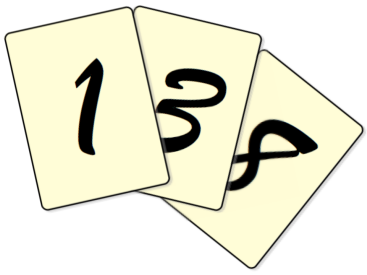
Intuition, guesswork or skill …?



1. Look at these ten plates positioned around the board. They are numbered 1 to 10, but they have been placed in a random order around the board. You have to decide if the number on the next plate will be higher or lower than the previous plate.

2. You are now going to play "Higher or Lower" with your partner. You will be given 10 cards numbered 1 to 10. Shuffle them and then place the cards face down on the table in a random order. In pairs, turn the first card over and ask your partner if they think the next card will be higher or lower than the first. If they get it right, they keep going; if not, they lose.

3. After you have played the game a few times, discuss why you chose to go higher or lower each time.



1 Do you think this is a game of skill or a game of pure chance?

Heard in passing ...



My brother says that it is always frosty when he wakes up on his birthday. When might his birthday be?

My sister asked my Dad a question. His reply was, “It's more than likely.” What might her question have been?

A reporter on the TV news yesterday said that there was a fifty/fifty chance of something happening. What might she have been talking about?

Amber's mum said, “There is no chance of us winning the jackpot.” What might she have been talking about?

Last summer, Matthew's dad said, “If we hurry, there's still a slight chance we might make it.” What might he have been talking about?

My younger bother said at school they had to design a spinner with three colours and then spin it lots of times. He said, “It landed on blue most of the time, red some of the time and only once on yellow.” What might his spinner have looked like?

Chance

Benjamin Franklin, President of the USA over 300 years ago, once said, “Nothing can be said to be certain, except death and taxes.”

In fact,almost everything we do is based on the chances of success. Big business, medicine, the weather and politics all weigh up the options and their chances of success before making any decisions or predictions.

Probability is the formal study of the laws of chance. But it is also the driving force behind a very common human pastime – gambling.

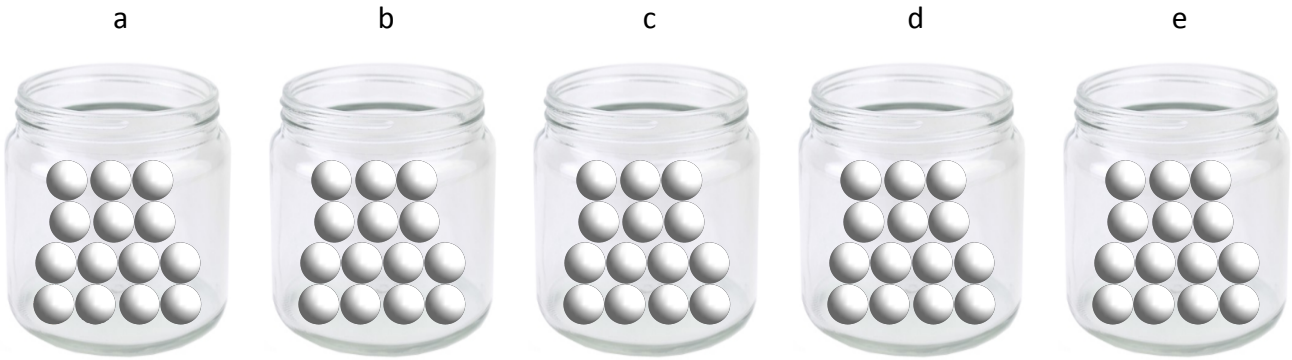
In modern society, it is essential to understand how probability can help you avoid gambling pitfalls by estimating your realistic chances of winning.

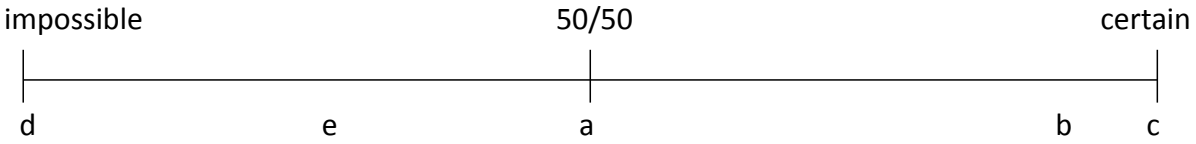
Gambling has been a human addiction for over 2000 years, and even Emperor Claudius (10BC – 54AD) wrote the first known book on gambling entitled, “How to win at dice.”

