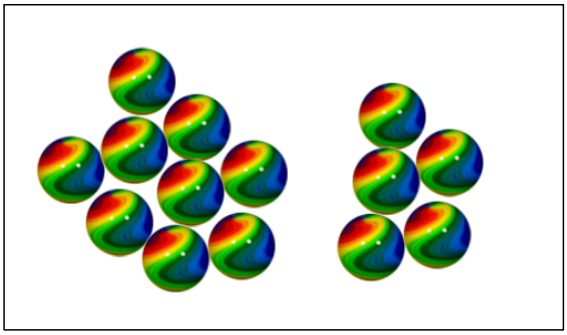
What does it mean to “even things out”?

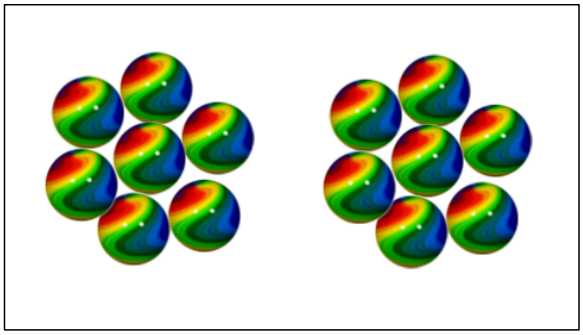
















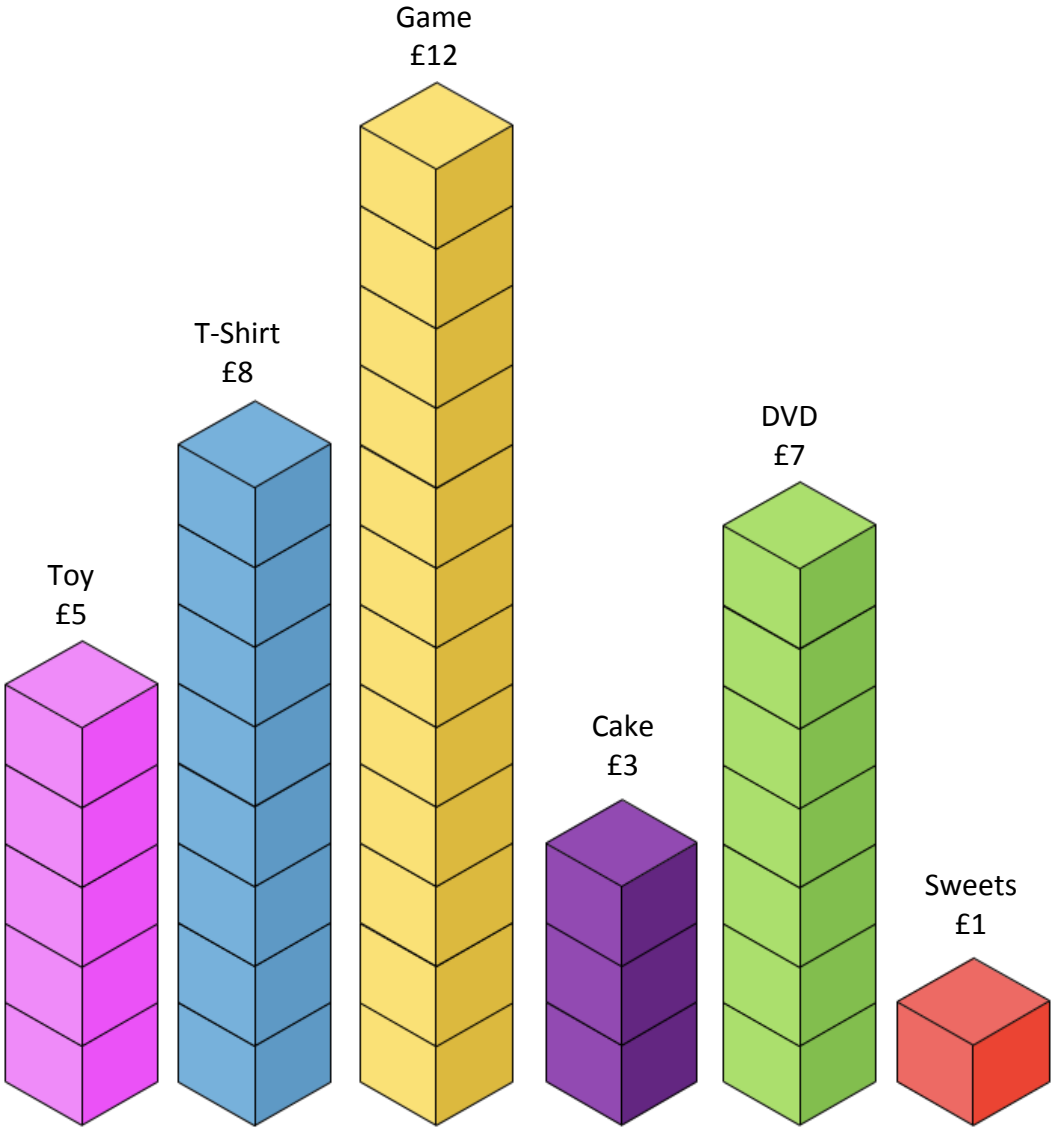
When we make unequal groups equal, we say we are “evening things out” or “averaging them out”.

