What do you know about probability?

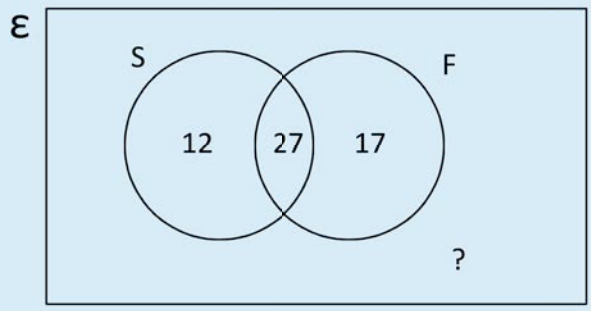
If the probability of winning a game of tennis is $\frac{1}{3}$, what is the probability of not winning?

If the probability of winning a game is 0.7. Roughly how many times would you expect to win if you played 20 games?

Alice spun a spinner 40 times.
Here are her results.
What might her spinner have looked like?

Colour	Tally marks	Total
Green	𝍸	10
Blue	𝍸 𝍸 ////	14
Red	𝍸 𝍸 𝍸 /	16

65 people were surveyed and asked "Have you ever been to Spain or France for a holiday?". Their answers are shown on this Venn diagram.



a) How many people had not been to either country?

b) How many people had been to both Spain and France?

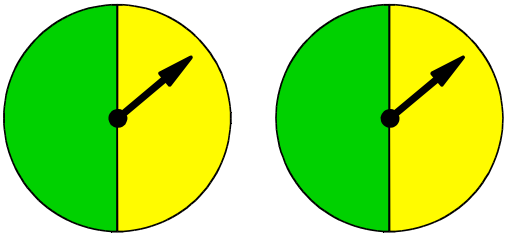
c) What is the probability that a person chosen at random from the group has only been to Spain?

Two spinners



Mr T

Look carefully at these two identical spinners. If both spinners are spun at the same time, what is the probability that they both end up pointing at yellow?





Alice

The probability of getting yellow on each spinner is $\frac{1}{2}$.



Matthew

In which case, the probability of getting yellow on both spinners must also be $\frac{1}{2}$.

1 Do you agree with Matthew's comment? Explain your thinking.



Amber

I am not sure, but we could try it out for ourselves by making two spinners.



Jon

If we have 100 goes then, if Matthew is right, we should yellow/yellow about 50 times.

2 What do you think happened when they tried it out?



Sarah

I think they got exactly 50 yellow/yellows.



Evie

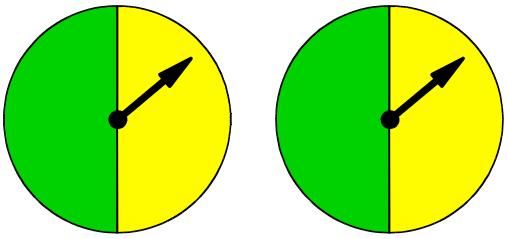
I think they got about 50 yellow/yellows.

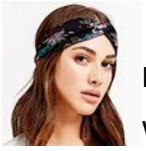


Taro

I actually think they got a lot less than 50.

Two spinners

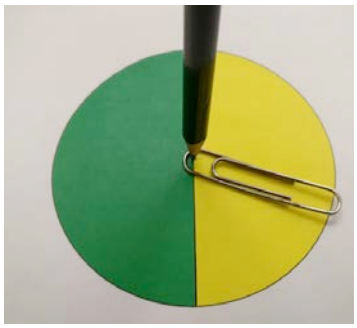




Miss F

I wonder if you would get the same sort of results?

We are going to make our own spinners and find out.



You will each need the template supplied, a large paper clip and a pencil as shown in the picture. You will then work in pairs so that you have two spinners. Spin your spinners 50 times and record the results.

Jon and Amber's results

Colours	Tally marks	Total
Two greens	𝍸𝍸𝍸𝍸///	23
Two yellows	𝍸𝍸𝍸𝍸𝍸/	26
One of each	𝍸𝍸𝍸𝍸𝍸 𝍸𝍸𝍸𝍸𝍸/	51



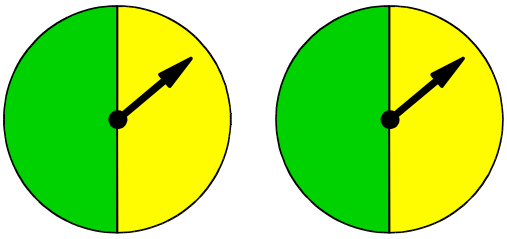
Amber

So it looks like the probability of getting two yellows is about $\frac{1}{4}$.

3

Where you results similar? What about if you recorded everyone's results?

Two spinners



Their class results

Colours	Total	%
Two greens	163	23.2%
Two yellows	174	24.9%
One of each	363	51.9%
Total	700	100%



Amber

So it definitely looks like the probability of getting two yellows is about $\frac{1}{4}$.