2 How would you describe the chances of throwing a six with a fair six-sided dice?

The probability line

1. These jars all contain some sweets. They may be red or black. One sweet is selected at random. How might you colour them in to match their position on the probability line if the probability line shows the chance of selecting a red sweet?

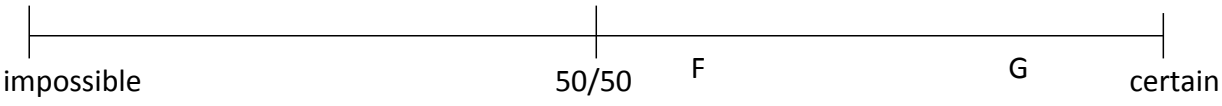




2. What are the chances of spinning blue on each of these spinners?
Mark your thoughts on a probability line.



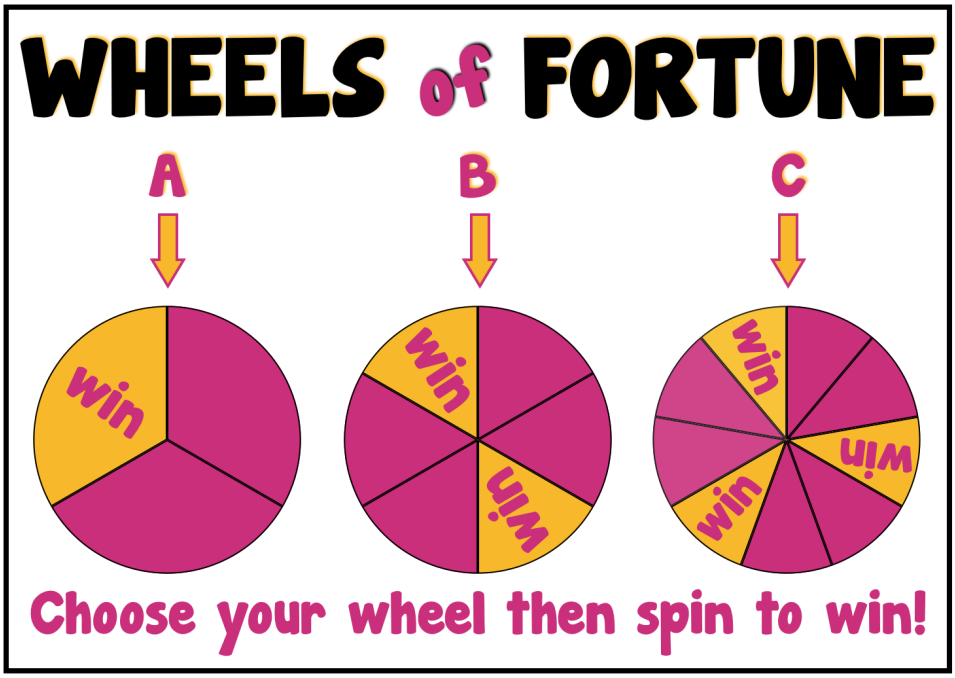
A B C D E



Chances of spinning blue

3. Design two extra spinners that match the points F and G on the probability line above.

What are your chances?





Amber: I think wheel C is best, as there are three ways of winning...



Sarah: No, wheel A best as there are only two ways of losing...



Evie: If I have 3 goes on wheel A, I am sure to win at least once!



Jon: The wheels are all equal. You either win or lose, so it is 50/50

1. What do you think about these statements?
2. Which wheel do you think gives the best chance of winning, and why?
3. Who will come off best in the long run – the person running the game or the players? Or both?
4. Make your own spinners and play!

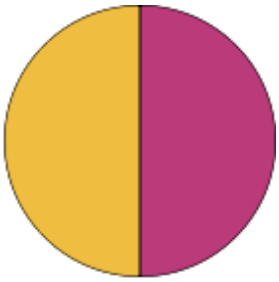
Roll Up! Roll Up!

1. Look at the six spinners shown below.
2. Which game would you like to play? And why?
3. Which game would you like to run? And why?



