A library classifies its books as fiction, non-fiction and reference books.
   The ratio of fiction books to non-fiction books is 2:3.
   The ratio of non-fiction books to reference books is also 2:3.
   What is the ratio of fiction books to reference books?

2. The ratio of boys to girls in 8X is 4:5 and the ratio of boys to girls in 8Y is 1:2. If both classes contain the same number of pupils, what is the ratio of boys to girls if the classes are merged for PE?

3. Alan baked some cookies. He shared them out between himself and his sister in the ratio 3:1. Later, he shared the cookies he had left with his brother in the ratio of 3:1 again. If Alan sister received 2 more cookies than Alan's brother, how many cookies did Alan initially bake?

4. Riya's young sister has a bag of red, blue and green blocks. Riya watches his sister make towers with the blocks and realises that the ratio of the number of red blocks to the number of blue blocks is 3 : 5. She also sees that the ratio of red blocks to the number of green blocks is 4 : 7. If Riya's sister has 3 more green blocks than blue blocks, how many red blocks has she got?

5. Some apples, oranges and pears were taken to market. The ratio of apples to oranges was 4 : 5. The ratio of pears to the total number of apples and oranges was 5 : 6. The first customer bought some apples and oranges. The number of apples and oranges left was then equal to the number of pears. There were now 120 pieces of fruit left to sell. How many apples and oranges did the first customer buy?

6. Mr Lim spent $\frac{3}{8}$ of his salary on rent. Miss Cutter spent $\frac{2}{7}$ of her salary on rent.
   If they both spent the same amount of money on rent each month, what was the ratio of Mr Lim's salary to Miss Cutter's salary in its simplest form?

7. The Head of PE assembled all of the pupils in Yrs 7, 8 and 9 together in the sports hall to organise the teams for sports day. $\frac{5}{8}$ of the pupils were involved in individual events and were sent off to their events.

   $\frac{1}{6}$ of the remainder were helping with judging events and they were also sent off to their events. The remainder of the pupils were sent to the spectators' area.

   a) What fraction of the pupils were spectators?

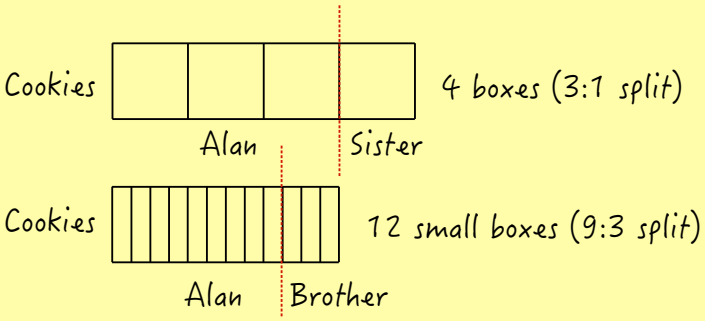
   After the final event, 225 competitors and judges joined the spectators for the presentations but the ratio of those present at the end to those present at the beginning was 7 : 8.

   b) How many pupils initially assembled in the sports hall?



Amber

Amber's solution to Q3



Alan's sister has one big box = 4 small boxes
Alan's brother has 3 small boxes.
So 1 small box holds 2 cookies.
So Alan made 16 x 2 = 32 cookies originally



Taro

Taro's solution for Q4

The ratio of red to blue is 3 : 5 or 12 : 20
The ratio of red to green is 4 : 7 or 12: 21
So there are 21 green boxes and 20 blue boxes for every 12 red boxes.
But there are 3 more green cubes than blue cubes so each box contains 3 cubes.
The number of red cubes = 12 x 3 = 36