4 Can you think of a reason for their results?

Probability and games

Almost everything we do, is based on the chances of success. Big business, medicine, the weather and politics all weigh up the options and their chances of success before making any decisions or predictions.

Probability is the formal study of the laws of chance. But is also the driving force behind a very common human pastime – gambling.

In modern society, it essential to understand how probability can help you to avoid the pitfalls of gambling by estimating your realistic chances of winning.

Gambling has been a human addiction for over 2000 years and even the Emperor Claudius (10BC – 54AD) wrote the first known book on gambling entitled, “How to win at dice.”

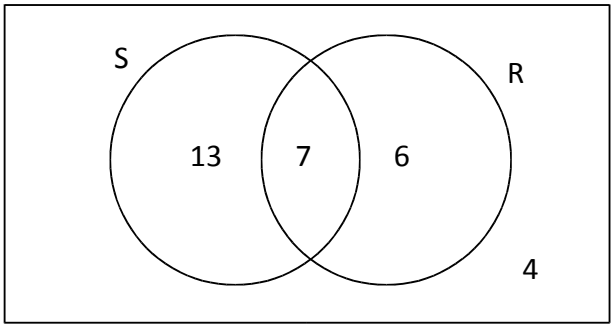
One of the best ways of getting a feel for probability is by playing games of chance.

5 Do you usually win at games? Would you describe yourself as “lucky”.

Let's think about weather patterns

Bognor Regis prides itself on being the sunniest place in Britain – although this is disputed by various other coastal towns. But that doesn't mean it doesn't ever rain.
For example, this simple Venn diagram gives an overview of the weather in April 2018.
S represents days that was a significant about of sunshine and R represents days when there was a significant amount of rain.







Fitness check

1. How many days are there in April?
2. What is the probability that a randomly chosen day is
   a) Sunny all day?
   b) Rainy all day?
   c) Cloudy all day?
   d) Has large periods of sunshine and rain?

Building a probability model

Studying probability helps us to makes sense of random phenomenon. For example, throwing two dice and recording their sum appears to be a random event. But, by looking at a sample space, we have discovered that sum totals are more likely to arise than others. So perhaps the events are not quite as random as we first thought.

Weather forecasters make use of probability models to make their forecasts.

We are going to build a very simple probability model that will simulate the weather in Bognor Regis in April – using a specially designed spinner.

6 Think about how could we build a spinner that would model the weather patterns in Bognor Regis in April?



Alice: Well there are 30 days, so perhaps we should have 30 sectors?



Taro: I think it could be done in a simpler way if you think about pie charts.

Probability 4



Taro

Taro's spinner

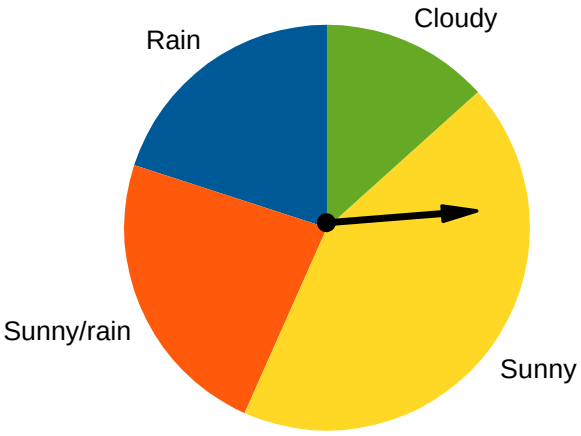
My calculations

Sunny $\frac{13}{30} \times 360 = 156^{\circ}$

Sunny/rain $\frac{7}{30} \times 360 = 84^{\circ}$

Rain $\frac{6}{30} \times 360 = 72^{\circ}$

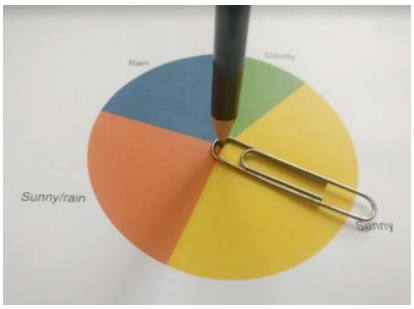
Cloudy $\frac{4}{30} \times 360 = 48^{\circ}$





Miss F

You are now going to test Taro's spinner by making your own spinner from a template.
Spin your spinner 30 times and record your results.





Taro

Taro's results

Outcomes	Tally marks	Totals
Sunny	𝍸 𝍸 ///	13
Sunny/rain	////	4
Rain	𝍸 ////	9
Cloudy	////	4
	Total	30

The law of large numbers

In probability games and experiments, the law of large numbers (LLN) is a theorem that describes the result of performing the same experiment a large number of times.

According to the law, the the cumulative results will get closer to the theoretical results the more trials you have.

So although Taro's results can give us a rough idea of whether the model gives a reasonable prediction of what the weather might be like next April, we actually test the model by generating lots more results.



So to test the model, we need a lot more results. In that case, let's collect up everyone's results. We can then test the accuracy of our model.