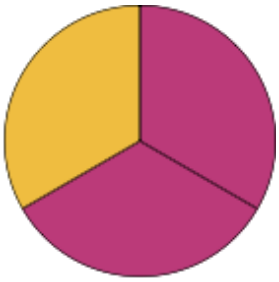
Megan's Big Wheel

10p a go, 15p if you win!



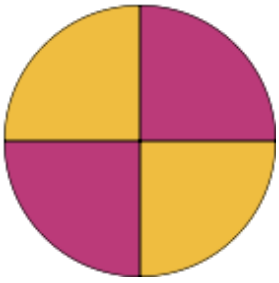
Raj's Winners

50p a go, £1 if you win!



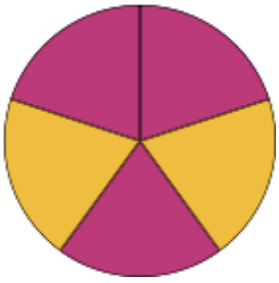
Matthew's Max Win

£1 a go, £3 if you win!



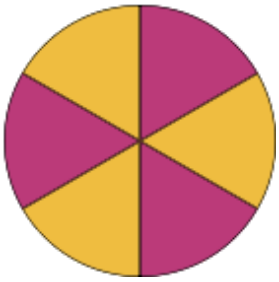
Tilly's Treasure

£5 a go, £8 if you win!



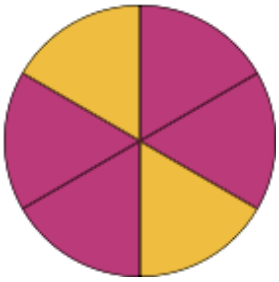
Amber's Amazing Arc

£1 a go, £2 if you win!



Taylor's Treat

£5 a go, £20 if you win!



What's in the bag?



Mr T

I have put 10 cubes in a bag.
You have to decide what colour they are without looking.
You should draw one cube out at a time, look at it, record the colour and then put it back.
After every 5 goes, you must make a prediction of what is in the bag.

3 How many goes do you think you will need before you are confident of your prediction?



Jon

10 ?

No ...definitely more than 10



Alice



Amber

Why would you need more than 10 goes?

4 How would you answer Amber's question?

Sarah's working



Sarah

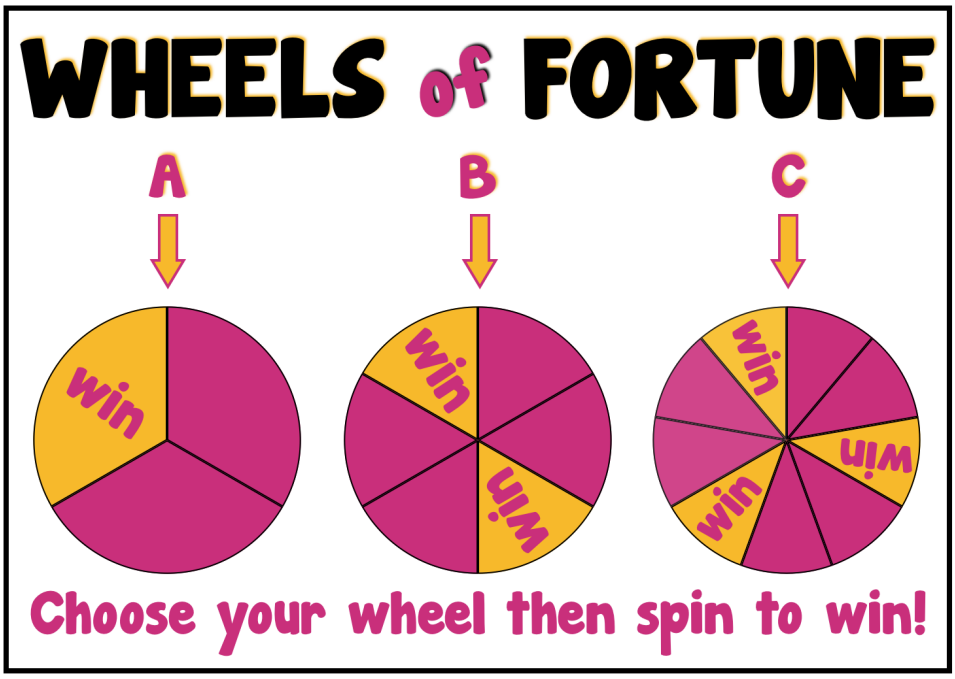
Ok, my first 5 goes were red, blue, red, blue, red
So I think there are 6 reds and 4 blues.
Next 5 goes - red, red, blue, blue, red.
So now I have 6 reds and 4 blues – that is my prediction.
That's no change.
Next 5 – red, blue, red, red, red.
That's 10 red and 5 blue.
That doesn't fit my pattern, but I still think 6 reds and 4 blues.
Next 5 – red, red, red, red, red.
Wow that 15 reds and 5 blues.
So I think it might actually be 7 reds and 3 blues.