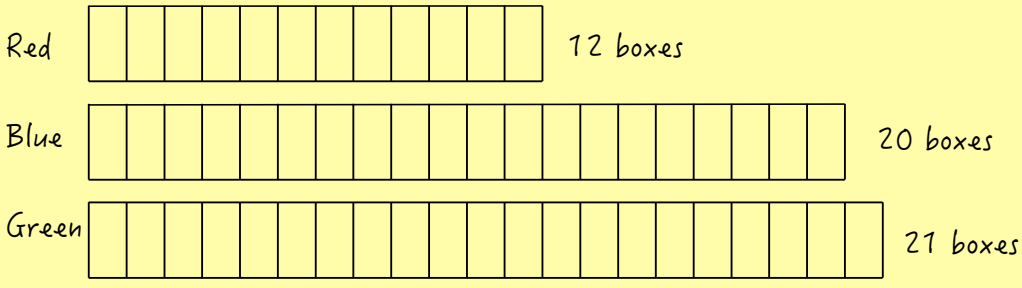
I just made the number of red cubes the same in both ratios.



Sarah

Sarah's solution for Q4

I did the same thing but I drew a diagram to help me think!



1 more green box than blue boxes.
So each box represents 3 cubes
So 12 x 3 = 36 red cubes

But how much did Amber spend?

At the start of a new term, Jon, Sarah and Amber went to the corner shop to stock up on pens and pencils.



Jon

I bought 3 pens and 5 pencils for £3.40.



Sarah

I spent £2.15 on 2 pens and 3 pencils.

I just bought one pen and one pencil.



Amber

4

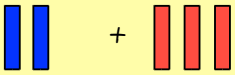
Can you work out how much Amber spent?



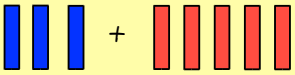
Evie

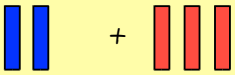
Evie's solution

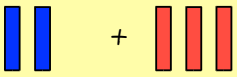
Jon  = £3.40

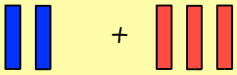
Sarah  = £2.15

Jon  = £3.40

 = £3.40

Sarah  = £2.15

 = £2.15

 = £2.15

I decided to make the number of pens the same in both cases.

6 pens and 10 pencils would cost £6.80

6 pens and 9 pencils would cost £6.45

So 1 pencil must cost £6.80 - £6.45 = 35p

So thinking about Sarah's purchase

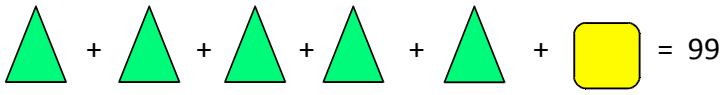
3 pencils cost 3 x 35p = £1.05

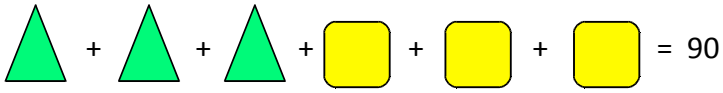
So 2 pens cost £2.15 - £1.05 = £1.10

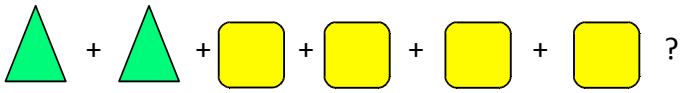
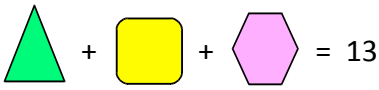
So 1 pen cost £1.10 ÷ 2 = 55p

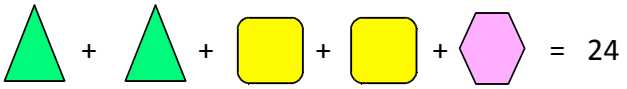
Amber spent 55p + 35p = 90p.

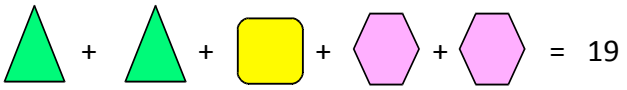
Some more questions

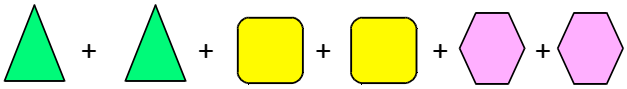
1. An ice cream and two lollies cost £2.70. Three ice creams and four lollies cost £6.60. How much does five ice creams and five lollies cost?
2. If  = 99