Miss F

All of our results for April

Outcomes	Totals
Sunny	183
Sunny/rain	95
Rain	85
Cloudy	57
Total	420

Mean results for April

Outcomes	Totals
Sunny	13.1
Sunny/rain	6.8
Rain	6.1
Cloudy	4.1
Total	30.1

Rounded to whole days

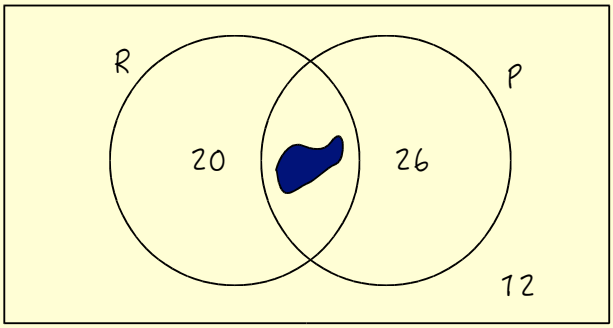
Outcomes	Totals
Sunny	13
Sunny/rain	7
Rain	6
Cloudy	4
Total	30

7 Are you surprised how accurate the results turned out? It looks like the law of large numbers works too!

Free papers and the library

The local library stocks two free local newspapers, the Recorder and the Post, in their reading room . They are delivered each Thursday at 9am just as the library opens. The librarian, who had a keen interest in statistics, was asked to keep a record of how many people came into the reading room and whether they took away a paper with them. In her report she included a Venn diagram to make things clear.

These are the my results for January. On average 72 people came into the reading room each Thursday and many of them took away the free papers. My average weekly analysis is shown in the Venn diagram below.



R = people who took the Recorder

P = people who took the Post

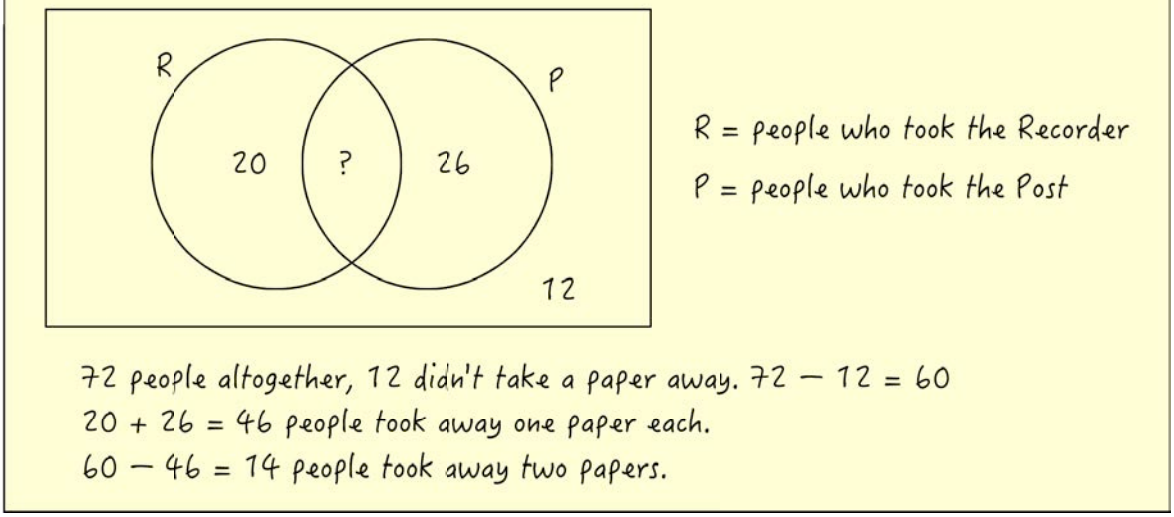
8 On average, how many people took both free papers from the library?

Free papers and the library



Evie

Evie's solution



Now think about designing a spinner to model the number of papers taken by each person who entered the reading room.

1. How many sectors will be needed?
2. What size is each sector?
3. Construct your spinner.
4. Now spin it 20 times and record your results.
5. Interpret your results.

Fitness Check

An MOT station tested 60 vehicles on a particular day. 30 vehicles passed. Of the vehicles that failed, 23 vehicles Venn diagram.

What is the probability that a particular vehicle

1. Passed?
2. Failed on both brakes and lights?
3. Failed on brakes only?
4. Failed on lights only?

Now design a spinner to model the results.

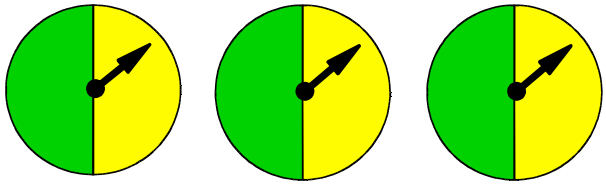
5. Spin your spinner 30 times and record your results.
6. Interpret your results.

Is this a fair game?