Mr T

Now let's have a look in the bag ...

The Probability scale





Taro

We could use fractions to express the chances of winning on each spinner. For example, the chances of winning on spinner A is ...

5 How would you finish Taro's sentence?

1. What are the chances of winning on spinner B?
2. What are the chances of winning on spinner C?
3. Which spinner gives you the best chance of winning?



Amber

Amber's working

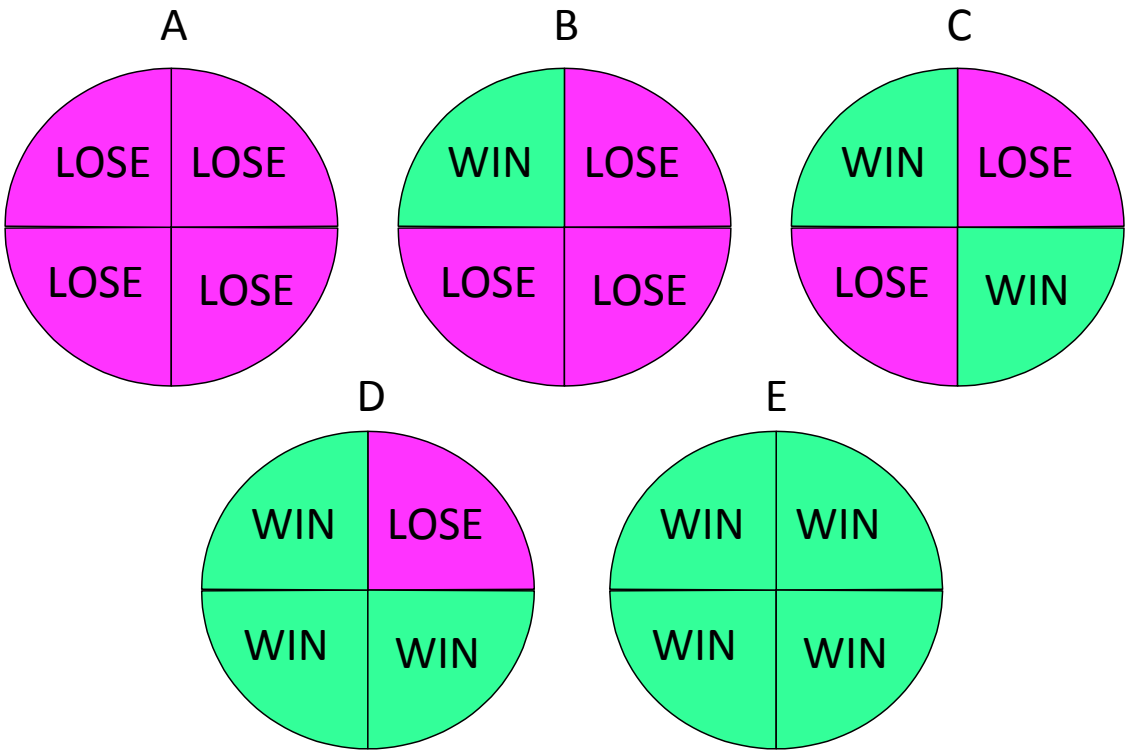
The chances of winning on spinner A is $\frac{1}{3}$

The chances of winning on spinner B is $\frac{2}{6} = \frac{1}{3}$

The chances of winning on spinner C is $\frac{3}{9} = \frac{1}{3}$

So the chances of winning are exactly the same in each case.