   And  = 90

   What is  ?
3. At a local tea shop, Ellen noticed that 3 cups of tea and 2 cakes cost ££6.40 but 2 cups of teas and 3 cakes cost £6.60. How much would 1 cup of tea and 1 cake cost?
4. 4 T-bars and 7 C-bars weigh 11.5 tonnes. 5 T-bars and 4 C-bars weigh 10.1 tonnes. How much heavier is 1 T-bar than 1 C-bar?
5. If  = 13

   and  = 24

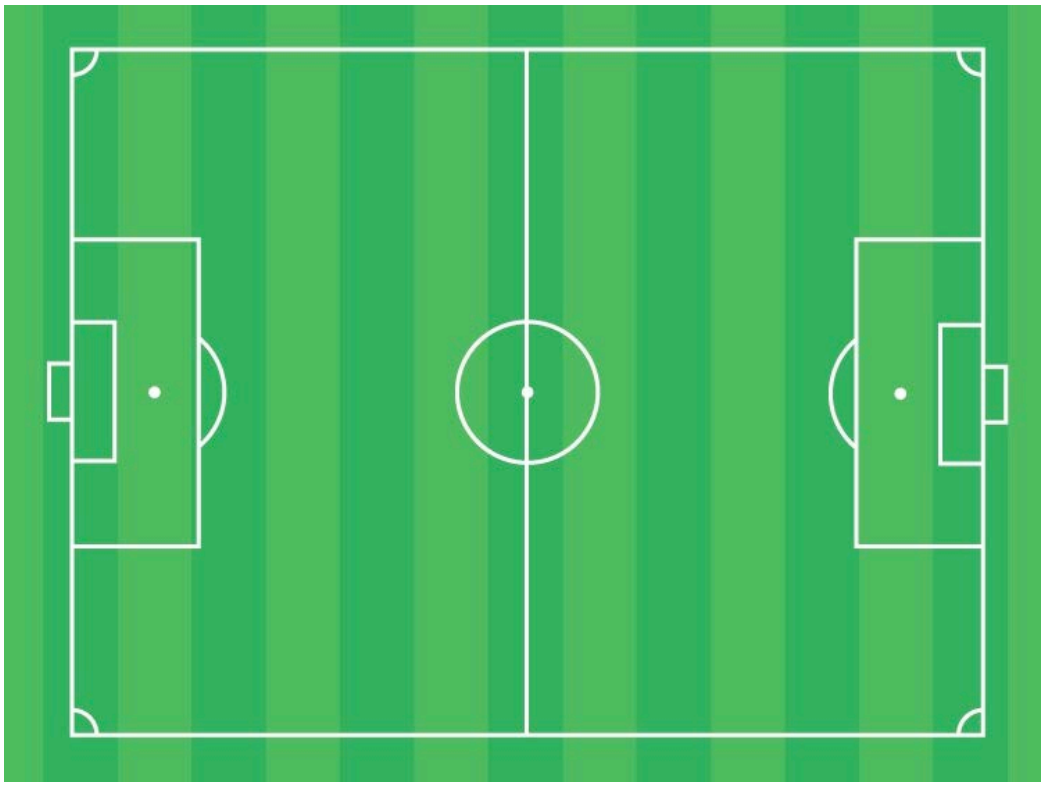
   and  = 19

   What is  ?



Question creation - Have fun creating some difficult problems of your own for your friends to solve

How big is this soccer pitch?





Evie

In the diagram, he actual length of the pitch is 12 cm.

… and the width is 9cm.



Alice

4 Using these results, work out the dimensions of the actual pitch if the scale is 1cm represents 9m.



Taro

You could use ratios to express the scale. For example, in this case the scale is 1: ...

5 What do you think Taro was about to say?



Sarah

Sarah's suggestion

The scale is 1 cm represents 10m.

As ratios do not include the units, the quantities must be in the same units.