What does it mean to “even things out”?



Alice

This is how much I spent on Christmas presents last year:



1 On average, how much did Alice spend on each present?



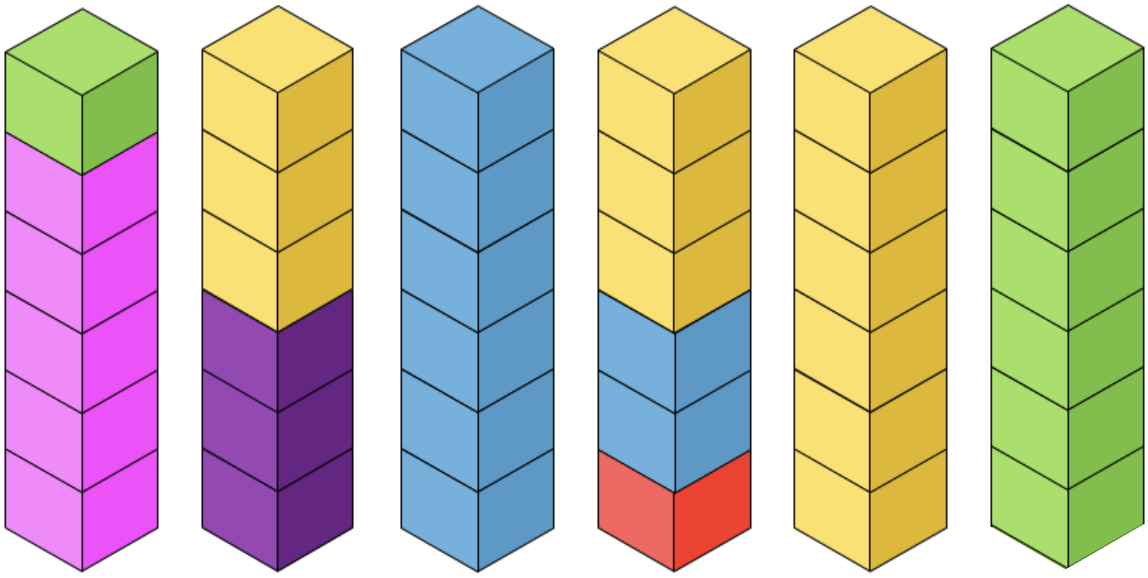
Evie

We can rearrange the cubes to make the stacks even.

There has to be lots of ways to make even stacks.



Jon

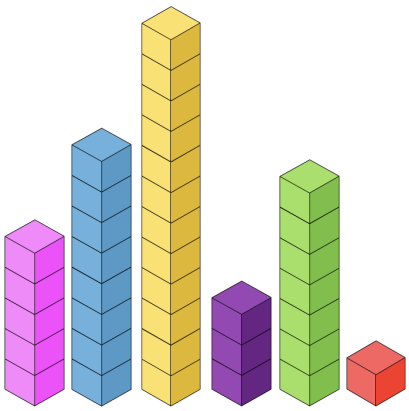


When the stacks are equal, the height is called the **mean** height of the bars.