More spinners



Each of these spinners has 4 equal quadrants.

What is the probability of winning in each case written as a fraction?

Mark your results on a probability line with a suitable numeric scale.



Jon

Jon's working

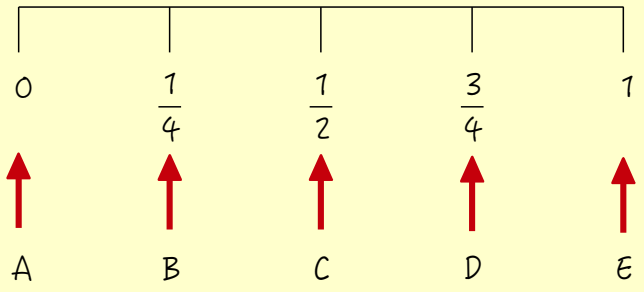
The chances of winning on spinner A is $\frac{0}{4}$

The chances of winning on spinner B is $\frac{1}{4}$

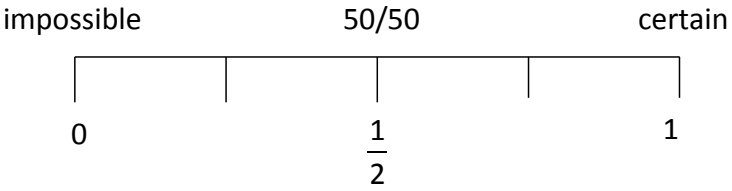
The chances of winning on spinner C is $\frac{2}{4} = \frac{1}{2}$

The chances of winning on spinner D is $\frac{3}{4}$

The chances of winning on spinner E is $\frac{4}{4} = 1$



Using the probability scale





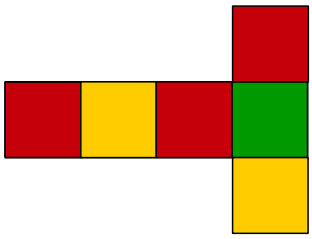
Miss F

This line is sometimes called the probability line. Why must it start at zero and end at one?



Mr T

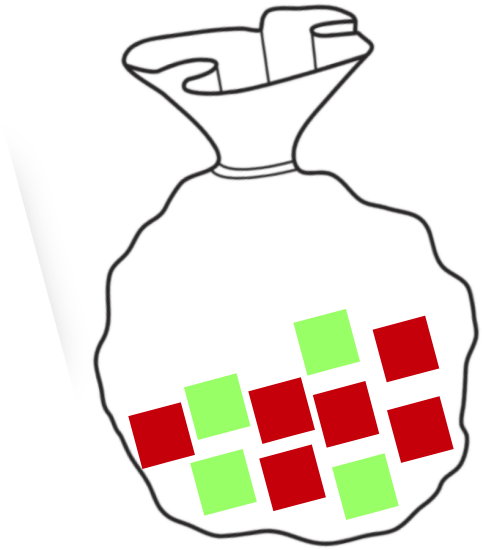
Look at the net of this 6-sided multicoloured dice.



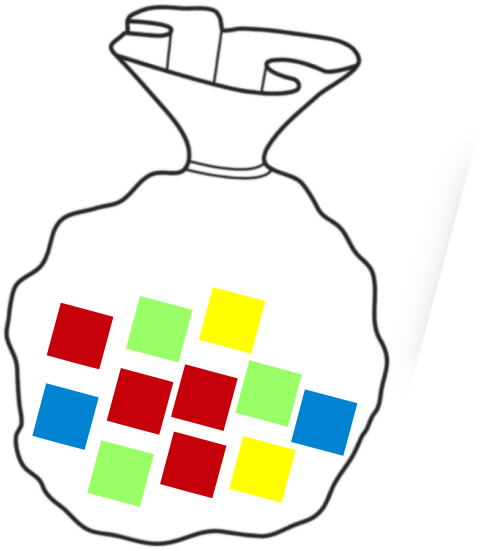
1. If you rolled the dice, which colour is most likely to end up on top?
2. What is the least likely colour to end up on top?
3. What is the probability of red ending up on top?
4. What is the probability of yellow ending up on top?
5. What is the probability of green ending up on top?
6. Mark your answers to Q3 – 5 on a probability line.
7. What is the probability of it not being yellow on the top?
8. What is the probability of it not being green on the top?
9. What is the probability of it not being red on the top?
10. What is the probability of green or yellow ending up on top?
11. Why must the answers to Q9 and Q10 be the same?
12. Design a 6-sided, multicoloured dice of your own so that the probability of getting red is $\frac{1}{3}$, the probability of getting yellow is $\frac{1}{3}$ and the probability of getting green is $\frac{1}{6}$.
13. Why is it not possible to design a 6-sided, multicoloured dice so that the probability of getting red is $\frac{1}{4}$?