1m = 100cm, so

10m = 1000cm

So, as a ratio, the scale 1 : 1000

Fitness check

1. Rewrite these ratios in the correct form (without units). Give your answer in the simplest form.

a) 10 minutes : 1 hour

b) 15mm : 2cm

c) 20p : £1

d) 4 hours : 1 day

e) 3 litres : 1200ml

f) 3 days : 6 hours

g) 2 minutes : 100 seconds

h) 3km : 800m

i) 280cm : 420m

j) 600kg : 2 tonnes

k) 12 seconds : 1 week

l) 5cm : 3km

2. Rewrite these ratios in the simplest form.

a) 3.5 : 1

b) 0.6 : 40%

c) $\frac{1}{3} : 2$

d) $\frac{3}{5} : \frac{5}{6}$

e) $\frac{3}{4} : \frac{5}{8}$

f) $0.35 : \frac{11}{20}$

3. On a map the scale is 1 : 40000.

a) What distance is represents by 3cm on the map, Give you answer in metres.

b) The distance from Penzance in Cornwall to Exeter in Devon is 5.8cm on the map. How far is that in real life. Give your answer in km.

c) The distance from Southampton to Edinburgh in real life is approximately 570km. What would be the distance on the map?

4. A sheet of A4 paper measures 210mm by 297mm. Alice wants to draw a map of the United Kingdom on a sheet of A4 paper. Can you suggest a sensible scale for her map. Explain your reasoning.

Buy two, get one free?





6 What is the normal price of one tin of Christmas Treats?



Amber

£2.40 divided by three.
Thats 80p each.

That's not right. It should be £2.40 divided by two. So that's £1.20 each.



Matthew

7 What would you have to pay if you wanted 4 tins?
What about 5 tins? Or 6 tins? Or 8 tin? Or 9 tins?



Alice

Alice's suggestion

4 tins would cost £2.40 (special offer) + £1.20 (1 tin) = £3.60

5 tins would cost £2.40 (special offer) + £2.40 (2 ins) = £4.80

But for that amount I could get 6 tins!

£2.40 (special offer) + £2.40 (special offer)

8 tins would cost £2.40 + £2.40 + £2.40 = £7.20

But that is the same cost as 9 tins.

For the best value you should buy in multiples of 3!

Unit price

Supermarkets generally express the cost of goods in terms of a simple unit to help shoppers compare prices.

For example

Luxury Christmas Cake
(1kg)



£7.50

75p per 100g

Iced Christmas Cake
(900g)



£5.00

56p per 100g

Mini Christmas Cake
(150g)



£2.00

£1.33 per 100g

Fitness check

In each of the following questions, which offers the better value? You must explain your reasoning.

1

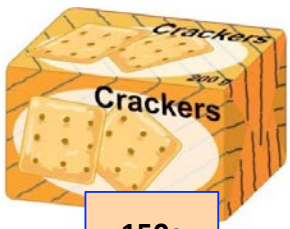


1.5L
£1.75

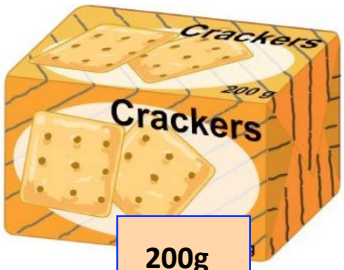


800ml
£1

2



150g
£1.02



200g
£1.38

3



150g
63p



180g
74p

4



Diet Coke
8 x 330ml
£3.50



Buy 2
Get one
FREE

Diet Coke
1 x 330ml
70p

Some more questions

1. The Corner Shop sells Salt 'n Vinegar crisps in a bumper pack of 8 packets for £1.84. The local supermarket sells the same crips in packs of 10 for £2.35. Which shop offers the best value?



2. The ingredients needed to make 8 pancakes are 250ml milk, 1 egg, 140g flour and 5g of butter. How much of each ingredient would be needed to make 50 pancakes for a youth group?

3. If Taro only has 400g of flour but plenty of the other ingredients, how many pancakes could Taro make?

4. 

The ingredients needed to make chocolate brownies to serve 12 people are 100g butter, 200g dark chocolate, 4 eggs, 250g golden caster sugar, 100g plain flour, 1 teaspoon of baking powder and 30g of cocoa. How much of each ingredient is needed to serve 80 people?