1. What would happen to the mean if we added 6 to each original bar?

2. What if we multiplied each original bar by 10?

Item	**Price**	**+6**	**x10**
Toy	£5	£11	£50
T-Shirt	£8	£14	£80
Game	£12	£18	£120
Cake	£3	£9	£30
DVD	£7	£13	£70
Sweets	£1	£7	£10



3. How do these ideas help you quickly to find the mean of:
   a) £45, £48, £52, £43, £47 and £41.
   b) £500, £800, £1200, £300, £700 and £100.

4. Suppose Alice bought one extra present. How much did it cost if
   c) the mean is unchanged?
   d) the mean goes up by £1?
   e) the mean goes up by £2

Evening things out

This idea of “evening things out” is a very useful in Statistics.

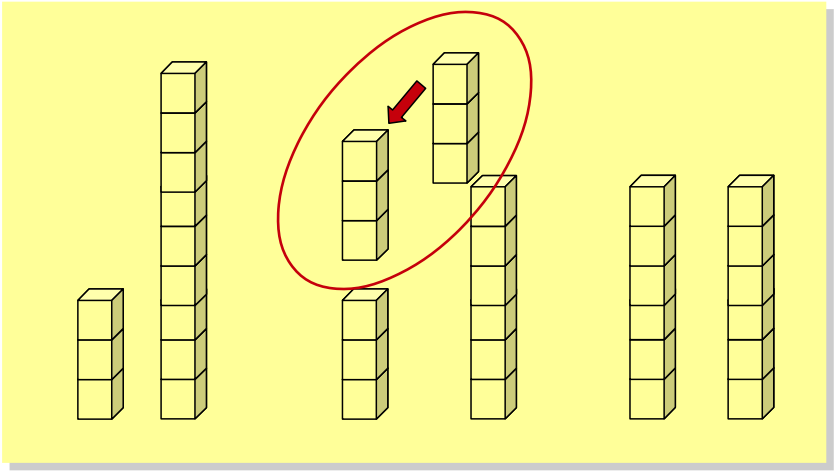
In each example below, explain how you can “even things out” by moving cubes between the stacks.

1. 2 Stacks - 3 high and 9 high.
2. 2 Stacks - 14 high and 18 high.
3. 3 stacks - 6 high, 7 high and 11 high
4. 4 stacks – 3 high, 9 high, 14 high and 18 high.

Sarah's working on Q1



Sarah





In Food Technology, the pupils have been making pure orange juice by squeezing oranges. Each pupil was given six oranges, and their teacher Mrs MacDonald asked them, “On average, how much juice do you get from an orange?”



Matthew

Matthew's results

My results were 76ml, 79ml, 84ml, 71ml, 75ml and 77ml.
The least amount is 71ml, and the most is 84ml, so the mean must be between 71ml and 84ml.
I've now got to even things out.
How can I do that?

2 Can you work out how to “even things out”?

Calculating the mean

We can calculate the "evened out" or mean amount by calculating the total amount and sharing it equally.


Sarah

Sarah's work

The total amount of juice is
76 + 79 + 84 + 71 + 75 + 77 = 462
462 ÷ 6 = 77
The mean amount is 77ml.

Fitness check



1. Amber's results were 72ml, 88ml, 84ml, 78ml, 90ml and 80ml. Calculate the mean amount of orange juice per orange for Amber's results.
2. Suppose the mean of Jon's results is 81ml of orange juice. If 4 of his results are 78ml, 82ml, 85ml and 74ml. What could the other results be?
3. The mean of the first 4 of Alice's results is 76ml. The mean of the last two results is 81ml. What is the mean of her 6 results?
4. Amber brought in a box of 6 eggs from a local supermarket. On the box, it states that the average weight of the eggs is 55g. What should the total weight of the eggs be if the claim is correct?

Summarising data

So far, we have been looking at how to summarise data by finding the mean. But this isn't the only measure used to summarise data.
In this section, we will be looking at two other statistics used to summarise given data. These are called the median and the mode.