Taro

Jon and I have been playing a game with three spinners. I win if they are all point to the same colour, otherwise Jon wins.Unfortunately Jon seems to be winning most of the time.



9 Can you explain why this is not a fair game?



Alice

Alice's explanation

I have listed all of the possible results below

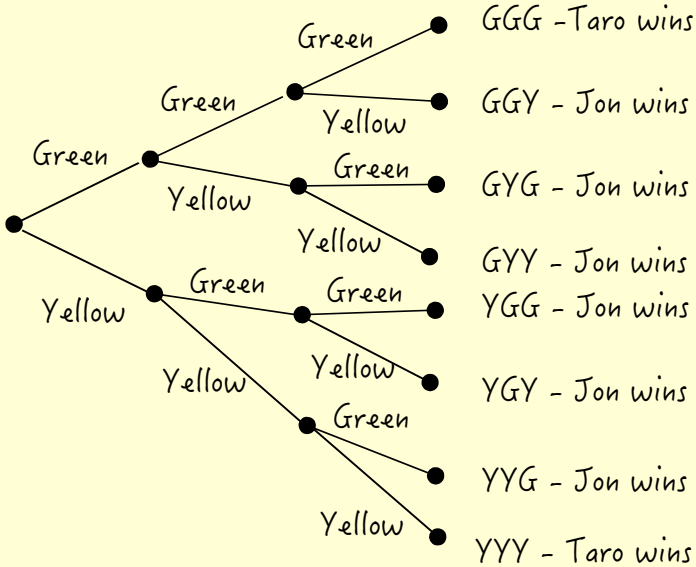
Green Green Green – Taro wins	Yellow Green Green – Jon wins
Green Green Yellow – Jon wins	Yellow Green Yellow – Jon wins
Green Yellow Green – Jon wins	Yellow Yellow Green – Jon wins
Green Yellow Yellow – Jon wins	Yellow Yellow Yellow – Taro wins

So Jon will win about 3 times as often as Taro.
It isn't a fair game!



Matthew

Matthew's diagram



Jon wins six times out of 8. So Jon will win in 75% of the games in the long run. It is not a fair game.

"Overs and Unders"

This game is very popular in Australia as a fund raising game at school fetes. It is played with 2 standard 6 sided dice,

In this game you "bet" one chip on whether the total of the two dice will be "Over 7", "Under 7" or "Equal to 7".

You receive 2 chips if you win on "Overs" or "Unders". But if "seven" comes up you get 5 chips in return.

Try to decide on a useful strategy for playing the game but, of course, the best way to find out is to play the game.




Sarah

I think that betting on 7 is probably the best idea.

I'm not so sure, 7 doesn't come up that often. I think I might bet on "Unders". Then I will win if I get 2, 3, 4, 5 or 6.


Matthew


Amber

In that case, I think I might bet on "Overs". Then I will win if I get 8, 9, 10, 11 or 12.

In that case, one of you will win each time but two of you will lose, so the bank will usually win.


Taro

10

Play the game 36 times and record your results but before you do, write down what you expect to happen. This is your gut feeling.

Sarah, Matthew and Amber's results

Score	Tally	Frequency	Score	Tally	Frequency
2	///	3	8	///	3
3	/	1	9	~~////~~ /	6
4	////	4	10	////	4
5	///	3	11	//	2
6	~~////~~	5	12		0
7	~~////~~	5			

"Overs and Unders"

11 Look at the results for Sarah, Matthew and Amber on the previous page. How much did they each win or lose?




Sarah

I won 5 times. So altogether I won 25 chips but spent 36 chips, so I lost 11 chips altogether.


Matthew

I won 16 times, so I won 32 chips. But it cost me 36 chips to play, so I lost 4 chips altogether.


Amber

I won 15 times, so that 30 chips. So altogether I lost 6 chips.

I acted as the banker. Altogether I won 21 chips. Thank you for playing.


Taro

Making decisions

In most of the games included in this unit, you have to make decisions on your strategy. This is mirrored in real life. Unfortunately most people rely on their **"gut feeling**" and seldom think things out logically. This is also the reason carnival games are seldom fair and the "odds" of winning are stacked against you.

In probability based situations you effectively have three choices. You can rely on your **gut feeling**, carry out an experiment to determine the **experimental probability** or construct a sample space diagram to find the **theoretical probability**.

Relative frequency

When an experiment can be repeated, then the **experimental probability** of an event happening is the proportion of times the event happened in the long run.

The **relative frequency** of an event happening is equal to the number of times the event happens ÷ the total number of trials.

In the example above the experimental probability of Sarah winning was $\frac{5}{36}$

The experimental frequency of Matthew winning was $\frac{16}{36}$

And the experimental probability of Amber winning was $\frac{15}{36}$

Of course, what influenced Sarah's choice was the hope of winning more.

The law of large numbers

The **law of large numbers** says that in the long run the experimental probability or relative frequency will get closer and closer to the **theoretical probability**.

In reality this means the more times you play, the more confident you can be in your results.