5. If Alice can use 14 eggs, how much would she need of all of the other ingredients and how many people could she then serve.

6. At the local fun fare 4 rides on the Big Wheel and thee rides on the Octopus cost £9.35. 5 5 rides on the Big Wheel and 4 rides on the Octopus cost £12. How much does a ride on the Big Wheel cost?

Ratio 4 – Mastery questions

In questions 1 – 12, four possible answers are given. Only one of them is the correct. Record the letter which you think indicates the correct answer. You must be able to give reasons for your choice.

1. Express 12 : 42 in its simplest form.

A	6 : 21	B	2 : 7	C	1 : 3.5	D	0.56: 1

2. Write 54cm to 2.16m in its simplest form?

A	0.54 : 2.16	B	54 : 216	C	3 : 12	D	1 : 4

3. Charlie and Olivia share out their sweets in the ratio 3 : 7. If Olivia gets 40 more sweets than Charlie, how many sweets does Charlie receive?

A	12	B	16	C	28	D	30

4. What is the ratio of 35 minutes to 2.5 hours in its simplest form?

A	35 : 150	B	35 : 25	C	7 : 30	D	1 : 4.29

5. The ratio of girls to boys in 9X is 3 : 5. Which of these numbers can not be the number of pupils in 9X?

A	16	B	24	C	30	D	32

6. If a : b = 5 : 7, and b : c = 2 : 5. What is the ratio of a : c?

A	5 : 35	B	10 : 35	C	14 : 35	D	7 : 12

7. Farmer Coombe harvests pears. He splits his crop into Premium, Budget and Reject pears in the ratio 4 : 7 : 2. Calculate the number of pears he picked if he has 42 more Budget pears than Premium pears.

A	74	B	84	C	126	D	182

8. Molly is making a fruit drink using orange juice, pineapple juice and blackcurrant juice in the ratio 6 : 3 : 2. She has lots of orange juice and blackcurrant juice but only 300ml of blackcurrent juice. How much fruit drink can she make?

A	600ml	B	900ml	C	1500ml	D	3600ml

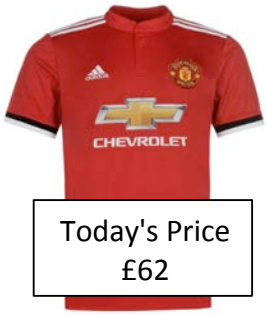
Ratio – Mastery questions

9. A lasagne recipe for 6 people requires 300g of pasta, 750g of mince and 2 celery sticks. Which of these statements calculates how much mince is needed for 7 people?

A	$\frac{300}{6} \times 7$	B	$\frac{750}{6} \times 7$	C	$\frac{750}{6} + 1$	D	$\frac{750}{7} \times 6$

10. Which of these offers represents the best value for money?

A	Buy one, get one free
B	Buy two, get one free



Today's Price £62

C	50% off today
B	£32 discount if you pay cash.

11. A small box of screws contains 32 flat head screws and 48 cross head screws. How many cross head would be in a similar box if there are 180 screws altogether in the box?

A	72	B	80	C	100	D	108

12. Sarah has been collecting stamps for a number of years. She splits here stamps into European, Asian and American in the ratio 5 : 3 : 4. If she has 504 non-European stamps, how many Asian stamps does she have?

A	360	B	216	C	288	D	504

In Q13 – 15, you must show all of your working.

13. James has a large collection of action sports photographs. The ratio of male to female is 5 : 4. For his birthday, his sister gave here 6 more female action photographs. The ratio of male to female photographs was then 8 : 7. How many female action photographs has James now got in his collection?

14. Maisie had saved £114, but her sister Suzzie had saved only £44. After they both earned the same amount of money cleaning cars one weekend, Maisie then had twice as much money as Suzzie. What was the combined total they earned by cleaning cars?

15. Mike and Amy went shopping with a total of £196. After Mike spent 4/7 of his money and Amy spent £52, they both had the same amount of money left. How much money did Amy have at first?