Earlier, Mrs MacDonald asked her pupils the question, "On average, how much juice do you get from an orange?"
We can now begin to answer her question by putting some of the results together.

The results so far:

Matthew's results: 76, 79, 84, 71, 75, 77
Amber's results: 72, 88, 84, 78, 90, 80
Jon's results:78, 82, 85, 74, 80, 87
Alice's results: 78, 72, 79, 75, 78, 82

3 Can you sort out this data into a sensible form for us to work on?

Sorting the results

Alice's work



Alice

I used an unsorted stem and leaf diagram first

7	1 2 4 2	(4)
7	6 9 5 7 8 8 9 8 5 8	(10)
8	4 4 0 2 0 2	(6)
8	8 5 7	(3)
9	0	(1)
	Count	(24)

7	1 2 2 4	(4)
7	5 5 6 7 8 8 8 8 9 9	(10)
8	0 0 2 2 4 4	(6)
8	5 7 8	(3)
9	0	(1)
		(24)

Then I sorted it into order by rows.

Summarising the data – the mode

The **mode** is simply the most common answer to the question, "How much orange juice is there in an orange?" In this case, the answer is 78 and so the mode is 78ml.

Summarising the data – the median

The **median** is the middle point of our results if they are first sorted into size order.

4 Where is the "middle point" in our results?

Jon's idea



Jon

There are 24 results, so we have to split them into 2 groups of 12.

71, 72, 72, 74, 75, 75, 76, 77, 78, 78, 78, 78,

79, 79, 80, 80, 82, 82, 84, 84, 85, 87, 88, 90

So I think the middle point is between 78 and 79.

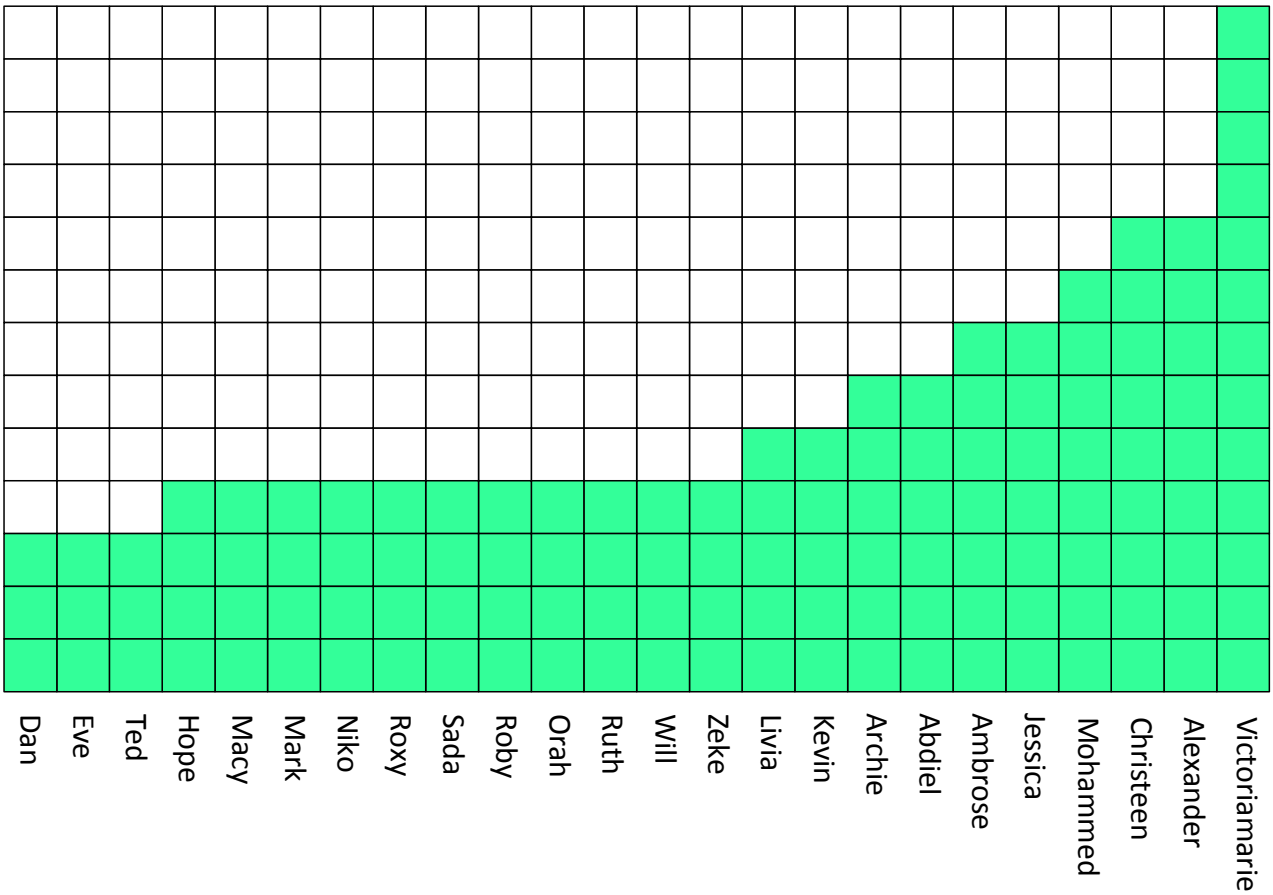
So I think the median is 78.5ml, but that isn't actually one of our results. Is that allowed?