Fitness check

1. A bag contains 6 red and 4 green cubes. You select one cube at random. What is the probability that it is:
   a) Red?
   b) Green?
   c) Red or green?
   d) Yellow?
   e) Not red?
   f) Not green?



2. A second bag contains 4 red, 3 green, 2 blue and 2 yellow cubes. You select one cube at random. What is the probability that it is:
   a) Red?
   b) Green?
   c) Red or green?
   d) Yellow?
   e) Blue?
   f) Not red?
   g) Not green?
   h) Neither red nor green?



3. There is a third bag containing some cubes. The chances of choosing a red cube is $\frac{1}{2}$. The chances of choosing a green cube is $\frac{1}{3}$. What might be in the bag? Explain your thinking.



Let's think about choices

1. What's for lunch?

The school canteen offers a choice of 4 main meals and 3 desserts.

How many different options are available, assuming you have a main meal and a dessert?



2. Heads or tails?

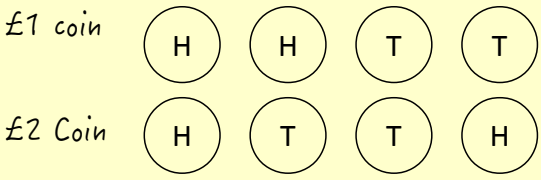
Suppose your toss a £1 coin and a £2 coin at the same time.

How many different outcomes are possible assuming the coins land flat.



Amber

Amber's solution to Q2.

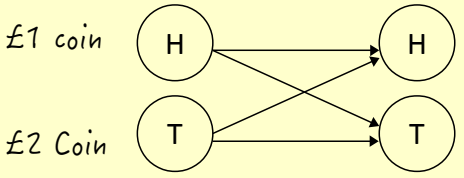


So 4 possible outcomes



Jon

Jon's solution to Q2.



2 ways starting with heads

and 2 ways starting tails

So 4 possible outcomes

Sample spaces

It is often helpful when thinking about probability questions to list all of the possible outcomes logically. This is exactly what Amber did in the heads and tails problem.

A **sample space** is a logical listing of all the possible outcomes in an experiment. It can be pictures, a table or a list.

6 Can you create a heads/tails sample space if there were three coins?

Let's think about recording the outcomes

Back to the three coin problem



Matthew

Matthew's solution

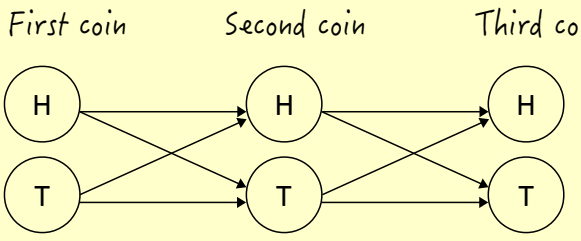
First coin	H	H	H	H	T	T	T	T
Second coin	H	H	T	T	T	T	H	H
Third coin	H	T	H	T	T	H	T	H

So 8 possible outcomes



Sarah

Sarah's solution



So 4 possible outcomes starting with heads and four possible outcomes starting with tails.

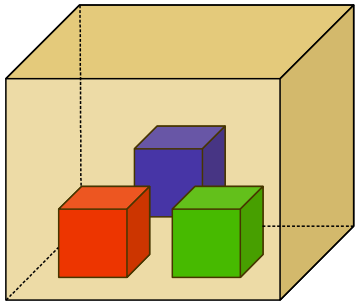
So 8 possible outcomes

Just two cubes

A box contains a red cube, a blue cube and a green cube. One cube is randomly drawn out, the colour recorded and then put back into the box.

A second cube is now drawn out, and the colour recorded.

Try to write down the sample space for this experiment.