12 As a class, combine all of your results and work out the experimental probability of each player winning.

8Z collected in their results from 10 groups.
The results are shown below.

8Z The results

Score	Frequency	Score	Frequency
2	6	8	48
3	26	9	35
4	36	10	26
5	43	11	19
6	48	12	11
7	62	Total goes	360

Player	Wins
Sarah	62
Matthew	159
Amber	139
Totals	360

Based on these results what is the experimental probability of each player winning?

Based on these results, how much did each player win or lose?

Based on these results, how much did the bank win or lose?

Theoretical probability

To find the theoretical probability of an event happening we need to draw a sample space diagram. This is a diagram that systematically lists all of the possible outcomes for an experiment.

13 Can you draw a suitable sample space diagram for this game?



Amber

Amber's idea

I am going to use a two way table to record the results

Second dice →

First dice ↓

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

14 Using this sample space diagram, what is the theoretical probability of each player winning?



Alice

Alice's working

Sarah would win 6 times, so the theoretical probability of Sarah winning is $\frac{6}{36}$

Matthew would win 15 times, so the theoretical probability of Matthew winning is $\frac{15}{36}$

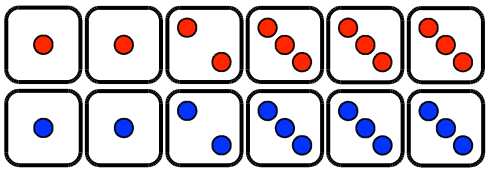
Amber would win 15 times too, so the theoretical probability of Amber winning is $\frac{15}{36}$

15 In theory, how much would each player win or lose in 36 games? How much would the bank win or lose in that time?

16 How did the experimental probability compare with the theoretical probability? Does this support the law of large numbers?

It's your choice

This game, called “It's your choice, is played with two special dice that have the numbers shown on their faces. Your score is the sum of the two dice.



17 At first glance what do you think will be the most common score? This is known as you “gut feeling”



Alice

I think 6 will be the most common score

I'm not so sure. We should play the game and see



Jon

18 Play the game 36 times and record your results but before you do, write down what you expect to happen. This is your gut feeling.

Our initial thoughts or “gut feeling”

	2	3	4	5	6	Total
Alice	4	4	11	3	14	36
Jon	4	1	12	7	12	36
Evie	3	3	9	6	15	36
Taro	5	3	16	4	8	36

Our actual results

Score	Tally	Total
2	///	3
3	////	4
4	~~////~~ ~~////~~ ///	13
5	~~////~~ //	7
6	~~////~~ ////	9
	Total	36

So I was wrong!



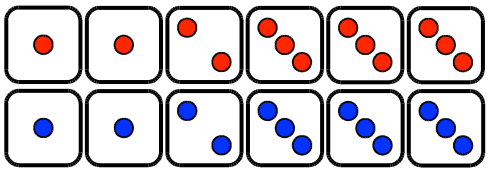
Alice

It's your choice



Matthew

So our "**gut reaction**" wasn't very accurate.





Alice

But if we played the game another 36 times, we would probably get a different result. So **experimental probability** is only a guide.

We could probably get a better guide by looking at the sample space.



Jon

19 Construct a sample space for this experiment.



Amber

Amber's idea

I am going to use a two way table to record the results

Second dice →

First dice ↓

+	1	1	2	3	3	3
1	2	2	3	4	4	4
1	2	2	3	4	4	4
2	3	3	4	5	5	5
3	4	4	5	6	6	6
3	4	4	5	6	6	6
3	4	4	5	6	6	6

There are 36 outcomes. 2 comes up 4 times, 3 comes up 4 times, 4 comes up 13 times, 5 comes up 6 times and 6 comes up 9 times.

20 How did your results compare with the sample space?.

It's your choice

Our **theoretical probability** can be worked out by summarising our sample space.

For example, the probability of scoring 3 is $\frac{4}{36}$

And so P(X = 3) = $\frac{4}{36}$

Score	Frequency
2	4
3	4
4	13
5	6
6	9
Total	36

Fitness check

What is …

1. P(X = 2) ?
2. P(X = 4) ?
3. P(X = 6) ?
4. P(X < 5) ?
5. P(X ≥ 3) ?
6. P(X ≠ 4) ?
7. P(2 < X < 6) ?
8. P(4 ≤ X ≤ 6) ?
9. P(1 < X ≤ 6) ?

Making decisions

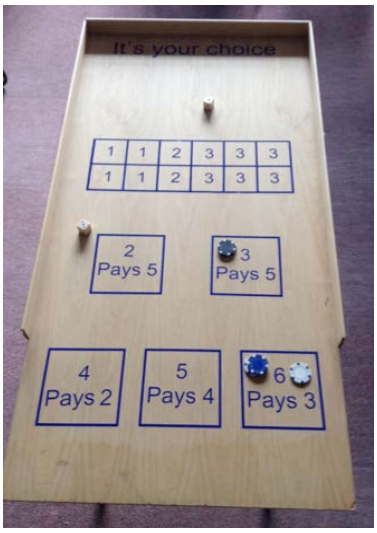
In most of the games included in this unit, you have to make decisions on your strategy. This is mirrored in real life. Unfortunately most people rely on their **"gut reaction**" and seldom think things out logically. This is also the reason carnival games are seldom fair and the "odds" of winning are stacked against you.