Jon is absolutely right. He has split the distribution exactly in half.

12 results are below 78.5, and 12 results are above 78.5.

What's in a name?

The graph below shows the names of 24 students, arranged in order according to the length of their names. The bars are shaded to represent the number of letters in each name.



1. What is the mean length for these names?

2. Draw a horizontal line through the graph to show the mean.

3. The number that occurs most frequently is the **mode.**
   What is the mode?

4. The **median** is the middle point when the data is listed in order.
   What is the median of the data above?

5. Which of the mean, median and mode do you think represents a 'typical' name length for the students in the graph above?

5. Using the graph on the previous page:
   a) How many people have a name longer than the mean?
   b) How many people have a name shorter than the mean?
   c) How many people have a name longer than the mode?
   d) How many people have a name shorter than the mode?
   e) How many people have a name longer than the median?
   f) How many people have a name shorter than the median?

6. Which of the mean, median or mode will divide the graph so that the same number of people are above and below that value?

7. What factors may cause the mean to be off-centre?

8. What factors may cause the mode to be off-centre?

9. Can the mean, median and mode of a set of data all be on centre? Explain your answer.

10. If Victoriamarie shortens her name to Victoria, will any of the types of average change? Explain your answer.

11. If she goes by the name Vicki, which will change? Explain your answer.

12. Explain under what set of circumstances changing one element of the data will change:
    a) The mean
    b) The median
    c) The mode

5

Summarise the names of the pupils in your group.
How do the results compare?

Fitness check



1. Sarah collected the following data; 5, 6, 7, 8, 8, 9, 4, 9 and said the mean was 8. Do you agree? Explain your answer.
2. At a shoe shop, which statistic would be most useful to the manager when recording shoe sales; the mean, median or mode? Explain your answer.
3. The class mean average on a spelling test was 27.5 out of 40. The 19 girls in the class scored 532 points. How many total points did the 11 boys have?
4. When 11 students took a test, the mean average score was 77.1. Two more students took the test, the sum of their scores was 130. What is the new mean average?
5. The mean average height of a Year 8 class is 134.7cm. The sum of all the heights is 3771.6cm. There are 17 boys in the class. How many girls are there?
6. The mean average score on a maths test for 58 pupils was 87.3. Twelve more students took the test. The mean average of the 12 students was 90.7. What was the mean average for all the students?
7. The class scores for two tests are shown below. On which test did Norman do better relative to the rest of the class? Explain your thinking.

Pupil	Test 1	Test 2
Norman	85	89
Steve	72	93
Penny	89	96
Raj	75	65
Nicki	79	79
Denise	86	60

8. The tickets for the school fashion show went on sale for 10 days. The pupils selling the tickets told the headteacher that the mean number of tickets sold for the first 5 days was 24. The mean number sold for the next 3 days was 22, and the mean number sold for the last two days was 31.