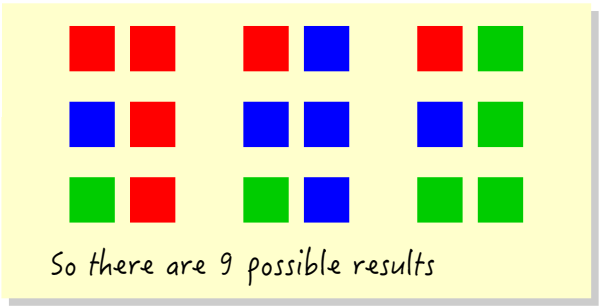


What are your chances?



Sarah

Sarah's sample space



1. Looking at Sarah's results, what is the probability that she got:
   a) Two the same colour?
   b) Two that were not the same colour?
   c) No red cubes?
   d) Only one red cube?
   e) At least one red cube?
   f) A green cube?

2. Suppose that instead of putting the cube back into the box after the first draw, you put the cube on the table before drawing the second cube.

   Create a new sample space for this experiment.

   What is the probability that you now have:
   a) Two the same colour?
   b) Two that were not the same colour?
   c) No red cubes?
   d) Only one red cube?
   e) At least one red cube?
   f) A green cube?

3. A fair six-sided dice is rolled once. Create a suitable sample space diagram.

   What are the chances of getting:
   a) a six?
   b) an even number?
   c) anything but a six?
   d) more than four?
   e) less than two?



And the Gold medal goes to ...


Evie


Sarah


Matthew



Evie, Sarah and Matthew were the three overall winners in the inter-class algebra competition. The medals were then awarded to them based on their individual scores.

There was a Gold medal for the overall winner, a silver medal for second and a bronze medal for third.

1. Draw a sample space to show how the medals might have been awarded.

   Hence decide on the probability that

   a) Evie won gold or silver

   b) Sarah did not win gold

   c) Matthew finished ahead of Evie


Taro

Taro's working

Gold	Silver	Bronze
Evie	Sarah	Matthew
Evie	Matthew	Sarah
Sarah	Evie	Matthew
Sarah	Matthew	Evie
Matthew	Evie	Sarah
Matthew	Sarah	Evie

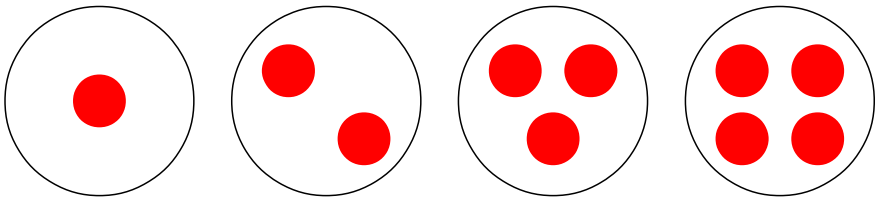
There are six different ways the medals could be awarded.

a) The probability that Evie won gold or silver is $\frac{4}{6} = \frac{2}{3}$

b) The probability that Sarah did not win gold is $\frac{4}{6} = \frac{2}{3}$

c) The probability that Matthew finished ahead of Evie is $\frac{3}{6} = \frac{1}{2}$

Lu-Lu



The game of Lu-Lu originates from the Hawaiian islands and is played with 4 “stones” or “coins”. As shown above, these stones are about 2.5cm in diameter and are marked with red dots on one side only.

How to play

The stones are thrown or shaken and let drop to the floor. You count the number of spots that are face up.

If all of the pieces land face up, you have a second go with all of the stones. Your score for the round is the total score from both drops.

If all of the pieces do not land face up, the player has a second throw but only throws the stones that landed face down. The scores from the second throw are added to the score from the first throw.

Play then moves to the second player.

A round ends when everyone has had their first turn.

The players now play a second round.

This procedure continues until the end of the round, in which one person reaches 60 points.

Now play the game!

1. What are all the possible scores a player can get in one turn?
2. How many different ways can each score be obtained?
3. How many turns would a player need to reach 60 if he got the maximum number of points per turn?
4. What is the best way of keeping track of the score?