In probability based situations you effectively have three choices. You can rely on your **gut instinct**, carry out an experiment to determine the **experimental probability** or construct a sample space diagram to find the **theoretical probability**.

Clearly in the game "It's your choice" at first glance 6 seems the most likely outcome but both the experiment and the sample space show that 4 is the most likely outcome.

Having now worked out the theoretical probabilities you would expect 4 to turn up the most in the game. It wouldn't be much of a game if that was all you had to consider.

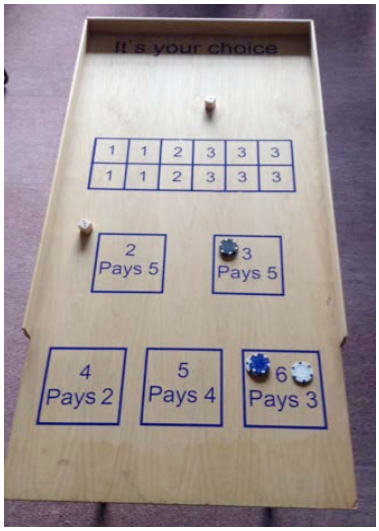
The game is made more attractive and intriguing by giving prizes which vary on the number you choose. Look carefully at the "It's your choice" board.



It's your choice

Is the game fair?

1. Look carefully at the prize structure in this game.
2. Which number pays out the least?
3. Can you explain why?
4. Which numbers pay out the most?
5. Can you explain why?
6. Do you think the “bank” will win in this game?
7. Which number would you choose to “bet” on?
8. Play the game and see how much you win or lose.



8Z decided to try it out for themselves



I will stick with “2”

Jon



I will bet on “3”

Evie

I'll go with “4”



Alice



I'll go with “5”

Sarah



It's “6” for me.

Matthew

And I'll be the bank!



Taro



Who do you think will win?

Now play the game several times and see who wins.

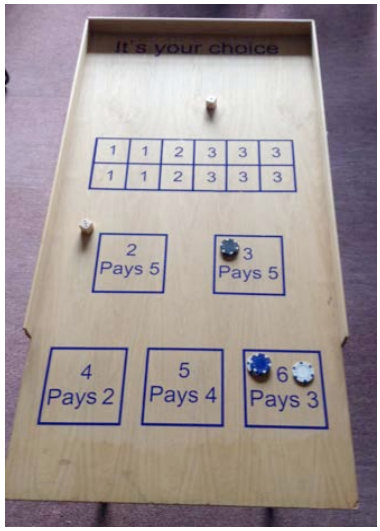
Miss F

It's your choice

The results

8Z played the game 36 times and here are their results.

Score	Tally	Total	Winnings
2 (Jon)	𝍸	5	25
3 (Evie)	////	4	20
4 (Alice)	𝍸 𝍸 𝍸	15	30
5 (Sarah)	//	2	8
6 (Matthew)	𝍸 𝍸	10	30
	Total	36	113



1. Based on these results what is the experimental probability of each player winning?
2. Based on these results, how much did each player win or lose?
3. Based on these results, how much did the bank win or lose?



I lost 11 chips

Jon



I lost 16 chips

Evie

I lost 6 chips



Alice



I lost 28 chips

Sarah



I lost 6 chips

Matthew

And I won 67 chips.
Thank you for playing



Taro

Wealth warning



In the long run it is generally only the banker who wins.

Gambling is for fools!

Let's think about who actually wins in "It's your choice".



Sarah

Suppose it is one chip a go, in theory how much will the "bank" win?

Perhaps we could use our sample space to help us to decide.



Evie

21 Can you use your sample space diagram to decide, in theory, how much the banker wins every 36 games.



Jon

Jon's working

Suppose I put 1 chip on each number every go. Then I would "win" every go. It would cost me 5 chips per go. So in 36 goes, it would cost me 180 chips. So how much would I win in theory.

Score	Frequency	Prize	Winnings
2	4	5	20
3	4	5	20
4	13	2	26
5	6	4	24
6	9	3	27
Total	36		117

So I would actually lose 180 – 117 = 63 chips.
So the bank would win 63 chips that doesn't seem fair!
The bank is making 35% profit.
I think I'll be the banker in future.

Wealth warning



Gambling is a mug's game.

In the end you lose, the bank always wins!

Probability 4 Mastery

1. If the probability that it will rain tomorrow is 0.4, what is the probability that it will not rain tomorrow?

A	B	C	D
0.2	0.4	0.6	Not enough information

2. There are two outcomes to an experiment, X, and Y.

   If P(X) = 3P(Y).