   What was the mean number of tickets sold over the 10 days?

   Construct a dataset of your own for tickets sold each day over the 10 days. It should fit the information given by the pupils to the headteacher.

WIPED OUT

1. 1 2 3 4 5 6

One of these numbers is wiped out, and the mean of what is left is **3.6**

Which number was wiped out?

2. 1 2 3 4 5 6 7

One of these numbers is wiped out, and the mean of what is left is **4.0**

Which number was wiped out?

3. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

One of these numbers is wiped out, and the mean of what is left is **7.714285714**

Which number was wiped out?

4. 1 2 3 4 . . .

You have a set of consecutive numbers beginning at 1, but you don't know the last number.

One number is wiped out, and the mean of what is left is **6.833333333….**

Which number was wiped out and what was the last number?

5. 1 2 3 4 . . .

You have a set of consecutive numbers beginning at 1, but you don't know the last number.

One number is wiped out, and the mean of what is left is **25.76**

Which number was wiped out and what was the last number?

6. 1 2 3 4 . . .

You have a set of consecutive numbers beginning at 1, which ends in an even number.

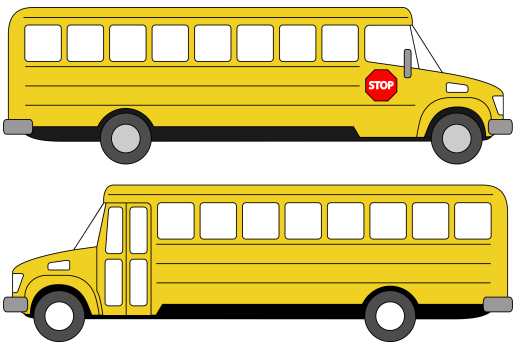
One number is wiped out, and the mean of what is left is **an integer (whole number).**

Which numbers could have been wiped out? Explain your thinking.

On the buses

A class recorded how long they were on the school bus in the afternoon.

1. Find the mean, median and mode for the data below.
2. If the bus breaks down before the last stop. Matthew, who is the last to get off, will spend 90 minutes onboard instead of 30. How will this change the averages?
3. If instead, Matthew remembered he had rugby practice after school and got off the bus after only one minute. What effect will this have on the averages?
4. In general, what effects do extreme results have on the "averages"?
5. Which of the three averages do you think best represents the data?



Name	Time
Ben	6
Zach	6
Amber	8
Sarah	9
Evie	9
Jon	9
Molly	10
Rhys	12
Georgie	12
George	12
Tarquin	!2
Mohamed	15
Peter	15
Edward	17
Scott	20
Josephine	22
Jane	25
Chris	26
Rachel	27
Matthew	30

6

Find the average time for your group to get to school in the morning. Do the same for going home. Do you expect the results to be the same?

Who would you choose for your team?



Long before ten pin bowling became popular, there was skittles on the green.
To capture the excitement of forgotten games, the school decided to have a skittles competition.



There are four players in a team, and your four best friends all want to be in your team, but with you as captain that makes five.
You decide the only way to be fair is to hold an elimination competition. Each player has 5 goes, and their scores are shown below:

The scoreboard

	Round 1	Round 2	Round 3	Round 4	Round 5
Alex	7	8	4	5	6
Chloe	9	8	1	5	7
Brad	4	4	6	7	9
Jess	6	6	6	6	6

7 Who would you choose for the team? Explain your thinking.


Amber

We could find their “average” score.

But which average would you use?


Jon


Sarah

Well, the mean's no help because ...

Maybe we should go for the most consistent?


Alice

Evie's thoughts



Evie

	Round 1	Round 2	Round 3	Round 4	Round 5
Alex	7	8	4	5	6
Chloe	9	8	1	5	7
Brad	4	4	6	7	9
Jess	6	6	6	6	6

	Total	Mean	Median	Mode	Spread
Alex	30	6	6	n/a	4
Chloe	30	6	7	n/a	8
Brad	30	6	6	4	5
Jess	30	6	6	6	0

Their totals are all the same, so I don't think any of the averages are much help.