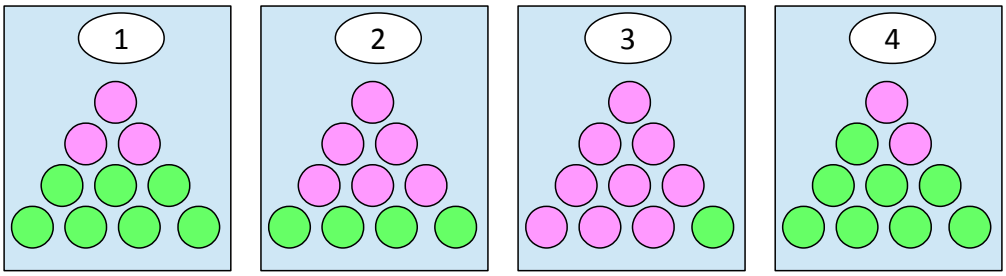
Matthew

This game is fun! I wonder what would happen if we added another stone?

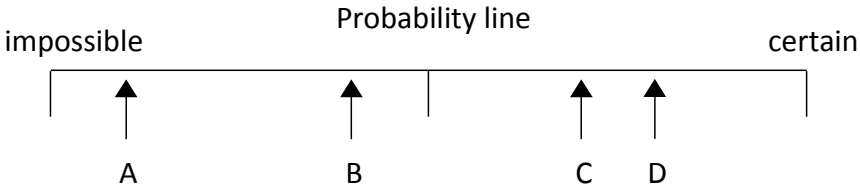
5. Try remaking the game with an extra stone. What should an extra stone look like?
6. Introduce your own new rule and try playing the game again.
7. How does the new rule affect the game?

Probability 2: Mastery questions

Look at these bags of marbles. Each one contains 10 marbles. One marble is chosen at random from each bag. The probability line represents the chance of choosing a green marble.



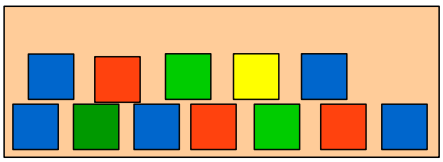
Questions 1 to 4 refer to these bags and to the probability line below.



1. Which arrow represents bag 1?
2. Which arrow represents bag 2?
3. Which arrow represents bag 3?
4. Which arrow represents bag 4?
5. Which word best describes the chances of the sun setting tomorrow?

A	B	C	D
Certain	Unlikely	Likely	Even chance

6. Matthew has some multilink in a box and chooses one at random. What is the probability that it is blue?

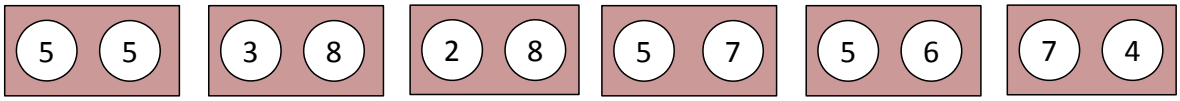


A	B	D	
$\frac{5}{8}$	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{7}{12}$

7. What is the probability of throwing a multiple of 3 with a fair six-sided dice?

A	B	C	D
$\frac{1}{6}$	$\frac{3}{6}$	$\frac{2}{3}$	$\frac{2}{6}$

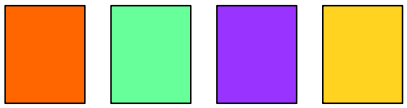
8. Each of these cards has 2 numbers on them.



Amber chooses a card at random. She adds up the 2 numbers. What is the most likely total of the numbers on her card?

A	B	C	D
5	10	11	12

9. Look at this set of cards.



Jon chooses three of them. How many different ways can he do this if the order doesn't matter?

A	B	C	D
4	6	12	24

In Questions 10 and 11, you must explain your thinking.

10. You toss a 50p piece, a £1 coin and a £2 coin at the same time.

a) Draw a sample space to show all of the possible results.

What is the probability that you get

b) 3 heads?

c) 3 the same?

d) 2 tails and 1 head in any order?

11. Alice has designed a spinner with several equally sized red, blue, green and white sectors. She then created a table to show the probability of getting each colour, which is shown below.

Red	Blue	Green	White
$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$

a) Assuming Alice is correct, how many sectors of each colour might she have designed?

b) Can you find another solution?

c) Could Alice's spinner have 30 sectors? Explain your answer.