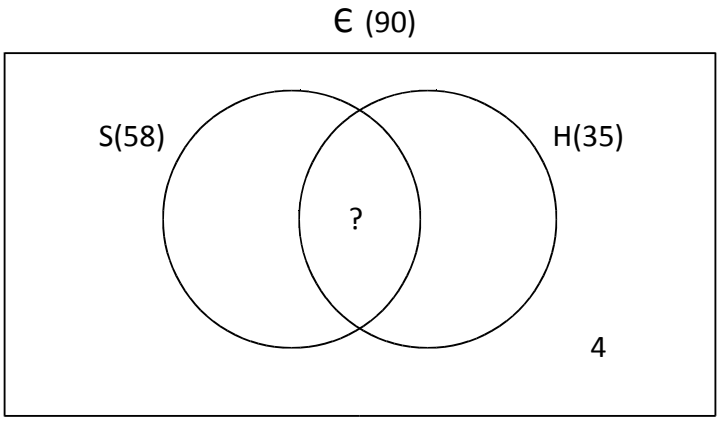
   What is P(Y)?

A	B	C	D
0.2	0.25	0.4	0.75

3. Given that P(A) = 0.35, What is the P(A')

A	B	C	D
0.35	0.45	0.55	0.65

4. 90 PE students were asked if they belonged to the Soccer club (S), the Hockey club (H) or both. The results are shown in the Venn diagram below.

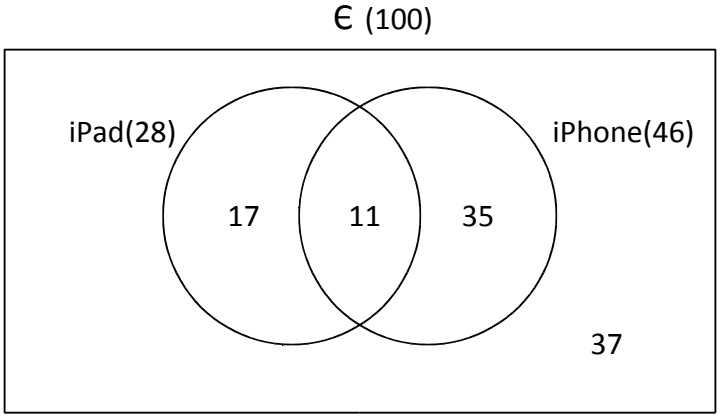


   How many students belonged to both clubs?

A	B	C	D
7	23	86	93

100 students were asked if owned an iPad or iPhone. Their responses are shown on the Venn diagram below.

Q5 – 8 all refer to this diagram.



5. What is the probability that a student chosen at random had both an iPad and an iPhone?

A	B	C	D
$\frac{11}{28}$	$\frac{11}{46}$	$\frac{11}{63}$	$\frac{11}{100}$

6. What is the probability that a student chosen at random had an iPad but not an iPhone?

A	B	C	D
$\frac{17}{28}$	$\frac{17}{63}$	$\frac{52}{100}$	$\frac{17}{100}$

7. What is the probability that a student chosen at random did not have an iPhone?

A	B	C	D
$\frac{54}{100}$	$\frac{28}{63}$	$\frac{28}{100}$	$\frac{37}{100}$

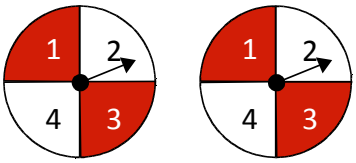
7. What is the probability that a student chosen at random had either an iPhone or an iPad but not both?

A	B	C	D
$\frac{52}{100}$	$\frac{52}{63}$	$\frac{63}{100}$	$\frac{37}{100}$

9. You toss two coins. What is the probability of getting one tail and one head in any order?

A	B	C	D
$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{1}{2}$

In a game, two identical spinners are spun at the same time as shown below and the two numbers added together to give you your total.



Questions 10 – 13 refer to this game.

10. When the pairs of numbers are listed systematically, how many different outcomes are possible?

A	B	C	D
4	7	8	16

11. When the scores are added together, how many different totals are possible?

A	B	C	D
4	7	8	16

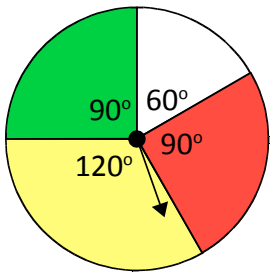
12. When the scores are added together what is the probability that you score 5 in total?

A	B	C	D
$\frac{1}{7}$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{1}{8}$

13. What is the probability that your final score is more than 5 in total?

A	B	C	D
$\frac{3}{7}$	$\frac{3}{8}$	$\frac{10}{16}$	$\frac{6}{16}$

14. A spinner is made up of four unequal sectors as shown.

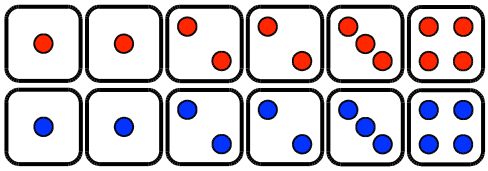


If the spinner is spun 600 times, approximately how many times would you expect the spinner to land on yellow?