I then looked at how consistent they were by finding the spread.

To calculate the spread, I took the lowest score from the highest score.

In which case Chloe is the most inconsistent, so I think we should leave her out of the team.

Measuring spread

The spread of results is another way of thinking about data. It is not an "average" or "typical value" but it does tell you a lot about how consistent your data is. This may or not be important. It depends on the circumstances.

There are various measures of spread, but we will only be thinking about the **range,** which is what Evie was measuring.

The **range** is the difference between your highest result and the lowest result. It is always a single positive number, but it may not be a whole number.

Earlier, we looked at the scores in two tests. They were

Test 1 85, 72, 89, 75, 79, 88

Test 2 89, 93, 96, 65, 79, 60

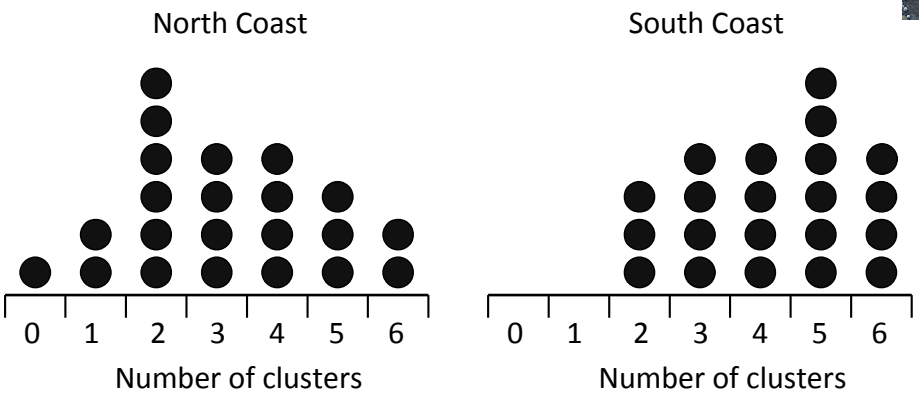
8 What was the range for each test? Does that tell you anything about the tests? Explain your thinking.

Fitness check

1. These are the ages of the male and female members of a local tennis club.
   Summarise the data and explain the similarities and differences between the two groups.
   Male members: 32, 44, 48, 52, 56, 62, 63, 64, 68, 70, 78
   Female members: 45, 47, 49, 52, 58, 64, 64, 65, 71, 72

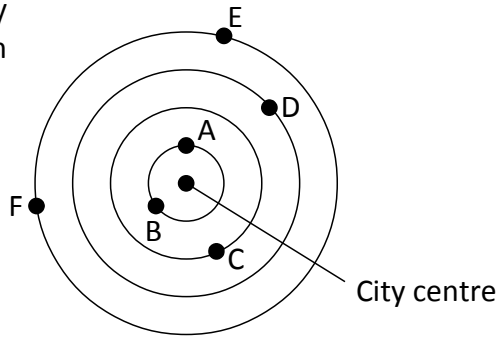


2. These two dot plots show the number of clusters of limpets found in rock pools on Cornwal's North and South Coasts..



Summarise the data and explain the similarities and differences between the two groups.

3. This diagram shows the distance of 6 hotels from the city centre. The circles show distances of 50 m, 100 m, 150 m and 200m.
   Write the letters of four possible hotels whose distance from the town centre
   a) Have a mean of 125 m and a range of 150 m.
   b) Have a mean of 100m and a range of 150 m.



4. In Evie's IT project, she had to record the hours she spent on social networking, listening to music or playing games. These are her results:.

Technology	Mon	Tues	Wed	Thur	Fri	Sat	Sun
Social networking	4	7	5	5	2	3	3
Listening to music	1	2	1	1	1	2	3
Gaming	1	0	1	0	3	4	2

Summarise the data and explain the similarities and differences between the three groups.