A	B	C	D
120	150	180	200

In question 15 and 16, you must show all of your working.

15. A game is played using two identical six sided dice that have numbers shown on their faces as shown. Both dice are thrown together and your "score" (X) is the sum of numbers shown.

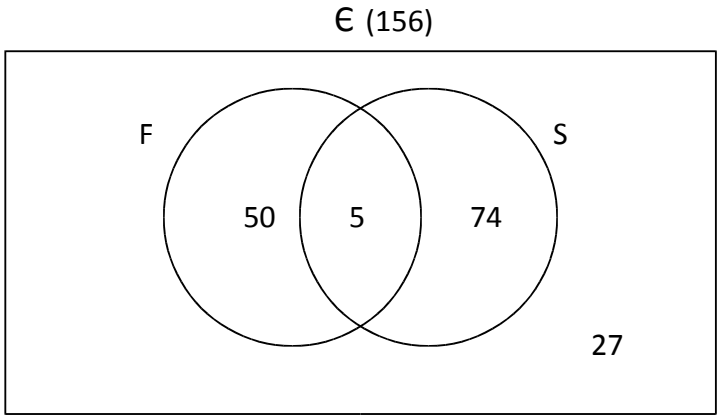


By first drawing a sample space of the game, calculate

a) P(X=5)

b) $P(2 \leq X \leq 6)$

16. The Venn diagram below shows the number of Year 9 pupils which selected to study French (F) or Spanish (S) at GCSE.

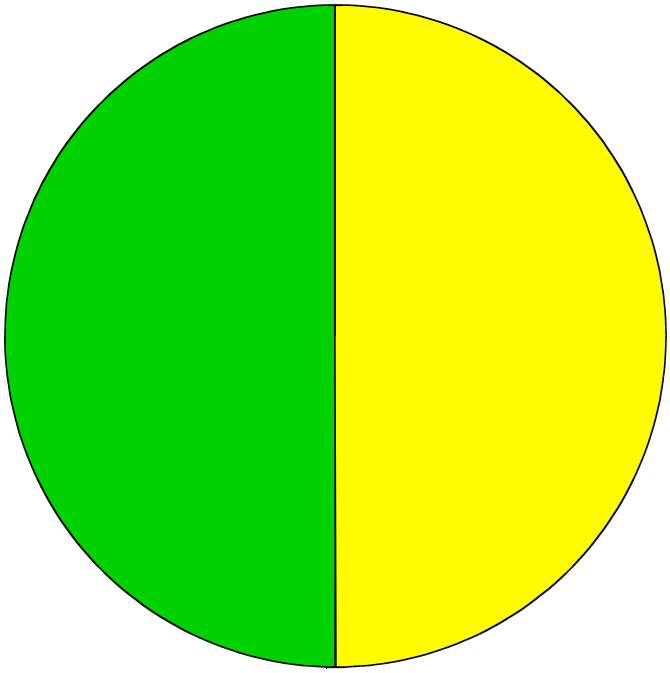


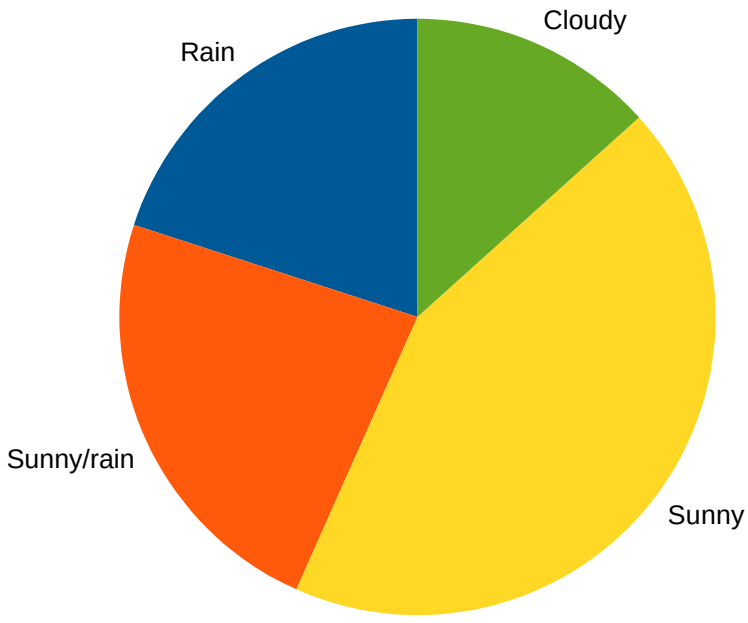
What is the probability that a pupil selected at random is

a) Not studying Spanish?

b) Not studying either Spanish or French?

c) Only studies one language?




Cloudy
Rain
Sunny/rain
Sunny