An Average Challenge

1. Find four numbers with the mean equal to 5
2. Find six numbers with the mode equal to 2
3. Find five numbers with the median equal to 7
4. Find five numbers with the mean equal to 3 and the mode equal to 4
5. Find six numbers with the mean equal to 5 and the range equal to 4
6. Find five numbers with the median equal to 5 and the mode equal to 2
7. Find five numbers with the mean equal to 7 and the median equal to 5
8. Find four numbers with the median equal to 9
9. Find six numbers with the range equal to 8 and the median equal to 10
10. Find four numbers with the mean equal to 4, the median equal to 3.5 and the range equal to 5.
11. Find six numbers with the mode equal to 3, the median equal to 3.5 and the range equal to 7
12. Find four numbers with the mean equal to eight and the median equal to six.



Miss F

In each case, "Can you find another set of numbers?" and "What about another set?".

Extend this to "How many sets are there in each case?"

13. Find the mean of the following set of numbers 2, 5, 4, 7, 8, 2, 4, 3, 1, 0
    a) If you added one more number to this list, what could the number be so that the mean stays the same?
    b) What if you added two more numbers to the list? What numbers could they be so that the mean stays the same?
    c) What about three or more numbers? Can you state under what conditions you can add n numbers such that the mean of the entire set of numbers doesn't change?
14. Find a set of numbers that has a mean of 3.6 and
    a) includes only positive integers
    b) includes only integers
    c) includes only improper fractions
    d) includes only fractions whose denominators are 3 and 4

Statistics 7b: Mastery questions

1. What is the mean of these 5 numbers: 9, 16, 15, 7, 13?

A	13	B	15	C	60	D	12

2. The mean age of 4 players in a 5-a-side team is 17. If the 5th player is 22, what is the mean age of the whole team?

A	17	B	18	C	19	D	20

3. What is the median of these 5 numbers: 9, 16, 15, 7, 13?

A	13	B	15	C	60	D	12

4. What is the median of this dataset: 7, 8, 5, 5, 9, 6, 8, 8, 7?

A	9	B	7	C	8	D	5

5. What is the mode of this dataset: 7, 8, 5, 5, 9, 6, 8, 8, 7?

A	9	B	7	C	8	D	5

6. What is the range of this dataset 7, 8, 5, 5, 9, 6, 8, 8, 7?

A	4	B	0	C	9	D	5

7. Which of these datasets has a mode of 6?

A	4, 3, 6, 7, 3	B	3, 4, 3, 6, 6
C	7, 6, 6, 6, 4	D	3, 4, 6, 9, 9

8. This table shows the summary statistics of 7X and 7Y in a recent science assessment. Which group were the more consistent in their scores?

	Mean	Range
7X	55	45
7Y	52	30

A	7X because their range was greater.	B	7X because their mean was higher.
C	7Y because their range was lower.	D	7Y because their mean was lower.

9. Jess choose 5 different positive integers and said that their mean was 8. What was the highest number Jess could have chosen?

A	40	B	30	C	36	D	28

10. Jamie's overall score in Science was 34. This was the mean of his scores in his Physics, Chemistry and Biology. If he scored 35 in Physics, 32 in Chemistry, what was his Biology score?

A	34	B	35	C	36	D	37

11. The table below shows the number of tries scored by the 12 teams in Premiership Rugby matches over a particular weekend. What was the mode for the number of tries scored?

Tries scored	Teams
0	1
1	2
2	2
3	4
4	3

A	1
B	2
C	3
D	4

12. Using the same dataset as Q11, what was the median for the number of tries scored?

A	1	B	2	C	3	D	4

13. Using the same dataset as Q11, what was the mean for the number of tries scored?

A	1.5	B	2	C	2.5	D	3

14. This stem and leaf plot gives the scores of 7Z in a Geography quiz. What was the modal score?

0 |	4 4	(2)
0 |	5 5 6 6 7 7 8	(7)
1 |	0 1 2 2 2 3 4 4	(8)
1 |	5 5 6 6 7 7 8	(7)

Count (24)

Key 1 \| 5 = 15

A	12
B	2
C	8
D	18

15. Using the same dataset as in Q14, what was the median score?

A	12	B	2	C	8	D	11

16. Using the same dataset as in Q14, what was the mean score?

A	12	B	2	C	8	D	11